VOL. 34, NO. 8 JUNE 1967

Public Roads

A JOURNAL OF HIGHWAY RESEARCH

PUBLISHED
BIMONTHLY
BY THE BUREAU
OF PUBLIC ROADS,
FEDERAL HIGHWAY
ADMINISTRATION,
U.S. DEPARTMENT
OF TRANSPORTATION,
WASHINGTON

Thomas Creek Bridge on the Oregon Coast Highway, U.S. 101, Curry County in Southern Oregon.

IN THIS ISSUE

Aerial Analytic Triangulation Investigation, Wyoming Interstate 80, *by J. R. Chaves*..... 151

Linear Shrinkage Test, *by G. W. Ring III*..... 160

Design Use of the Capacity Manual, *by D. W. Loutzenheiser, W. P. Walker, and D. B. Lewis*. 166

New Publications.......................... 169

Errata.................................. 165

U.S. DEPARTMENT OF TRANSPORTATION
ALAN S. BOYD, Secretary

FEDERAL HIGHWAY ADMINISTRATION
LOWELL K. BRIDWELL, Administrator

BUREAU OF PUBLIC ROADS
F. C. TURNER, Director

Public Road

A JOURNAL OF HIGHWAY RESEARC

Vol. 34, No. 8 *June 19*

Published Bimonthly

Muriel P. Worth, ***Editor***
M. Patricia Coccaro, ***Editorial Assista***

THE BUREAU OF PUBLIC ROA

WASHINGTON OFFICE
1717 H St. NW., Washington, D.C. 20235

FHA REGIONAL OFFICES

No. 1. 4 Normanskill Blvd., Delmar, N.Y. 120
Connecticut, Maine, Massachusetts, New Ha shire, New Jersey, New York, Rhode Isla Vermont, and Puerto Rico.

No. 2. 1610 Oak Hill Avenue, Hagerstown, M 21740.
Delaware, District of Columbia, Maryland, O Pennsylvania, Virginia, and West Virginia.

No. 3. 1720 Peachtree Rd. NW., Atlanta, Ga. 303
Alabama, Florida, Georgia, Mississippi, No Carolina, South Carolina, and Tennessee.

No. 4. 18209 Dixie Highway, Homewood, Ill. 60
Illinois, Indiana, Kentucky, Michigan, and W consin.

No. 5. Civic Center Station, Kansas City, 64106.
Iowa, Kansas, Minnesota, Missouri, Nebras North Dakota, and South Dakota.

No. 6. 819 Taylor St., Fort Worth, Tex. 76102.
Arkansas, Louisiana, Oklahoma, and Texas.

No. 7. 450 Golden Gate Avenue, Box 36096, Francisco, Calif. 94102.
Arizona, California, Hawaii, and Nevada.

No. 8. 412 Mohawk Bldg., 222 SW. Morrison Str Portland, Oreg. 97204.
Idaho, Montana, Oregon, and Washington.

No. 9. Denver Federal Center, Bldg. 40, Den Colo. 80225.
Colorado, New Mexico, Utah, and Wyoming.

No. 10. Post Office Box 1648, Juneau, Alaska 99
Alaska.

Eastern Federal Highway Projects Office—Region 15.
1000 N. Glebe Rd., Arlington, Va., 22201.

No. 19. Apartado Q, San Jose, Costa Rica.
Inter-American Highway: Costa Rica, Guatem Nicaragua, and Panama.

Public Roads, A Journal of Highway Research, is sold by the S intendent of Documents, Government Printing Office, Washin D.C. 20402, at $1.50 per year (50 cents additional for foreign mai or 25 cents per single copy. Subscriptions are available for 1-, 3-year periods. Free distribution is limited to public officials ally engaged in planning or constructing highways and to instru of highway engineering. There are no vacancies in the free li present.

Use of funds for printing this publication has been approved b Director of the Bureau of the Budget, March 16, 1966.

Contents of this publication may be reprinted. Mention of source is requested.

Aerial Analytic Triangulation Investigation, Wyoming Interstate 80

Y THE OFFICE OF
NGINEERING AND OPERATIONS
UREAU OF PUBLIC ROADS

Reported by [1,2] JESSE R. CHAVES,
Highway Engineer, Aerial Surveys Branch,
Highway Standards and Design Division

The potential for the use of analytic (mathematical) aerial triangulation to establish supplemental control points for large scale highway mapping can be developed, according to the results of the investigation reported in this article. At present, investigation of all facets of photogrammetry as aids for highway location, route selection, design, and construction is being urged. As more use is made of aerial photography, a more expeditous method of mapping from aerial photographs is being sought. The analytic aerial triangulation method, in which computers are used to establish supplemental control points and thereby curtail the number of ground survey points otherwise required, has been applied to small scale mapping of projects to a limited degree. But, little investigation has been made as to the accuracy that might be obtained if this method were to be applied to large scale mapping of projects.

Although aerial analytic triangulation is a potentially useful method, operational techniques and hardware should be refined, according to the author. He notes that present aerial cameras are neither designed nor calibrated for analytical photogrammetry. To take full advantage of the mathematical techniques, some slight design changes in aerial cameras and related equipment will be needed.

Introduction

NTEREST OF highway location and design engineers in the use of analytic (mathe- atical) aerial triangulation prompted the vestigation reported here on this supple- ental control used in photogrammetric map- ng. Conventional analog aerial triangulation th first order bridging instruments and ound control surveys heretofore have been ed to provide the ground position and eleva- n points—the supplemental control—need- l for the absolute orientation of stereoscopic odels in photogrammetric instruments for rge scale topographic mapping with small ntour intervals. Now electronic digital mputers have made it possible to use bridg- g for supplemental control for small scale pographic mapping. Although analytic idging was understood and used by a few gineers before 1960, only recently have ghway engineers become interested in its plication to highway engineering mapping. need existed, therefore, to evaluate the alytic approach for extending surveyed ound control to determine how accurate e computed control data would be for aerial apping by photogrammetric equipment and aterials presently available to highway ganizations.

The decision to investigate the use of alytic aerial triangulation for highway rveys was made because:

- Conventional ground control surveys are pensive, time consuming, and sometimes fficult to make. Analytic extension of ound control points would minimize the ed for so many ground control points and nit the number of positions and elevations at would have to be surveyed on the ground.
- Electronic digital computers are available r use of most highway organizations.
- The monocomparator required in the rial analytic triangulation procedure to easure the x and y photographic plate co- dinates can be bought for much less than e conventional analog, optical train bridg- g instruments. Also less training is required r the monocomparator operators to successfully measure the x and y coordinates of image points on glass plate transparencies of the aerial photographs than for the operation of the optical train bridging instruments.
- The aerial analytic triangulation method has the potential required to provide data for accurate photogrammetric mapping for highway location and design. Also, this method is potentially more flexible than the conventional analog types of bridging instruments.

Preliminary Evaluation

Results obtained in a preliminary evaluation of analytical photogrammetry encouraged the author to continue the investigation reported here. The mathematical procedures developed by the Coast and Geodetic Survey (*1*)[3] were used in the preliminary evaluation; they were modified slightly and programed in the Fortran language. The mathematical procedures (*1*) have since been replaced by a method called three-photo aerotriangulation (*2*). The preliminary evaluation (*3*) of the analytic method was made in 1964 on an 18,000-foot segment of Interstate Highway 66 in Fairfax County, Va., and reported by the author in his master's thesis, *Survey Control Extension by Analytic Aerotriangulation for Highways,* submitted to Syracuse University, September 1965. In this work, seven photographs, taken with a Wild 6-inch focal length aerial camera at a scale of 1:8,400, were analytically bridged. A monocomparator was used to measure x and y coordinates of natural images and targeted points on the photographic glass plate transparencies of the aerial photographs. Second-degree cantilever strip adjustments were made by using three horizontal and six vertical control points. The resultant root mean square errors (RMSE) on test points were 0.41 foot for the horizontal and 0.71 foot for the vertical ground coordinates.

Objectives

The results of the preliminary evaluation of the analytic method encouraged the author to undertake the research reported in this article. Eight major objectives were established for the investigation of aerial analytic triangulation on Wyoming Interstate Highway 80, as described in the following statements.

- To analytically bridge 11 photographs (10 stereoscopic models) taken at a scale of 1:6,000—500 feet to 1 inch—with a 6-inch focal length aerial camera lens.
- To evaluate the effect of analytically bridging photographs that had been drilled with a Wild PUG point transfer device.
- To develop computer programs written in the Fortran language, to perform a coordinate transformation of image points on the photographic glass plates that have side fidu-

---

[1] Presented at the 46th annual meeting of the Highway esearch Board, Washington, D.C., Jan. 1967.

[2] Mr. Chaves is now a Highway Engineer in the Engineer- g Systems Division, Office of Research and Development.

[3] Italic numbers in parentheses identify the references listed on page 159.

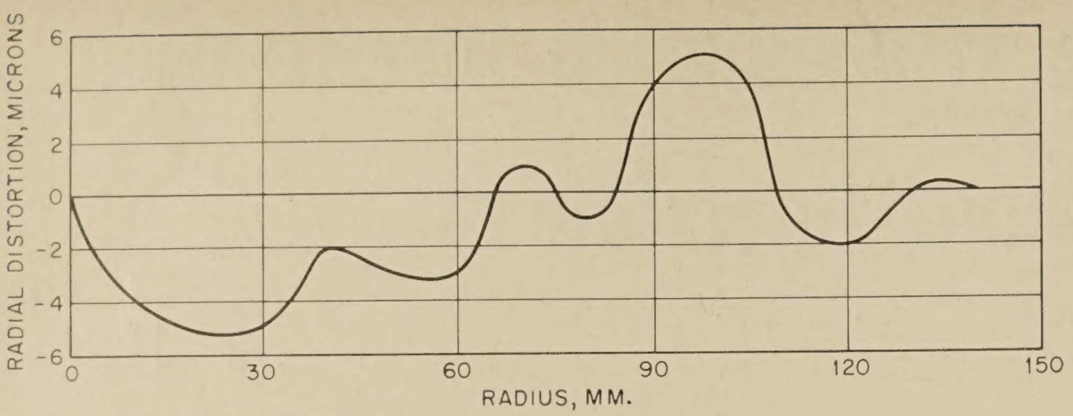


Figure 1.—Average radial distortion curve for Zeiss Pleogen lens.

cial marks, and to apply a polynomial curve-fitting technique to compensate for the effects of radial lens distortion.

- To determine the density and distribution of ground control needed for adequately adjusting a strip of 10 stereoscopic models.
- To determine the degree of strip adjustment needed for a strip of 10 stereoscopic models.
- To analyze photographic materials and photogrammetric instruments, equipment, and methods as possible sources of error in the analytic system of bridging.
- To revise the cantilever adjustment program originally written for use on an IBM 1401 for the IBM 7010 system.
- To make recommendations for improving and implementing the analytic method and suggest needed research.

Conclusions and Recommendations

The aerial analytic triangulation method is potentially useful as a method for obtaining accurate supplemental data that can be used in the compilation of topographic maps for highway location and design, according to results obtained in the investigation reported here. The author cautions, however, that operational techniques and hardware should be refined to overcome the four main sources of error in the computation of the X, Y, and Z coordinates of selected points on the ground. These four sources of error that affected the accuracy of results obtained in the investigation were: Film deformation, pass point transfer, x and y coordinate measurements of points on the glass plate transparencies, and ground control.

Most present aerial cameras are neither designed nor calibrated for analytical photogrammetry. To take full advantage of mathematical techniques to compensate for errors, some slight design changes in aerial cameras and related equipment undoubtedly will be needed.

In future investigations of aerial analytic triangulation for highway mapping, the author recommends that consideration be given to use of the equipment and methods listed here: (1) color diapositive plates; (2) aerial cameras equipped with eight fiducial marks or a reseau grid; (3) stereocomparator (*4*); (4) photograph scales of 1:12,000 or smaller; (5) other mathematical methods of aerial analytic triangulation described in references (*2*, *5*, *6*, *7*, and *8*), and different point transfer and marking devices.

The conclusions and recommendations were based on the findings discussed in the summary at the end of this article.

Test Procedures and Results

Eleven photographs at a scale of 1:6,000 were selected from a flight strip taken in July 1963 by the Continental Engineers, Inc., Denver, Colo., for mapping a corridor for Wyoming Interstate Highway 80 between Green River and Rock Springs in southwest Wyoming. These 11 photographs covered a strip of topography that was approximately 4,500 feet wide and 20,000 feet long and that had a light to moderate brush cover. The photographs were taken, from an average flight height of 3,000 feet, with a Zeiss RMK A 15/23 aerial camera equipped with a Pleogon lens having a calibrated focal length of 15 45 mm. ±0.02 mm. and a maximum apertur of f/5.6. The average radial lens distortion b ed on determinations made on two radii did ot exceed ±5 microns, as shown in figur 1. The distortions were determined within an accuracy of 2 microns. The distance betv en the fiducial marks in both directions as 226.00 mm. ±0.02 mm. The negative im used had an estar base from which diaposi ve plates (Kodak Aerographic Positive Pl es, Improved, 0.25 inch thick) and photogra ic prints were made with a LogEtronics Cl 18 automatic dodging printer.

Image selection

The image points used in the triangula on experiment were those for which grc nd control data were available. These cor ol points had been surveyed for use in comp ng topographic maps of a corridor along Ir state Highway 80 in Wyoming. The pc ts measured were images of targets and im es of natural objects that had been selecte in accordance with the mapping needs of he project. Pass points for each stereosc ic model were selected in the usual rectang ar pattern in the six classical locations. 'wo or three additional points were selected in he area of triple overlap of the photographs or each two adjacent stereoscopic models s as to ensure that enough acceptable points re available for the scaling of one stereosc ic model to another in the cantilever assem y. All image points, targeted and natural, ed in the triangulation were predrilled wit a Wild PUG 3-point transfer device equi ed with drills having a diameter of 60 micr s.

Figure 2.—Nistri monocomparator, model TA1/P and accessory equipment.

Table 1.—Errors in horizontal coordinates from third degree adjustments

Point	Errors in horizontal coordinates from third degree adjustments—									
	1		2		3		4		5	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
Number	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
5-1	0.95	−0.68	0.86	−0.74	0.91	−0.58	−0.96	−0.64	[1] −0.02	−0.05
SW-45	[1] −0.02	−0.09	[1] −0.05	−0.18	[1] −0.04	−0.09	[1] 0.00	−0.14	0.71	0.53
SW-44	0.14	0.31	−0.02	0.18	−0.05	0.18	[1] 0.00	−0.19	−0.29	0.74
SW-43	0.31	0.45	[1] 0.10	−0.29	[1] −0.03	0.22	0.02	−0.33	−0.14	0.67
2-1	−0.29	−0.61	−0.44	0.47	−0.42	0.45	−0.42	0.52	−0.46	−0.97
2-2	−0.30	1.21	−0.38	−0.01	0.04	0.79	−0.28	0.88	−0.38	−1.22
SW-41	[1] −0.08	0.28	[1] −0.13	0.08	[1] 0.08	0.15	[1] 0.00	−0.07	[1] −0.13	0.22
1-2	−0.80	0.10	0.75	−0.09	−0.36	−0.36	−0.49	−0.32	−0.88	−0.22
0-1	−0.73	0.12	−0.69	−0.12	−0.38	−0.47	−0.50	−0.39	−0.89	−0.08
8-1	−0.88	0.91	−0.80	0.80	−0.30	−0.41	−0.42	0.46	−1.21	−0.37
9-1	0.23	0.21	0.29	0.07	0.77	−0.22	0.64	−0.18	[1] −0.08	−0.27
SW-38	[1] −0.01	−0.49	[1] 0.09	−0.56	−0.49	−0.97	0.40	−0.93	−0.49	−0.91
SW-37	[1] 0.05	0.30	[1] 0.02	0.37	[1] 0.00	0.02	[1] 0.00	0.01	−0.72	0.11
BMC-131	0.68	−0.01	0.71	0.33	−0.85	0.08	0.78	0.06	[1] 0.13	−0.09

[1] Horizontal control points used to adjust strips.

Pnt transfer and marking

The accuracy with which pass points are trnsferred to a photographic strip materially afects the results. Obviously, precise x and y masurements made for a point but inaccately transferred are of no value. At the psent time considerable research is being cducted on different aspects of the measuremnt procedure, including evaluation of the tpes of point transfer and marking devices (At present no general agreement exists aong those concerned as to the efficiency ad reliability of the different types of instuments and devices. When using a monocur comparator it is a practical necessity to u some form of point transfer and marking stem. Point transfer should be precise and steoscopically correct. Measurement of pte coordinates is considerably facilitated ad the possibilities of blunders from misidtification of corresponding images are mimized. Targeted points, however, can bocated and measured readily without their lctions having been premarked on the dpositive plate.

Primarily because of the inexperience in use o the PUG point marking and transfer instment, some errors were introduced into t analytic system on the test project. Dtermining the magnitude of these errors, hvever, was considered impracticable. Most o he drilled holes were nonuniform in shape ad size. Some targeted control points were dled off center. Observation made with t Kelsh instrument of each stereoscopic mdel in the strip of photographs showed lack o stereoscopic correspondence of the drilled hes with the ground surface for a significant nmber of the points. Based on this stereospic analysis, digging or floating points were n used as control points for the strip adjtments. A comparison was made between t sign of the errors in the computed elevatns and their positions were noted in the sreoscopic models as being either on, above, obelow the ground surface.

There was a definite correlation between t sign of the errors in elevation and the oerved elevation of the holes in the stereospic models. No actual elevation measurements were made because a significant number of the drilled holes could not be seen in the stereoscopic model. However, the transfer of points is only one source of error known in the investigation reported here.

Measurements

The x and y coordinate measurements were made with the Nistri Monocomparator, Model TA1/P, which provided both digital readout and typewritten outputs. This monocomparator is shown in figure 2. The monocomparator had been calibrated a few months before measurements were made. The smallest reading possible on this monocomparator is 1 micron. The diapositive plates were measured with the emulsion side down under a 10× magnification. The objective lens on this particular instrument was equipped with a 20-micron measuring mark. Measuring marks of other sizes are available from the manufacturer of the instrument. The coordinate output of this particular monocomparator was in a left-handed system. Provision was made in the coordinate transformation program to change the coordinate system into a right-handed system, so that all coordinates increased along the y axis away from the observer and to the right along the x axis. A simple wiring modification can be made at the factory to produce output directly in the right-handed system. Measurements were made in an air-conditioned room at 72° F. Periodic checks were made for possible instability. The instrument and accessory equipment had excellent stability throughout the measurement operations. It took about 1 hour to measure an average of 25 drilled holes per photographic plate and the 4 fiducial marks. Each of the drilled holes was measured three times and each of the fiducial marks was measured six times. The mean of these measurements was accepted as true x and y coordinates for each point, respectively. The measured points were always approached with the measuring mark from the same direction to avoid the possibility of screw backlash, although screw backlash had been determined to be only 2 or 3 microns in magnitude.

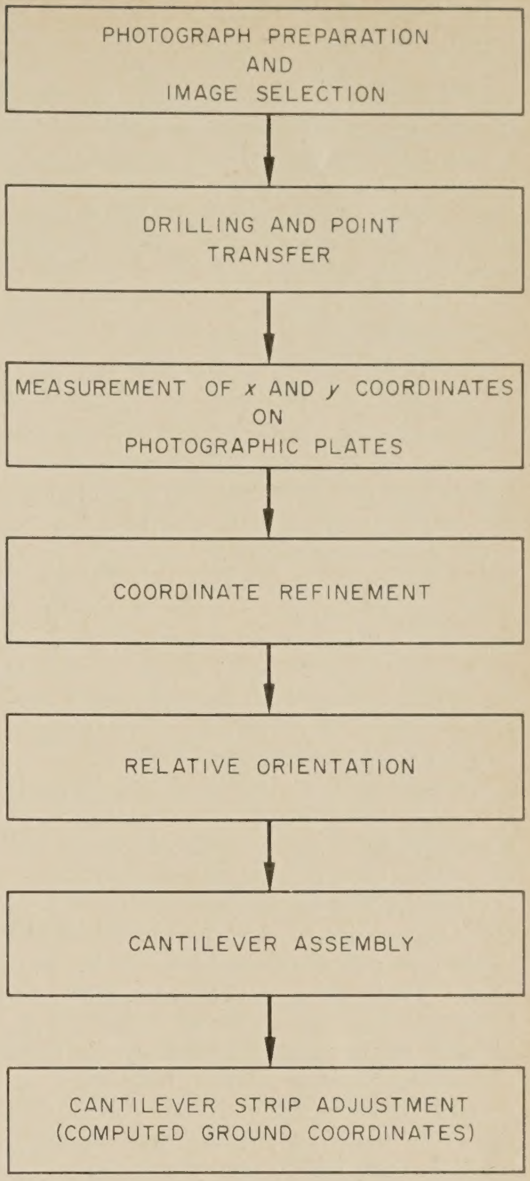


Figure 3.—Flow chart of aerial analytic triangulation.

Computations

Two electronic digital computers were used to make the mathematical computations of the analytic bridge. The cantilever strip adjustment program, which provided the X, Y, and Z ground coordinates of each measured point, was used in an IBM 7010 computer, which had a 60K digital storage capacity; all other programs were used in an IBM 1401, which had a 12K digital core memory.

Basic ground control was surveyed in a closed traverse approaching second order accuracy. Points identified in table 1 by the prefix SW were included in this traverse. All other ground surveyed points were assumed to be of at least third order accuracy, although no survey closure checks were actually made. The Electrotape and Tellurometer electronic distance measuring instruments and a Wild, T-2 Theodolite, were used to make the survey.

A generalized flow chart of analytic aerial triangulation used in the investigation reported here is shown in figure 3 and the basic computational concepts and procedures are described in the following paragraphs.

Certain mathematical operations must be completed to assure that the measured x and

y coordinates will be suitable for use in the analytic triangulation. These operations include the averaging of each set of coordinate measurements; converting the coordinate system from a left-handed to a right-handed system; performing a mathematical translation and rotation of the measured photographic plate coordinates; making compensation for film deformation; and correcting radial lens distortion.

Plate coordinate averages

The measurements made with the monocomparator are recorded in typed form. Card punching was performed directly from the typewritten record of the monocomparator measurements. Averages were computed with two separate computer programs for the three measurements made on each of the image points (drilled holes) and for the six measurements made on each of the fiducial marks. In the computer programs, the monocomparator measurements of the left-handed coordinate system were converted to a right-handed system by subtracting all x coordinates from an arbitrary constant.

When diapositive plates are placed on the monocomparator stage for measurement, the x and y axes of the photographic plate must be oriented physically parallel to the respective axes of the monocomparator, or some means must be provided for mathematical rotation so the two coordinate systems will be coincident. The relation of these axes is shown in figure 4. The orientation of a plate so that its axes are precisely parallel to the monocomparator axes is a time consuming procedure. Consequently, it is expedient to place the plate on the monocomparator so that the respective axes are more or less parallel, and then mathematically translate and rotate them to coincidence. The mathematical procedure is based on the instrument measured coordinates of the measured fiducial marks. The mathematical rotations are made by using standard rotation equations from analytic geometry. The resultant translated and rotated coordinates for all points measured on each photographic plate can then be referenced to the principal point of the photograph.

Film deformation

Plastic films are subject to dimensional change between the time of photographic exposure in the aerial camera and the printing of diapositives on optically flat glass plates. Therefore, some means of compensating for the movement of images is necessary. For cameras equipped only with four side fiducial marks, the only compensation possible is a comparison of the distances between the marks in two directions on the printed diapositive with those in the aerial camera. The distances between fiducial marks in the aerial camera may be furnished by the manufacturer or it can be measured on a diapositive plate (flash plate) that previously has been exposed directly to the aerial camera. Two scale factors were developed for each plate based on the distances between the fiducial marks reported by the manufacturer and those determined for each plate in the x and y directions. The x and y coordinates of all measured points were then multiplied by the respective film deformation correction factors.

Results obtained by comparing distances between fiducial marks in both directions on the measured photographic plates with distance between the same marks recorded on the camera calibration certificate were remarkably uniform in the deformation of estar base photographic film. The dimensional change in both the x and y photographic plate axes was about the same, as shown in table 2. All the computed linear factors for film deformation were less than unity, which indicated the estar base photographic film had expanded rather than shrunk in both directions. The average computed factors from all of the x and y measurements of fiducial marks on the separate photographic plates were 0.999676 and 0.999746, respectively.

This method of correction, based on the distance between side fiducial marks is not as accurate a method as would be desirable because film deformation is random and nonlinear. No better method could be devised for compensating for film deformation when only the four side fiducial marks could e measured. Film deformation is one of e sources of error in the analytic system f aerial triangulation. Recently develo l, scale stable base films, such as estar base, e a factor in helping to curtail film deformat n and thus minimize this correction metho s a source of error.

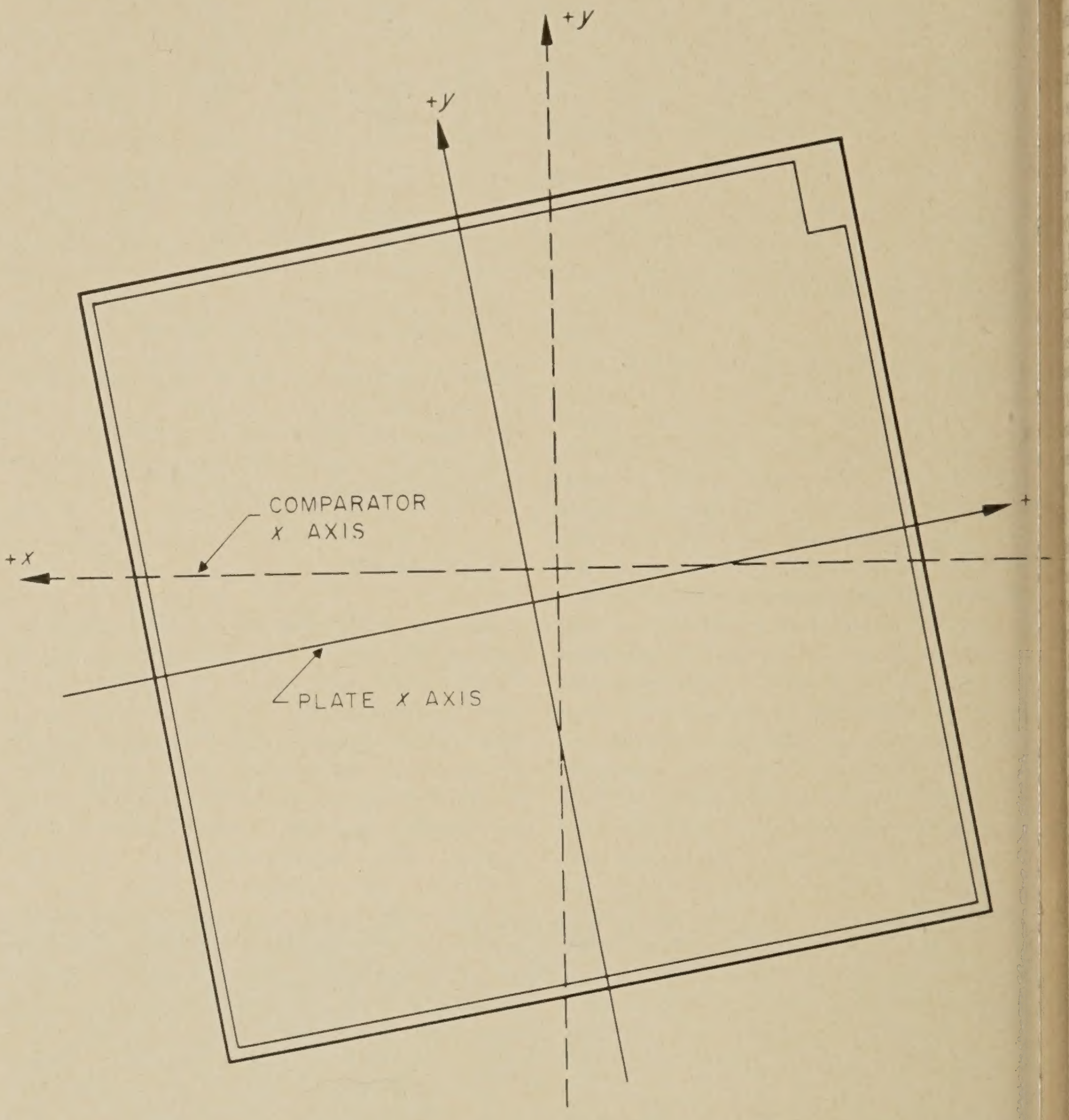


Figure 4.—Relation of diapositive plate to monocomparator axes at time of measurem t.

One possible solution to the problem of m deformation would be the use of glass x-posed directly in the aerial camera. is would eliminate need for film distor n compensation. However, no better accu y in locating points can be expected as g ss plate cameras use smaller formats and posi n accuracy is proportional to the scale of e photograph. The use of reseau equip d cameras might improve the accuracy ( ). However, some practical considerations t present limit the use of these two techniq s. One approach currently being employe to improve accuracy is the use of aerial cam as having eight rather than four fiducial ma s. This permits more adequate mathema al restitution of points displaced by film m ement (*11*). The distance between fiducial m ks reported by the manufacturer of the can ra used in this project was only to an accurac of ±20 microns, and the diameter of the fidu al mk holes was 250 microns. Locating the prise center of such fiducial marks with a msuring mark only 20 microns in diamet was somewhat uncertain.

he average radial lens distortion curve for th Zeiss Pleogon lens is shown in figure 1. Potive lens distortion covers the displacemt of an image radially outward from the ceer of the photograph; when distortion is netive the displacement is radially inward. Coections for lens distortion must be made tohe *x* and *y* coordinates of each measured pot on the photograph.

a equation for the lens distortion curve shn in figure 1 was determined by a polynoial curve fitting program. The program us. was one developed by W. R. Graves, IBI Corporation and modified by R. Larson, B, Region 15, and the writer for use on th IBM 1401. This program generates an apoximating polynomial using the least sqares technique. Coefficient terms of the ponomial curve are determined and they are us to compute the amount of radial lens diortion for any given radial distance from thcenter of the photograph. Actual computaon of the distortion is accomplished by a sht program in which the appropriate coficients are used. The *x* and *y* coordinates ofeach point are then corrected for the efts of image displacement caused by lens diortion. The particular distortion curve (fi 1) had many points of inflection. Conseently, the curve was divided into two sements—0 to 70 mm. and 70 to 140 mm.—to obin enough accurate polynomials. An ecation for each of the two segments of the tol curve was determined by using radii and diortion data for 15 and 30 points, respectily, as electronic computer input data. dequate results based on the reliability ofhe input information were obtained by us of the polynomial curve fitting technique. Ahough lens distortion data were available fonly 15 discrete radii at intervals of 10 mm. onthe plate, a smooth curve was plotted thugh these points (fig. 1), and it was acpted as the actual distortion curve. Eugh distortion data for specific radii were sected from the curve and used as input da for the curve fitting electronic computer pgram. All computed distortion results bed on the computer curve fell within less th 0.5 micron of the plotted curve. This tenique of radial lens distortion compensati. is well within the accuracy tolerances of ±2 microns given by the manufacturer.

Rative orientation

elative orientation may be defined as reonstruction of the geometric conditions exting between pairs of photographs at th moment of exposure (*12*). Relative ontation is the determination of three rotional (omega, phi, kappa) and two of the translational (*bx*, *by*, *bz*) elements that dne the attitude and positions of one photogph to another, providing there was sufficit common area of overlap in line of flight. he method of relative orientation in the eluation reported here depends upon enfoing a geometric condition where the potographic image, perspective center, and the object in the stereoscopic model are on a straight line (colinear). (Complete details on the mathematical formulation and derivations of the relative orientation and other parts of the analytic system are explained in reference *1*.) For a given stereoscopic model, there are many lines (a pair for each image). But, because of different errors, it was impossible to enforce all colinear conditions completely. As the left-hand photograph of a pair is assumed to have no tilt in this method of analytic relative orientation, small corrections were made to the measured *x* and *y* coordinates of the right-hand photograph in a least squares manner. Each stereoscopic model was oriented independently; 12 image points were used, three computer iterations were required for completion of the relative orientation. The third and usually last iteration determined the *x*, *y*, and *z* coordinates for each point in the stereoscopic model and also orientation data that later were used in the assembly of independently oriented stereoscopic models to form a strip. The lack of intersection of pairs of lines (corresponding rays) for all the image points in each stereoscopic model were printed out as residual *y* parallaxes. These residuals were reviewed at the completion of the orientation. Substitutions were made for points having unusually large residuals, and a new orientation was performed.

This procedure was continued until no residuals larger than 25 microns remained at any given point. Table 3 shows the average absolute residual *y* parallax remaining at each of the 12 points in each of the stereoscopic models that were oriented. Residual *y* parallaxes were as large as 50 microns for some points in the computed stereoscopic models. These larger residuals were primarily the result of errors introduced by the incorrect position of the drilled holes, but the inaccuracy attached to measuring the holes and the method of film deformation compensation also contributed to the large residual determinations. The method of independent relative orientation of models employed is dependent upon the intersection of only two rays (lines) from the respective photographs. Thus, no check is possible to determine the accuracy of computed elevations of points. Errors in *x* parallax are reflected as errors in the elevation of points on the ground. The elevations are computed in the final step of the analytic system of aerial triangulation.

Table 2.—Scale factors used to compensate for film deformation

Photographic plate	Compensation factor for axes—	
	x	*y*
1–40	0.999646	0.999646
1–41	0.999602	0.999611
1–42	0.999690	0.999690
1–43	0.999717	0.999788
1–44	0.999673	0.999805
1–45	0.999712	0.999797
1–46	0.999699	0.999814
1–47	0.999646	0.999735
1–48	0.999717	0.999761
1–49	0.999673	0.999766
1–50	0.999602	0.999797

Table 3.—Average absolute residual *y* parallax from relative orientation

Model	*y* parallax residual
Number	*Microns*
1	5.9
2	8.8
3	9.6
4	5.0
5	6.0
6	8.2
7	9.4
8	7.7
9	7.7
10	9.7

Table 4.—Scale factors for cantilever assembly computations

Models	Scale factors
1 and 2	0.91743098
2 and 3	0.96043167 0.95981338
3 and 4	0.95736429
4 and 5	0.91459298 0.91491979
5 and 6	0.95755554 0.95778659
6 and 7	0.91712976
7 and 8	0.91668318 0.91630765
8 and 9	0.91052712 0.91084330
9 and 10	0.90574928

Table 5.—Computed strip coordinates of points in triple overlap areas

Point	Computed strip coordinates		
Number	*x*	*y*	*z*
1–40–B	0.98092 0.98092	0.22099 0.22100	−1.60790 −1.60809
1–42–B	0.94122 0.94121	0.71363 0.71364	−1.59301 −1.59322
1–42–L	1.93575 1.93575	0.60043 0.60049	−1.49853 −1.49885
1–42–K	1.94781 1.94780	0.14704 0.14712	−1.57810 −1.57766
1–42–D	1.93855 1.93855	−0.49565 −0.49581	−1.54782 −1.54813
1–42–G	2.88542 2.88542	−0.17437 −0.17442	−1.54318 −1.54327
1–42–J	2.92091 2.92092	0.19175 0.19172	−1.52242 −1.52299
1–44–F	3.81770 3.81770	0.81302 0.81318	−1.51911 −1.51931
42–2	3.82037 3.82037	−0.14599 −0.14601	−1.53610 −1.53600
1–44–D	3.80635 3.80636	−0.64514 −0.64507	−1.53752 −1.53737
1–44–K	4.78011 4.78013	0.59229 0.59277	−1.49192 −1.49278
1–44–G	4.79534 4.79537	−0.01733 −0.01719	−1.51885 −1.51975
41–2	5.67728 5.67728	0.11931 0.11932	−1.49878 −1.49888
1–48–F	7.53923 7.53923	−0.20051 −0.20049	−1.53959 −1.53961
38–1	7.53990 7.53990	0.59161 0.59178	−1.51810 −1.51848
1–50–E	8.45788 8.45788	0.48609 0.48586	−1.48191 −1.48140
1–50–A	8.46795 8.46795	−0.11971 −0.11996	−1.53105 −1.53033

Table 6.—Summary of errors for five third degree adjustments

Adjustment	Control points		Test points		Errors for third degree adjustments by type of error for each coordinate											
	Horizontal	Vertical	Horizontal	Vertical	RMSE			Maximum			Minimum			Algebraic mean		
					X	*Y*	*Z*	*X*	*Y*	*Z*	*X*	*Y*	*Z*	*X*	*Y*	*Z*
					CONTROL POINTS											
Number	*Number*	*Number*	*Number*	*Number*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
1	4	12	--------	--------	0.05	0.32	0.59	−0.08	−0.49	1.28	−0.01	−0.09	0.00	−0.01	0.00	0.1
2	5	18	--------	--------	0.09	0.34	0.09	−0.13	−0.56	−1.93	0.02	0.08	−0.03	0.01	−0.12	−0.2
3	4	13	--------	--------	0.05	0.14	0.75	0.08	0.22	−1.78	0.00	0.02	0.25	0.01	0.30	−0.1
4	4	10	--------	--------	0.00	0.04	0.34	0.00	−0.19	−0.91	0.00	0.01	0.00	0.00	−0.09	−0.1
5	4	7	--------	--------	0.10	0.18	0.00	±0.13	−0.27	0.00	−0.02	−0.05	0.00	−0.02	−0.05	0.0
					TEST POINTS											
1	4	12	10	40	0.60	0.59	1.52	0.95	1.21	3.60	0.14	−0.01	0.12	−0.07	0.20	0.7
2	5	18	9	34	0.61	0.42	1.42	0.86	0.80	3.18	−0.02	−0.01	0.02	0.06	0.25	0.2
3	4	13	10	39	0.54	0.52	1.67	0.91	−0.97	3.94	0.04	0.08	−0.05	−0.11	−0.15	0.1
4	4	10	10	42	0.55	0.54	1.95	−0.96	−0.93	−4.63	0.02	0.06	0.01	−0.12	−0.09	0.0
5	4	7	10	45	0.69	0.69	1.89	−1.21	−1.22	4.61	−0.14	−0.08	−0.10	−0.48	−0.17	0.3

Table 7.—Summary of errors for five second degree adjustments

Adjustment	Control points		Test points		Errors for second degree adjustments by type of error for each coordinate											
	Horizontal	Vertical	Horizontal	Vertical	RMSE			Maximum			Minimum			Algebraic mean		
					X	*Y*	*Z*	*X*	*Y*	*Z*	*X*	*Y*	*Z*	*X*	*Y*	*Z*
					CONTROL POINTS											
Number	*Number*	*Number*	*Number*	*Number*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
1A	4	12	----------	----------	0.93	0.10	0.92	1.40	0.44	1.85	0.26	±0.05	0.10	0.00	0.00	0.0
2A	5	18	----------	----------	0.97	0.33	1.12	1.57	0.45	2.09	0.30	0.14	0.02	−0.01	−0.01	0.0
3A	4	13	----------	----------	0.41	0.12	1.25	−0.64	−0.17	2.02	−0.05	±0.01	0.07	0.00	−0.08	0.0
4A	4	10	----------	----------	0.51	0.10	1.21	−0.79	0.16	−1.94	−0.03	0.00	−0.39	0.29	0.00	−0.0
5A	4	7	----------	----------	0.81	0.23	0.84	1.20	0.38	−1.37	−0.22	0.02	0.17	−0.11	−0.11	−0.0
					TEST POINTS											
1A	4	12	10	40	1.26	0.69	1.54	2.01	1.27	4.06	0.35	−0.16	−0.01	0.22	0.55	0.8
2A	5	18	9	34	1.17	0.77	1.51	2.21	1.26	3.89	0.05	0.07	0.01	0.32	0.48	0.
3A	4	13	10	39	1.71	0.68	1.44	2.86	1.06	3.84	−0.15	−0.19	−0.03	0.58	0.10	0.
4A	4	10	10	42	1.62	0.60	1.38	2.56	0.94	3.67	−0.36	−0.01	−0.18	0.96	0.19	−0.
5A	4	7	10	45	1.61	0.61	1.46	−2.55	−1.47	3.84	0.43	0.09	−0.16	−1.17	−0.20	0.

Cantilever Assembly

After the successive completion of the relative orientation for each of the stereoscopic models, the individual models were tied together into a continuous strip. This assembly of models is accomplished by successive mathematical transformation of the model coordinates of each model in the strip. The three transformations, which were performed in order, were rotation, scaling, and translation. The strip coordinates, which are analogous to those obtained from an analog triangulation instrument, were obtained from these mathematical computations. The first stereoscopic model in the strip was arbitrarily considered to be at the desired scale and its coordinates in the proper system. Therefore, the mathematical transformations were performed only on the second and succeeding models. Scaling in this cantilever system was accomplished by comparing slope distances between two image points, which occur in adjacent stereoscopic models (the common overlap area of three photographs), and making an adjustment by a scale factor to the model being attached to the strip.

Because of the limited storage capacity of the IBM 1401 computer used in the investigation reported here, only 10 points per stereoscopic model could be accommodated in the cantilever assembly program. For this investigation, however, 10 points per model were considered enough. Table 4 contains a listing of scale correlation factors computed from points occurring in the triple overlap area. Wherever enough points were available, two scale factors were computed and the average value used. Scale factors should be in reasonably close agreement. Points causing the anomalies in scale factors were discarded, and substitute points used to recompute the strip coordinates.

The strip coordinates of points occurring in triple overlap areas, which are listed in table 5, were computed by using data derived from the independently oriented stereoscopic models of the strip. Averages of two sets of strip coordinates for each point were used; they were considered the most acceptable for the point. There was slightly larger disparity in computed strip elevations than in the horizontal strip coordinates.

Adjustments

The adjustment of cantilever strip coordinates is the last computational step; this computation provides the *X*, *Y*, and *Z* ground coordinates (*13*). The adjustment is fully applicable to strip coordinates derived from either analog optical train bridging instruments or to coordinates derived from measurements made with monocomparators. The method of adjustment used, attempts to correct for curvature of the strip (azimuth), twist or cross tilt, *BZ* fall off, scale change along the *x* and *y* axes of the strip, and lateral tilt of the strip in the *x* and *y* directions. Cumulative errors in a strip tend to be systematic and can be corrected by use of polynomial adjustments. Both second and third degree polynomial adjustments were applied to determine their effectiveness.

To accomplish the vertical and horizontal adjustment of the strip coordinates, data listed in the following statements are required

Table 8.—Errors in horizontal coordinates from second degree adjustments

Point	Errors in horizontal coordinates from second degree adjustments—									
	1A		2A		3A		4A		5A	
	X	Y	X	Y	X	Y	X	Y	X	Y
Number	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
45-1	1.89	−0.16	2.21	−0.54	1.72	−0.43	1.92	−0.32	[1] −0.22	0.02
SW-45	[1] 0.26	−0.05	[1] 0.60	−0.35	[1] 0.25	−0.17	[1] 0.59	−0.13	−1.18	0.17
SW-44	−1.04	0.25	−0.58	0.07	−0.60	−0.19	[1] −0.79	0.16	−2.04	0.26
SW-43	−1.54	0.30	[1] 0.90	0.17	[1] −0.64	0.01	−0.80	−0.01	−2.29	−0.19
42-1	−2.01	0.30	−1.47	0.24	−1.11	0.35	−1.22	0.22	−2.55	0.09
42-2	−1.39	0.83	−0.81	0.91	−0.15	−0.92	−0.36	0.74	−1.64	0.43
SW-41	[1] −0.87	0.05	[1] −0.30	0.14	[1] 0.44	−0.17	[1] 0.22	−0.04	[1] −1.05	−0.38
41-2	−0.35	0.43	0.14	0.54	1.12	0.52	0.87	0.29	−0.44	−0.18
40-1	−0.48	0.39	0.05	−0.23	0.92	−0.34	0.68	−0.56	−0.67	−0.94
38-1	0.70	1.27	1.02	1.26	2.20	1.06	1.94	0.94	0.43	0.22
39-1	1.29	0.95	1.71	1.00	2.86	0.93	2.56	0.74	[1] 1.20	0.13
SW-38	[1] 1.40	−0.44	[1] 1.57	−0.42	−2.69	−0.72	2.51	−0.81	0.81	−1.47
SW-37	[1] −0.80	0.44	[1] −0.98	0.45	[1] −0.05	−0.01	[1] −0.03	0.00	−2.11	−0.52
BMC-131	0.71	0.97	0.63	1.11	1.55	0.76	1.54	0.68	[1] −0.38	−0.22

[1] Horizontal control points used to adjust strips.

inputs: (1) A card containing the number vertical and the number of horizontal ground ntrol points used in the adjustments; (2) card containing the x and y cantilever ip coordinates of a point near the center of first stereoscopic model and another point r the center of the last model in the strip; the strip x, y, and z coordinates of all nts to be used as a basis for the adjustment which the ground X, Y, and Z coordinates known; (4) the horizontal and vertical ound control data for the points used in m 3; and (5) the cantilever strip coordinates ded to establish supplemental control. ree horizontal and five vertical control ints are the minimum number needed to ke a second degree adjustment, and four rizontal and seven vertical control points needed to make a third degree adjustment.

Computed Ground Coordinates

The errors summarized for the computed ound coordinates using second and third gree cantilever strip adjustments are shown tables 6 and 7. The horizontal and vertical rors for ground coordinates computed for 55 parate points in 10 trial adjustments are ted in tables 1, 8, and 9. The distribution ground control used in each of the adjust- nts is shown in figure 5. The third degree rizontal adjustments were the most accu- te. In table 6 the RMSE are given for the rizontal coordinates of the third degree justment. RMSE ranged from 0.42 and 39 of a foot and no significant difference was own between the X and Y coordinates. e use of five rather than four horizontal ntrol points did not significantly increase e accuracy of the computed horizontal ordinates.

The RMSE for the horizontal coordinates second degree adjustments are shown in ble 7; these RMSE ranged from 0.60 to 1.71 a foot. The magnitude of the error in the coordinates was about twice that for the coordinates.

The errors in the computed Y coordinates both second and third degree adjustments ere about the same. Errors in the computed coordinates of the third degree adjustment ere one-half the size of those for the second degree adjustments. Test point 45–1 tended to have slightly more error in horizontal coordinates in 8 of the 10 adjustments because of its position outside the confines of ground control point SW–45, located near the beginning of the flight strip.

A comparison of the vertical RMSE in tables 6 and 7 for the second and third degree adjustments shows no significant difference in accuracy for the first three sets of trials. Twelve or more vertical control points were used to adjust these flight strips. Comparison of the second degree adjustments, however, showed a significant improvement in accuracy over the third degree in the last two sets of adjustments, where vertical control points were not as dense.

An increase from 7 to 18 vertical control points had practically no effect on the RMSE of the second degree adjustments, but it did significantly reduce the RMSE of the third degree adjustments. The largest vertical errors were for test points located in areas where no vertical control points were present in their vicinity (adjustments 4, 4A, 5, 5A). The effects of density and distribution of vertical control points on ultimate accuracy are difficult to analyze. It is likely that some of the holes drilled for ground control points were not in stereoscopic correspondence with the ground surface. Only those control points observed to float or to dig in the Kelsh instrument were eliminated as control points for making the strip adjustments.

The algebraic mean errors listed in tables 6 and 7 for elevations of test points are positive for all trial adjustments except adjustment 4A. The positive sign of these mean errors tends to confirm the stereoscopic observations made of the drilled holes and the existence of systematic errors.

Summary

The author recognizes the fact that the method used to compensate for film deformation is inadequate, but it is the most suitable method that can be used for cameras equipped with only four side fiducial marks. It is, however, impossible to prove the amount of error in ground coordinates caused by film deformation. The estar base film is considerably more stable dimensionally than film bases previously available. At least three possible alternatives could be used for a more effective treatment of the film deformation problem. First, existing cameras could be equipped with four additional fiducial marks so that a more effective mathematical restoration of displaced images can be made. A computer program is available to accomplish this. Second, a network of small crosses (reseau) could be used in the focal plane of the camera. The crosses are recorded on the negative film at the instant of exposure. Displacement of images caused by film distortion or lack of film flatness at the instant of exposure could be compensated for by comparing the measured position of the reseau crosses with their calibrated position. Third, a glass plate aerial camera could be used; this would eliminate the need for compensation caused by film deformation.

When measuring plate coordinates with a monocomparator, some form of point transfer and marking technique is a practical necessity. The PUG point transfer device was used for the project discussed here. Primarily because of inexperience with the instrument, some error was introduced. A divergence of opinion exists at the present time regarding the inherent accuracy of several point transfer and marking devices. Research and evaluation of these techniques is being done by private firms, universities, and governmental agencies. Ground control and other points that are premarked with photographic targets can be reliably identified and measured without need for point transfer and marking.

The relative size of the measuring mark to that of the drilled hole appears to be a significant factor in the degree of accuracy of measurement. A 20-micron measuring mark, a fiducial hole having a diameter of 250 microns, or a drilled hole 60 microns in diameter are not desirable combinations for optimum accuracy of measurement. Better accuracy could be obtained by use of a larger measuring mark. Some doubt has also been expressed about the adequacy of drilled holes that are 60 microns in diameter for use in map compilation with the Kelsh instrument. At a 5:1 projection ratio, a significant number of holes could not be seen in the stereoscopic models. A drilled hole of at least 100 microns in diameter is needed for use with the Kelsh instrument.

Basic ground control surveys on which aerial analytic triangulation is based should be of second order accuracy (1/10,000 closing error in position) or better, so results of the triangulation will not be degraded because of inferior control. When this control is used, results of the triangulation as to accuracy can be more realistically evaluated. Results of the aerial analytic triangulation cannot be expected to be any better than the ground control survey on which the triangulation is based.

The polynomial curve fitting technique used to determine an equation for the radial lens distortion curve is considered an acceptable method. It is particularly adapted for use in determining the equations of curves that are

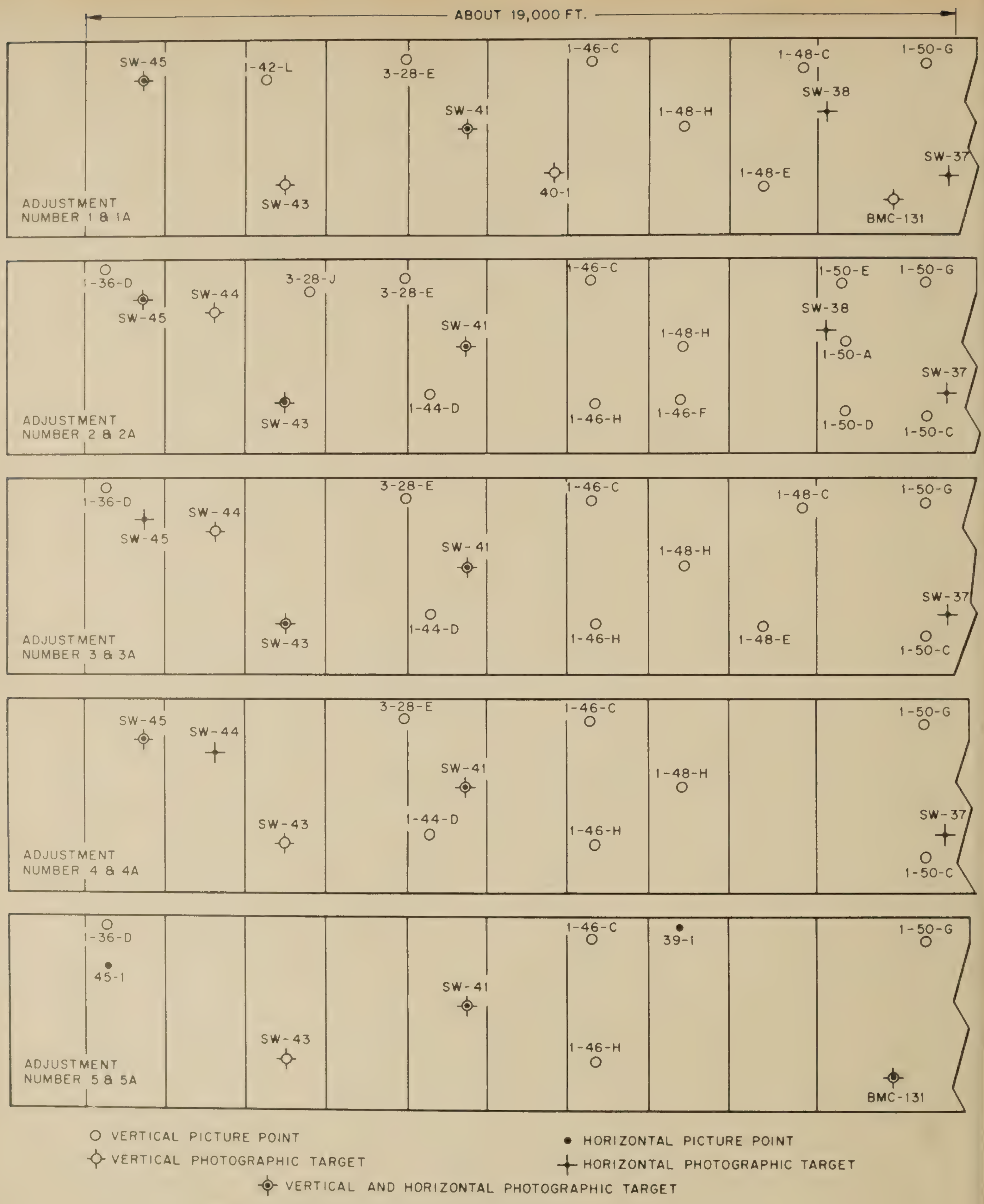


Figure 5.—Distribution of ground control used in different adjustments.

mooth and have only a few points of inflec- ion. Corrections made to the *x* and *y* easured coordinates, based on the computed urve, were within the accuracy tolerances for he manufacturer's determination of the lens istortion values.

Corrections for atmospheric refraction and djustments for earth curvature were not onsidered in the investigation reported here. isplacement of photographic images by at- ospheric refraction and earth curvature are egligible because of the relatively low flight eight from which the photographs were taken ad the short length of the flight strip. For ridging photographs taken from higher flight eights and for longer flight strips, appropriate djustments should be made to compensate r atmospheric refraction and earth curva- ure (*14*, *15*).

When small capacity computers are used r aerial analytic triangulation, considerable gmentation of the computer programs and xcessive card handling is required. Com- uters that have larger storage capacity and ore speed must be used if the analytic opera- ons are to be performed more efficiently.

Use of mapping scale photographs (1:6,000) r analytical bridging does not seem to be the ost accurate or economical approach for ob- aining supplemental ground control data for rge scale topographic mapping. Use of the rger scale photographs requires measurement f *x* and *y* for a larger number of stereoscopic odels for a given bridged distance. The rge number of intermodel ties needed tends deform the bridged strip and thus offset e advantages gained from having a larger ale.

Table 9.—Errors in vertical coordinates from second and third degree adjustments, 5 adjustments for each degree [1]

Point	Errors in vertical coordinates from the 5 adjustments—									
	1	1A	2	2A	3	3A	4	4A	5	5A
Number	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
1-40-B	−0.75	−1.36	−0.73	−1.67	−0.36	−0.85	−0.83	−1.84	−0.80	−2.09
1-42-B	−0.80	−0.73	−1.36	−1.00	−1.79	−1.27	−2.51	−1.26	−2.57	−1.26
1-36-D	1.61	1.00	[2] 0.71	[2] 0.62	[2] 0.70	[2] 0.30	0.31	−0.29	[2] 0.00	[2] 0.17
45-1	2.99	1.35	2.94	0.96	3.94	0.65	3.88	0.75	3.68	0.41
1-40-A	−0.50	−0.13	−1.43	−0.40	−1.22	−0.75	−3.02	−0.71	−3.17	−0.75
SW-45	[2] 0.59	[2] −0.12	[2] 0.23	[2] −0.45	0.47	−0.75	[2] 0.05	[2] −0.69	−0.10	−0.91
45-2	3.60	2.67	3.18	3.40	2.83	3.17	2.13	3.17	2.10	3.05
1-42-L	−1.78	[2] −1.09	−2.19	−1.27	−3.05	−1.42	−3.92	−1.49	3.85	−1.46
1-42-K	[2] −0.05	0.29	[2] −0.07	[2] 0.02	−0.60	−0.09	−1.32	−0.18	−1.23	−0.21
1-42-D	−0.12	−0.06	−0.19	−0.44	0.57	−0.50	0.05	−0.61	0.21	−0.70
1-40-C	1.36	−0.35	2.18	−0.73	3.77	−0.80	3.68	−0.79	3.83	−1.25
SW-44	−1.29	[2] −1.23	[2] −0.54	[2] −1.50	[2] −1.78	[2] −1.68	2.47	−1.70	−2.44	−1.83
1-42-C	−2.52	−1.99	−3.01	−2.20	−3.78	−2.41	−4.63	−2.44	−4.61	−2.48
SW-43	[2] −0.03	[2] −0.10	[2] −0.03	[2] −0.52	[2] 0.34	[2] −0.57	[2] −0.18	[2] −0.72	[2] 0.00	[2] −0.74
1-42-G	−0.54	1.13	0.23	0.81	−0.22	0.76	−0.87	0.57	−0.75	0.75
1-42-J	3.44	4.06	0.16	3.89	2.46	3.84	1.77	3.67	1.87	3.84
3-28-J	−1.03	−0.16	[2] −1.38	[2] −0.28	−2.46	−0.38	−3.33	−0.49	−3.27	−0.41
1-42-H	0.20	1.03	−0.06	0.93	−1.12	0.87	−1.91	0.74	−1.84	0.88
1-42-F	1.00	1.18	0.80	0.72	1.02	0.67	0.47	0.47	0.64	0.57
42-1	−0.66	−0.01	0.94	−0.24	−1.67	0.31	−2.35	−0.45	−2.24	−0.40
1-44-F	1.95	2.38	2.05	2.36	1.23	2.42	0.80	−2.26	0.78	2.40
42-2	1.39	1.97	0.02	1.64	−0.50	1.62	0.05	1.36	0.13	1.73
1-44-D	0.92	2.58	[2] −0.29	[2] 2.09	[2] 0.91	[2] 2.02	[2] −0.44	[2] 1.71	0.57	2.21
1-44-A	0.54	1.19	0.29	0.97	−0.40	0.95	−0.99	0.75	−0.93	0.98
3-28-F	1.62	2.23	1.47	2.09	0.68	2.08	0.12	1.90	0.16	2.10
1-44-B	1.88	2.01	1.13	1.92	0.14	1.90	−0.53	1.76	−0.49	1.90
3-28-E	1.28	[2] 1.85	[2] 1.29	[2] 1.81	[2] 0.37	[2] 1.84	[2] −0.16	[2] 1.69	0.16	1.83
1-44-K	1.85	1.75	2.11	1.67	1.79	1.81	1.81	1.59	1.72	1.81
1-44-G	0.89	1.05	0.67	0.73	0.35	0.70	0.27	0.46	0.30	0.94
3-26-D	2.33	2.38	1.62	2.35	2.13	2.48	2.02	2.29	1.94	2.45
SW-41	1.04	1.47	[2] −0.73	[2] 1.16	[2] −0.25	[2] 1.16	[2] −0.07	[2] 0.87	[2] 0.00	[2] 1.29
41-2	0.69	0.34	0.68	0.01	0.73	0.12	1.05	−0.19	1.02	0.29
3-26-E	1.86	0.39	2.24	1.29	2.25	1.48	2.58	1.25	2.43	1.48
40-1	[2] −0.20	[2] 0.13	−0.98	−0.52	−1.24	−0.61	−1.32	−1.03	1.07	−0.16
1-46-E	−0.72	−1.78	−0.21	−1.90	0.42	−1.57	−1.27	−1.79	0.99	−0.68
1-48-H	[2] −0.47	[2] −1.01	[2] −0.79	[2] −1.61	[2] −0.50	[2] −1.55	[2] 0.00	[2] −1.94	0.26	−1.19
1-46-F	0.31	0.17	[2] −0.61	[2] −0.77	0.60	−0.92	−0.36	−1.44	0.12	−0.20
1-46-C	[2] −0.85	[2] −1.47	[2] −0.44	[2] −1.58	[2] −0.29	[2] −1.36	[2] 0.16	[2] −1.59	[2] 0.00	[2] −1.37
1-46-H	0.78	0.97	[2] −0.12	[2] 0.20	[2] −0.31	[2] 0.07	[2] −0.28	[2] −0.39	[2] 0.00	[2] 0.64
1-46-D	0.56	−1.25	−0.45	−1.58	−0.11	−1.40	0.47	−1.71	0.40	−1.27
1-48-F	0.74	0.11	0.35	−0.60	−0.83	−0.50	1.46	−0.90	1.76	−0.13
1-48-E	[2] 0.20	[2] −0.12	−0.84	−1.26	[2] −0.72	[2] −1.45	−0.45	−2.02	0.40	−0.58
1-50-E	1.19	0.71	[2] 0.99	[2] 0.33	1.79	0.80	2.70	−0.59	2.58	0.60
1-50-A	1.35	1.05	[2] −0.72	[2] 0.25	1.19	0.42	1.74	0.05	2.23	0.73
1-48-C	[2] 0.25	[2] 0.54	0.25	−0.91	[2] 1.07	[2] −0.51	2.02	−0.76	1.89	−0.62
1-50-D	0.13	0.05	[2] −1.03	[2] −1.30	−1.00	−1.52	−0.91	−2.00	0.41	−0.54
1-50-G	[2] 0.00	[2] 0.72	[2] −1.93	[2] 0.23	[2] −0.55	[2] 0.80	[2] −0.03	[2] 0.62	[2] 0.00	[2] 0.48
1-50-C	2.33	2.93	[2] 0.86	[2] 1.30	[2] 0.49	[2] 1.01	[2] 0.04	[2] 0.41	2.08	2.14
BMC-131	[2] −0.01	[2] 0.26	−1.38	−1.32	−0.59	−1.61	−1.82	−2.23	[2] 0.00	[2] −0.48
1-50-B	0.84	0.98	−0.14	−0.05	−0.05	−0.03	0.30	−0.39	1.21	0.52
1-50-F	0.36	0.47	−0.22	0.03	0.39	0.55	1.12	0.36	1.08	0.29

[1] Second degree adjustment is indicated by the letter A after the number.

[2] Vertical control points used to adjust strip elevations.

Acknowledgments

The help of the following listed people in e work reported here is acknowledged: oseph C. Knittel, Continental Engineers, c., Denver, Colo. for providing photographic aterials and ground control, and drilling hotographic plates; Jack Friedman, OMI orp. of America, Alexandria, Va., for use of e Nistri Monocomparator; Robert Larson d Douglas Reid, Automatic Data Processing ection, Region 15, BPR; William T. Pryor, hief, Aerial Surveys Branch; Donald N. haw, Aerial Surveys Branch; and Robert J. arren, Region 9, BPR.

REFERENCES

(*1*) *Analytic Aerotriangulation*, by William . Harris, G. C. Tewinkel, and C. A. Whitten, .S. Coast and Geodetic Survey Technical ulletin 21, July 1963.

(*2*) *Three-Photo Aerotriangulation*, by Morton eller and G. C. Tewinkel, U.S. Coast nd Geodetic Survey Technical Bulletin 29, eb. 1966.

(*3*) *Results of U.S. Forest Service Stereo- iangulation Bridging on Virginia Highway hotogrammetric Test Project*, by Clair L. rneson, Highway Research Record 65, hotogrammetry and Aerial Surveys, 1963, p. 73–85.

(*4*) *Some Notes on Stereocomparators*, by . A. Harley, The Photogrammetric Record London, England), vol. IV, No. 21, Apr. 963, pp. 194–209.

(*5*) *Analytical Aerial Triangulation*, by G. H. Schut, National Research Council of Canada, AP–PR–7, Sept. 1957 (processed).

(*6*) *A Method of Relative Orientation in Analytical Aerial Triangulation*, by E. H. Thompson, The Photogrammetric Record (London, England), vol. II, No. 8, Oct. 1956, pp. 145–153.

(*7*) *Recent Developments in Analytical Aerial Triangulation at the Ordnance Survey*, by D. W. G. Arthur, The Photogrammetric Record (London, England), vol. III, No. 14, 1959, pp. 112–138.

(*8*) *Analytical Aerotriangulation by the Direct Geodetic Restraint Methods*, by Hugh F. Dodge, Photogrammetric Engineering, vol. XXV, No. 4, Sept. 1959, pp. 590–595.

(*9*) *Mono versus Stereo Analytical Photogrammetry—Theoretical Considerations and Experimental Results*, by H. M. Karara, Symposium on Spatial Aerotriangulation, University of Illinois, Urbana, Ill., Feb. 28–Mar. 4, 1966 (processed).

(*10*) *The Reseau as a Means of Detecting Gross Lack of Flatness of Film at the Instant of Exposure*, by G. S. Robinson, The Photogrammetric Record (London, England), vol. IV, No. 22, Oct. 1963, pp. 283–286.

(*11*) *Aerotriangulation: Image Coordinate Refinement*, by Morton Keller and G. C. Tewinkel, U.S. Coast and Geodetic Survey Technical Bulletin 25, Mar. 1965.

(*12*) *Manual of Photogrammetry*, 2d ed., 1952, American Society of Photogrammetry, p. 321.

(*13*) *Aerotriangulation Strip Adjustment*, by Morton Keller and G. C. Tewinkel, U.S. Coast and Geodetic Survey Technical Bulletin 23, Aug. 1964.

(*14*) *Computation of the Height Deformation in Stereoscopic Models Caused by Distortion of the Photographic Image*, by G. H. Schut, The Canadian Surveyor, vol. XIV, No. 1, Jan. 1958, pp. 19–24.

(*15*) *On Photogrammetric Refraction*, by A. Leyonhufvud, Photogrammetria vol. IX, No. 3, 1952–1953, pp. 93–113.

Linear Shrinkage Test

BY THE OFFICE OF
RESEARCH AND DEVELOPMENT
BUREAU OF PUBLIC ROADS

Reported by GEORGE W. RING I,
Highway Research Engineer,
Structures and Applied Mechanics Division

Introduction

RESULTS OF a study of the linear shrinkage test for soils are presented in this article. In another article, *Shrink-Swell Potential of Soils* (*1*) [1] linear shrinkage was shown to be closely related to the shrink-swell potential of a soil. The purpose of the study reported herein was to develop a modified linear shrinkage test procedure whose results would correlate even better with shrink-swell potential. Although a procedure for a better correlation was not obtained from the study, results showed how certain variations in test procedure affect the linear shrinkage of soils. The research information has been assembled for this article primarily for engineers interested in the linear shrinkage test.

Included is a brief review of three published linear shrinkage test procedures: Bureau of Public Roads test data, showing the variables that affect the linear shrinkage, including a quantitative amount of the variation; and a comparison of two correlations developed with shrink-swell potential. The linear shrinkage test procedure used for the *Shrink-Swell Potential of Soils* study is printed at the end of this article.

There are several published methods for conducting linear shrinkage tests. In one, *Determining the Shrinkage Factors of Soils*, AASHO T 92–60 (*2*), the linear shrinkage is calculated from the volumetric shrinkage of a circular disk dried from the field moisture equivalent. However, the field moisture equivalent is seldom determined in highway laboratories. Also, preliminary investigations in this study showed that for specimens molded at the field moisture equivalent, the volumetric shrinkage is very dependent on the molding moisture content. For these reasons, the AASHO method was not considered further.

In another method, the Texas Highway Department's linear shrinkage test—bar method—(*3*), the soil is molded into a ¾- by ¾- by 5-inch bar when the soil is just wet enough to flow into and close a groove of its own accord. Linear shrinkage is the decrease in length of the dried bar expressed as a percentage of its original length.

A third method, similar to the Texas method, has been proposed by Hondros (*4*). In this method the soil is mixed with enough water to provide a free-water surface and placed into a mold 10 inches long. This mold is shaped like one-half of a tube that has a radius of one-half inch. The Hondros test procedure has been used primarily to determine linear shrinkage of the fines in lateritic gravels of Australia.

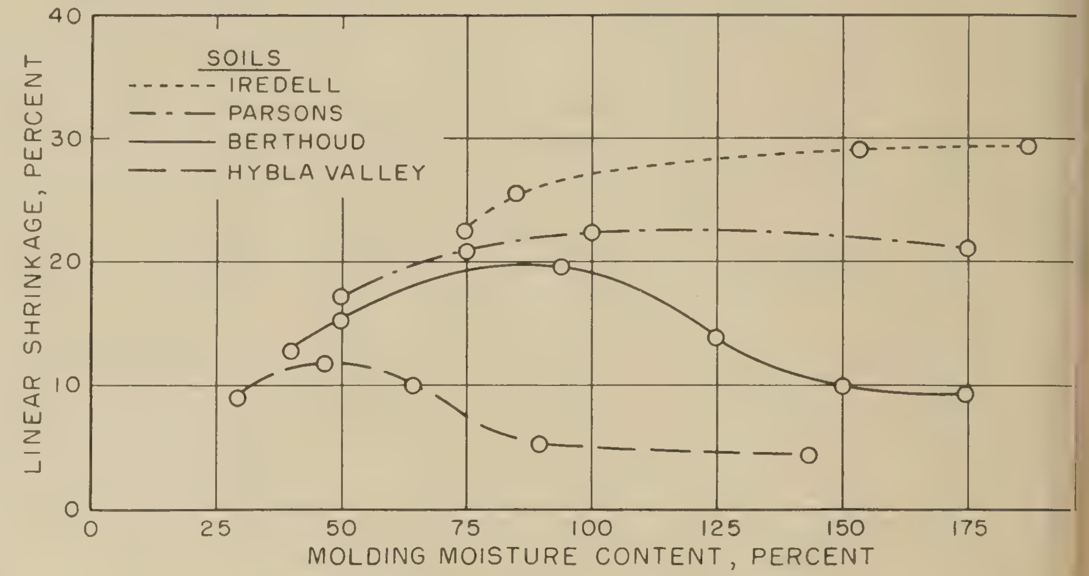


Figure 1.—Relation of linear shrinkage to molding moisture content of four soils.

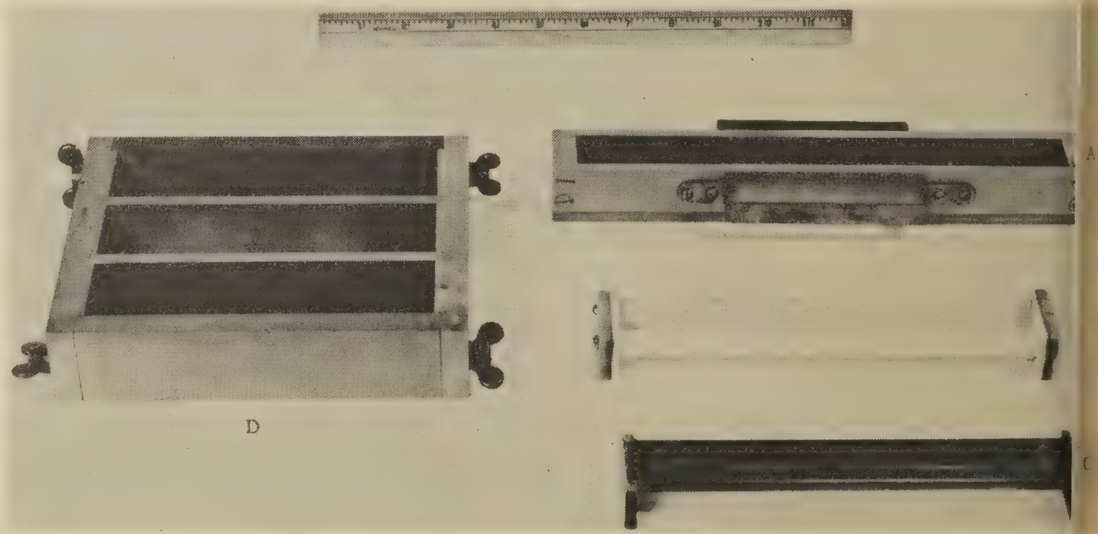


Figure 2.—Four types of linear shrinkage molds: A, steel and a square cross section; B, Teflon, semicircular; C, brass, semicircular; D, steel, square cross section.

From observations of linear shrinkage specimens molded at high moisture contents (1½ to 2 times the liquid limit) as in the method proposed by Hondros (*4*), it was noted that the specimens did not begin to shrink in the direction of their long dimension until enough water had evaporated for capillary forces to exceed the strength of the soil. The point at which shrinkage began, then, was not only a function of the soil's capacity to hold water but also the soil's clay activity and the resisting strength of the soil structure. In hopes that results of a linear shrinkage test procedure based on such high molding moisture contents would be more closely related to shrink-swell potential, tests were made on several soils to determine how best to obtain the maximum linear shrinkage for each soil. Some of the same 12 soils examined in the shrink-swell potential study (*1*, table 3, p. 100)

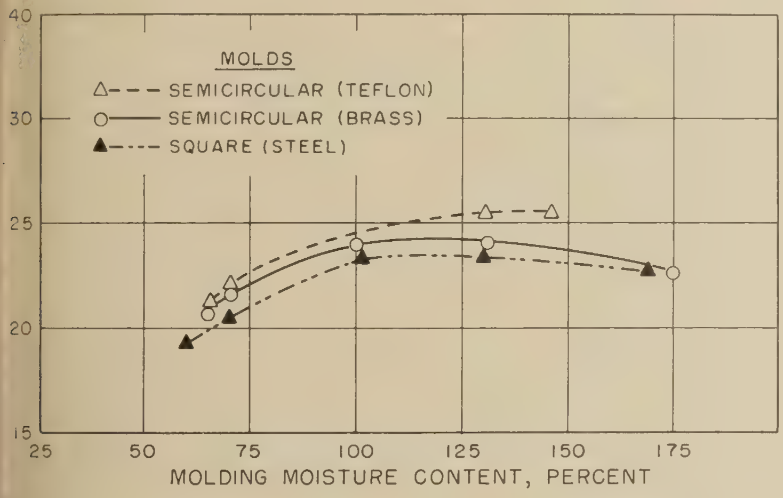
---

[1] References identified by italic numbers in parentheses are listed on page 165.

re used to determine the effect of the differ- variables on linear shrinkage. For most the testing, specific soils were selected to ain the probable maximum effect of a en variable on linear shrinkage. For ex- ple, a commercial grade halloysite is known be difficult to mix. Therefore, halloysite s used to determine the difference in linear inkage of test specimens prepared by hand xing and mechanical mixing. Except where ed, the soils were mixed with distilled ter. When one variable was studied, all other variables were held constant. Vari- es investigated included molding moisture tent, type of mold, mold lubricant, mixing thods, drying temperature, flocculation, d curing time of the mixture.

Molding Moisture Content

Linear shrinkage of some soils is affected by molding moisture content, as illustrated the curve in figure 1. Molding moisture tents for each soil tested ranged from below the liquid limit to saturation, which was considered to have been reached when free water was visible. In general, as molding moisture content is increased, the linear shrinkage increases to a maximum and then decreases. At high molding moisture contents, the linear shrinkage of two of the soils (Berthoud and Hybla Valley) was sharply reduced because of separation of the sand and clay-size particles, the sand settled to the bottom of the mold and the clay left on the top of the specimen curled and cracked badly. Elimination of this problem by flocculation of the soil moisture is discussed subsequently.



gure 3.—*Relation of type of mold to linear shrinkage of Winterset soil.*

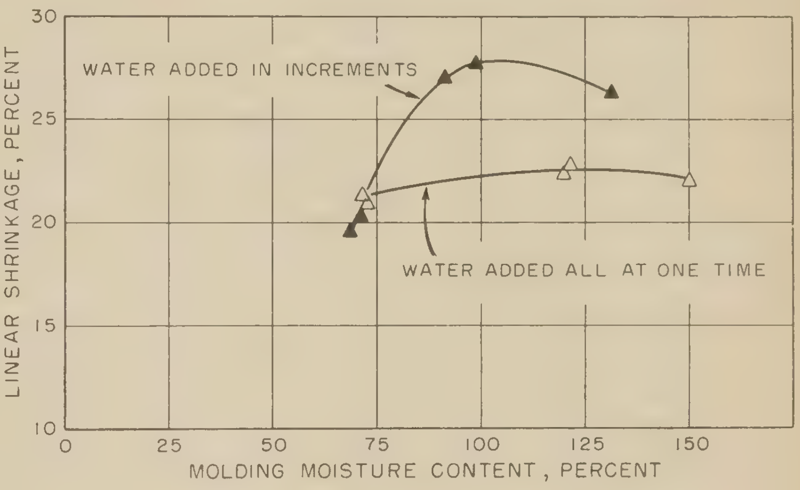


Figure 5.—Effect of mixing procedure on linear shrinkage of Winterset soil.

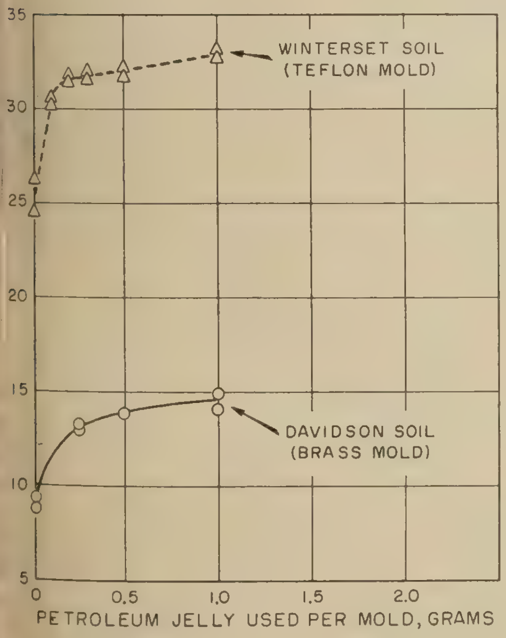
Type of Mold

Linear shrinkage molds in current use vary in cross section, length, and material. Some types of these molds are shown in figure 2. Mold A of steel has a square cross section; mold B of Teflon is semicircular; mold C of brass is semicircular; and mold D of steel has a square cross section for three specimens. Results of tests made in the three single molds, A, B, and C, with one soil show that the type of mold has a slight but noticeable effect on linear shrinkage, as shown by figure 3. The most linear shrinkage occurred when the semicircular mold made of Teflon was used, probably because Teflon has less affinity for water than for brass or steel. The Teflon mold also had a smaller surface area than the steel mold having the square cross sections. Limited investigations indicated that mold length has little or no effect on linear shrinkage test results but that specimens in the longer molds tend to crack more. Specimens molded at lengths longer than 10 times their diameters are difficult to measure after they are dry because they crack into many pieces.

Mold Lubricant

During drying, linear shrinkage specimens sometimes curl and crack, especially long specimens molded at high initial moisture contents. The broken and curled pieces are difficult to measure. Breaking and curling can be reduced by lubricating the mold. Petroleum jelly generally is used as a lubricant. Four other lubricants investigated in an attempt to find one more efficient than petroleum jelly were: Chassis grease, silicone release agent, Molycote type M, and Siliclad solution. None of the lubricants caused an increase in linear shrinkage or less cracking of the specimens than petroleum jelly, which is cheap and easy to obtain. Petroleum jelly was therefore used in all other tests.

It was determined that the linear shrinkage was dependent on the amount of lubricant. As shown in figure 4, the linear shrinkage of two soils increased considerably as the amount of petroleum jelly lubricant was increased from 0 to 0.25 gram per mold. Surface area of each of the molds was 16.5 square inches. A small, but consistent increase in shrinkage occurred when the amount of jelly used was increased over the initial 0.25 gram; however, 0.30 gram was selected as the proper amount of lubricant for all subsequent tests.



igure 4.—*Effect of amount of lubricant on mold on linear shrinkage of two soils.*

Mixing

Soil-water mixtures for linear shrinkage tests are generally prepared by hand mixing. As shown in figure 5, when all the dry soil was added to all the water at one time, less shrinkage occurred than when the water was gradually added to and thoroughly mixed into the soil.

When a milkshake machine was used to mix the soil and water, more linear shrinkage occurred than when the hand method was used for mixing. The effect of mixing methods on the linear shrinkage of a halloysite and Iredell clay is shown in figures 6 and 7. The halloysite required about 15 minutes of mixing with the milkshake machine to minimize the differences that might occur as a result of the mixing method, but the Iredell clay required only 5 minutes. The difference in the test results of duplicate tests is not readily understood.

Drying Temperature

Linear shrinkage specimens put immediately into a drying oven at 110° C. tend to

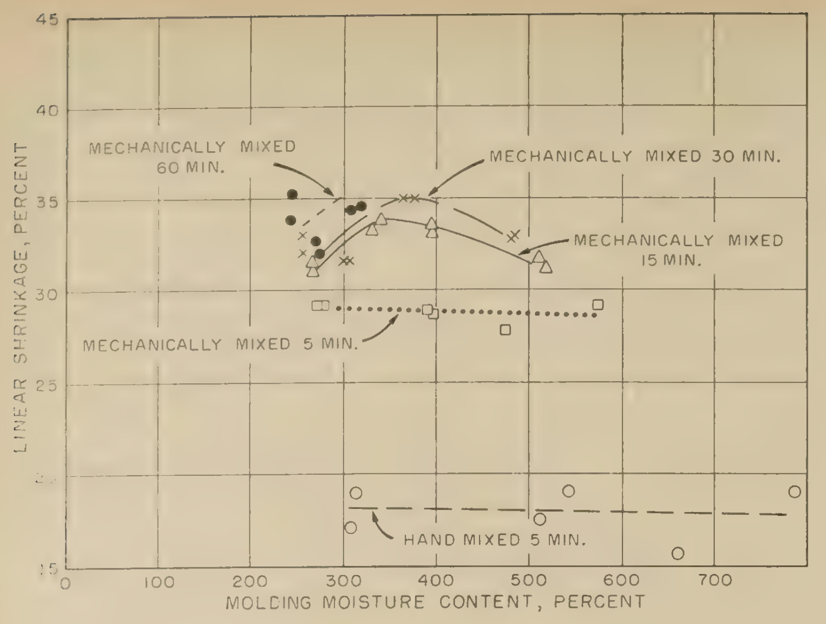


Figure 6.—Effect of mixing method on linear shrinkage of Halloysite soil.

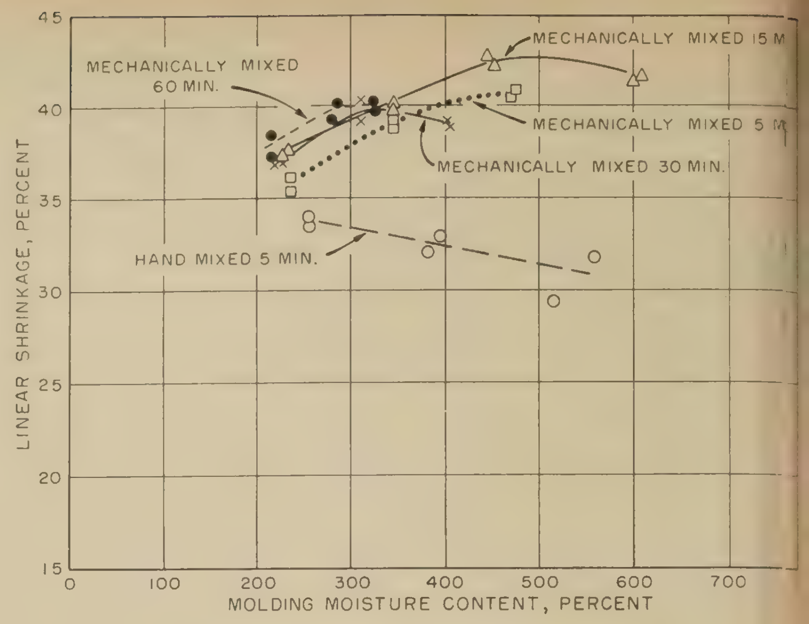


Figure 7.—Effect of mixing method on linear shrinkage of Irell clay.

crack more than those first dried at room temperature. Tests on two soils also indicate that for specimens in lubricated molds, the most linear shrinkage occurs when the initial drying temperature is between 60° and 80° C., as shown in figure 8. Results from tests made on one soil in unlubricated molds show that part of the change in linear shrinkage is caused by a change in the soil-water interaction at different temperatures. In the lubricated molds, other variations in linear shrinkage test results may have been the effect of different drying temperatures on the viscosity of the petroleum jelly. Although these tests indicate that 60° to 80° C. is the best temperature to use for oven drying, the linear shrinkage at 110° C. is not much less than at 60° to 80° C., and some laboratories may prefer to use the higher temperature so that the same oven can be used for other purposes.

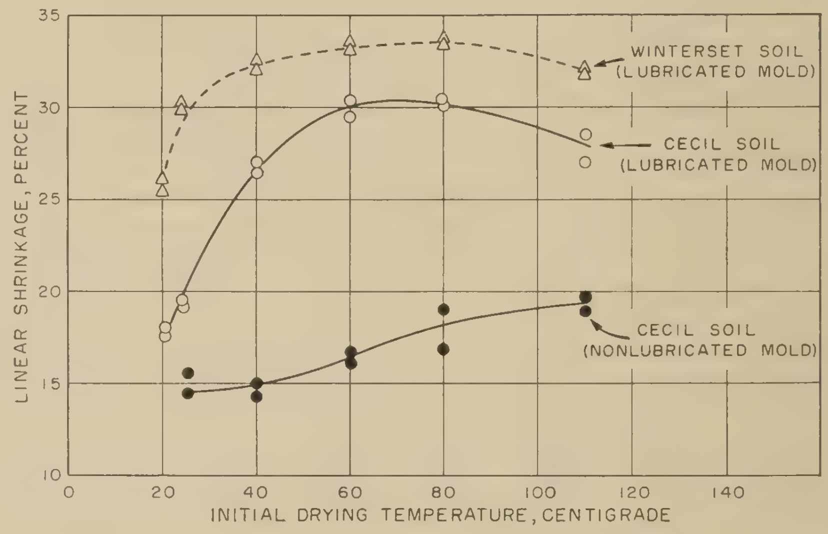


Figure 8.—Effect of drying temperature on linear shrinkage.

Type of Liquid

As mentioned in the section on molding moisture content, the sand particles in linear shrinkage specimens molded at high moisture content tended to settle out and a thick layer of clay was left at the top. The result was a linear shrinkage specimen that had a crumbly sandy bottom and a badly cracked top so that corresponding reductions occurred in linear shrinkage. It was reasoned that if the soil mixtures were flocculated, the separation of the different soil particle sizes might not occur, and a uniform and easily measured specimen would be obtained.

Some salts and acids are good flocculants for soil-water mixtures. Accordingly, tests were made first to determine a good flocculating agent and then to determine whether flocculation with this agent affected the test results adversely.

A series of linear shrinkage tests was made on the Parsons soil using different normal concentrations of $CaCl_2$, NaCl, HCl, and triethylamine hydrochloride (TEA). Soil mix-

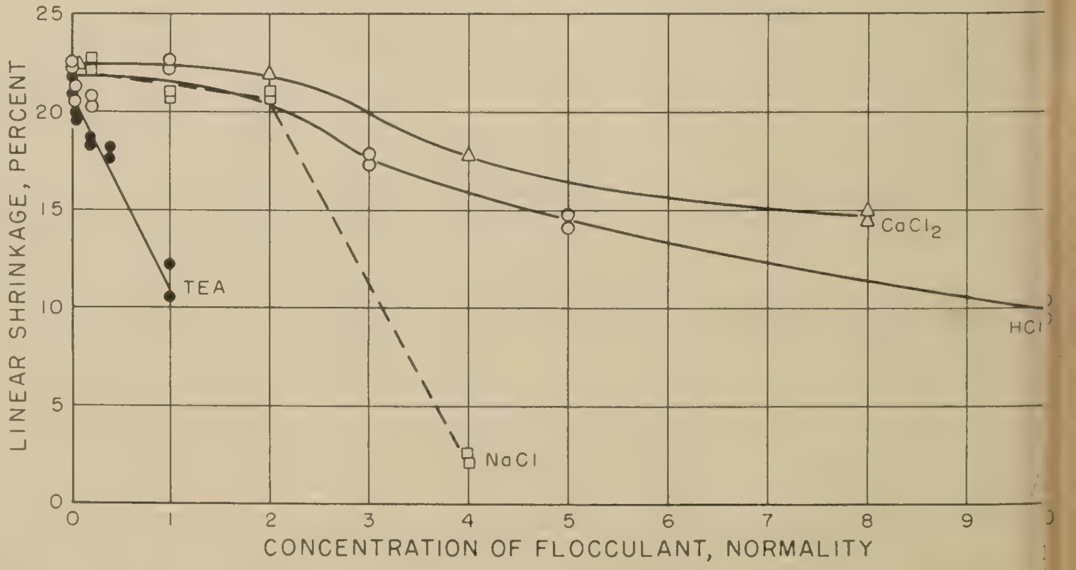
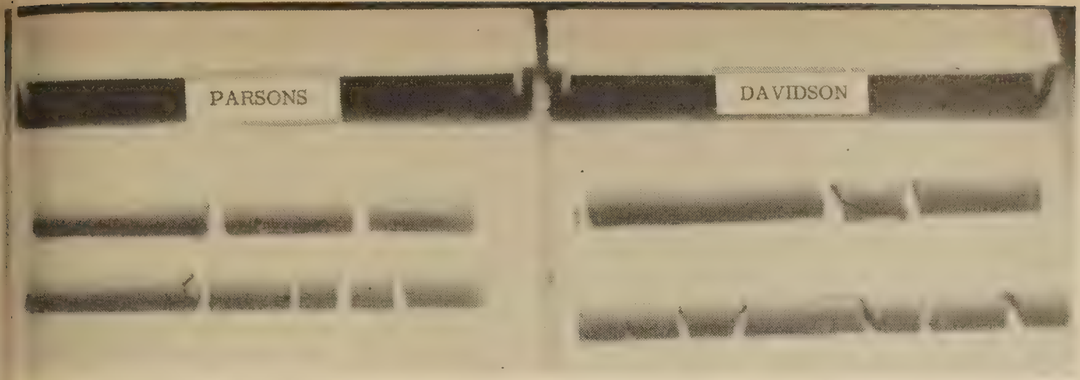


Figure 9.—Effect on linear shrinkage of Parsons soil when different concentrations of ur flocculating agents were used.



ure 10.—Cracking shown for two soils mixed with (upper) and without a flocculating agent.

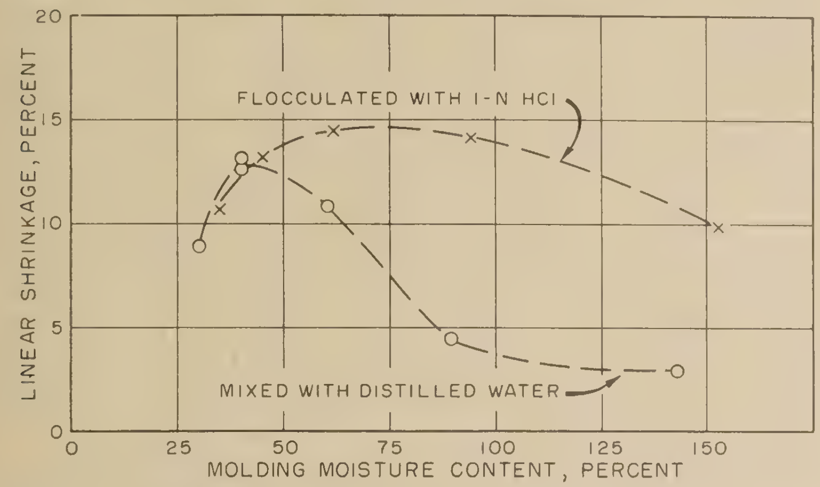


Figure 11.—Effect of flocculation on linear shrinkage of Hybla Valley clay.

Table 1.—Summary of variables that affected results of linear shrinkage tests

Variable and soil tested	Most change in linear shrinkage with change in variable	Maximum linear shrinkage	Change in linear shrinkage
Molding moisture content:	*Percent*	*Percent*	*Percent*
Winterset	5.3	22.5	+24
Hybla Valley	4.0	14.5	+28
Type of mold, steel, brass, and Teflon:			
Winterset	1.5	24.5	+6
Amount of lubricant:			
Davidson	4.7	13.9	+34
Winterset	7.3	32.0	+23
Mixing:			
Hand procedure:			
Winterset	5.4	27.6	+20
Milkshake vs. hand procedure:			
Halloysite	16.0	34.0	+47
Iredell	11.0	42.5	+26
Initial drying temperature:			
Cecil	12.7	30.3	+42
Winterset	7.5	33.5	+22
1-Normal solution of HCl vs. distilled water:			
Hybla Valley	10.3	13.8	+75
Portneuf	1.0	5.0	[1] −20
Winterset	2.0	18.0	[1] −6
Curing time:			
Halloysite	4.0	33.5	[2] −12
Iredell	1.0	43.4	[2] −2

[1] Flocculated specimens had less linear shrinkage than specimens prepared with distilled water.

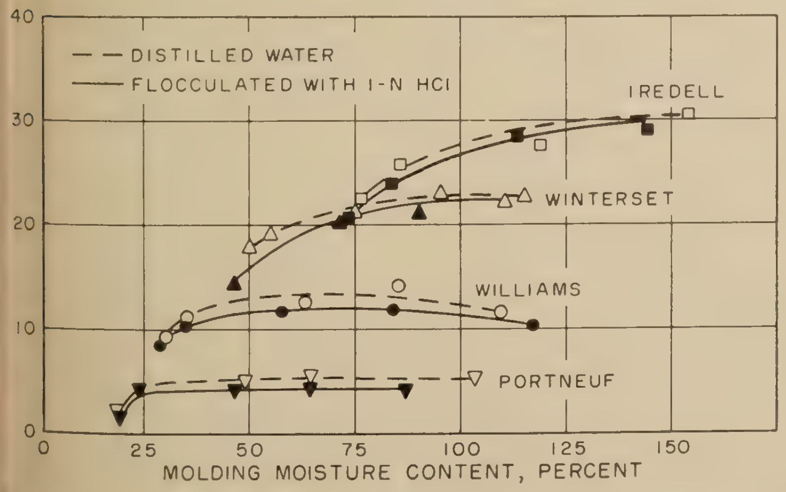
[2] Cured samples had less linear shrinkage than uncured samples.

t es containing the calcium and sodium sts and the hydrochloric acid all showed a rked reduction in linear shrinkage at conctrations above 2-Normal, as shown in fire 9. The TEA was not judged suitable bause of the reduction in linear shrinkage tt occurred even at low concentrations. e two salt solutions were not suitable bause the salt crystalization in the dried scimens produced rough specimens and de moisture measurements difficult. The l solution was deemed the most suitable fculant and a 1-Normal solution was used c tests with the 11 other soils.

The HCl solution reduced specimen cracking for all soils tested and thereby simplified the process of making length measurements. Examples of the reduction in cracking for the Parsons and Davidson soils are shown in figure 10. Specimens shown at the top of the figure were mixed with the 1–N HCl solution and those at the bottom were mixed with distilled water. Preventing separation of the clay and sand sizes by flocculation also increased the linear shrinkages at high molding moisture contents, as shown in figure 11. Comparisons of flocculated specimens to those mixed with distilled water are shown in figure 12 for four other soils. For these soils, which contained few sand size particles, the flocculated specimens generally had slightly less linear shrinkages.

Curing Time

After being mixed, samples sometimes were permitted to cure, either purposely or because of delay caused by other laboratory operations, before they were placed in the shrinkage molds. Tests on two soils showed that curing time had a small effect on linear shrinkage. Figure 13 shows that the linear shrinkage



igure 12.—Linear shrinkage of flocculated and nonflocculated soils.

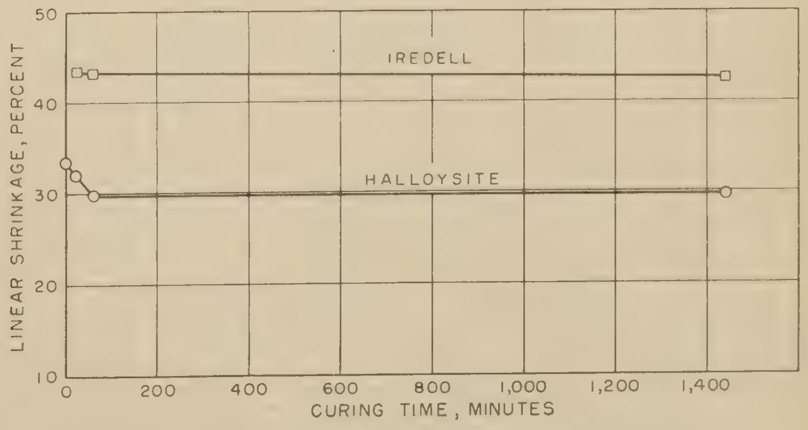


Figure 13.—Effect of curing time on linear shrinkage.

of halloysite was reduced at curing times of up to 60 minutes. The change in linear shrinkage of the Iredell clay in relation to curing time was hardly noticeable.

Summary of Study of Variables

All the variables investigated had an effect on the linear shrinkage of the soils tested. A summary of the variables and a quantitative measure of the amount of the variation is given in table 1. The amount of the variation was surprisingly large, especially for the mixing procedure, flocculation of sandy clays, amount of lubricant, and initial drying temperature. However, these variations were determined at high molding moisture contents and it is likely that specimens molded near the liquid limit would show less variation caused by mixing procedure and flocculation of sandy clays.

Based on the results of this detailed study, a modified linear shrinkage test procedure was devised in which each of the variables investigated was controlled to obtain maximum linear shrinkage for each soil. It was hoped that the maximum linear shrinkage values obtained by this modified procedure would be more closely related to the shrink-swell potential results than the linear shrinkage test results obtained by the test procedure used in the original study (*1*). Eleven of the twelve soils used in the original study were tested by the modified procedure discussed in the following paragraphs.

A 75-gram sample of air-dry soil passing the No. 40 sieve was mixed 15 minutes in a milkshake machine with enough distilled water for the mixer to easily stir the mixture. (Some of the sandy soils were mixed with 1-Normal HCl solution to prevent separation of the sand and clay particles). The mixture was poured into Teflon molds, 20.0 cm. long by 2.54 cm. diameter (semicircular cross section) that had been previously lubricated with 0.3 gram of petroleum jelly. Drying was done in an oven at 70° C. ± 5° C.

The test results were disappointing. The linear shrinkage for each soil was more than in the original study (*1*), as shown in figure 14. However, the correlation of these maximum linear shrinkage values with shrink-swell potential was not quite as good as was obtained from specimens molded at the liquid limit. No further efforts were made to improve the correlation. The correlation results are reported here primarily to save the time and effort of other researchers who mi consider trying the same approach tow improving the correlation. Also, the qual tive and quantitative measure of varial affecting linear shrinkage test results m prove useful to other researchers investigat the linear shrinkage test.

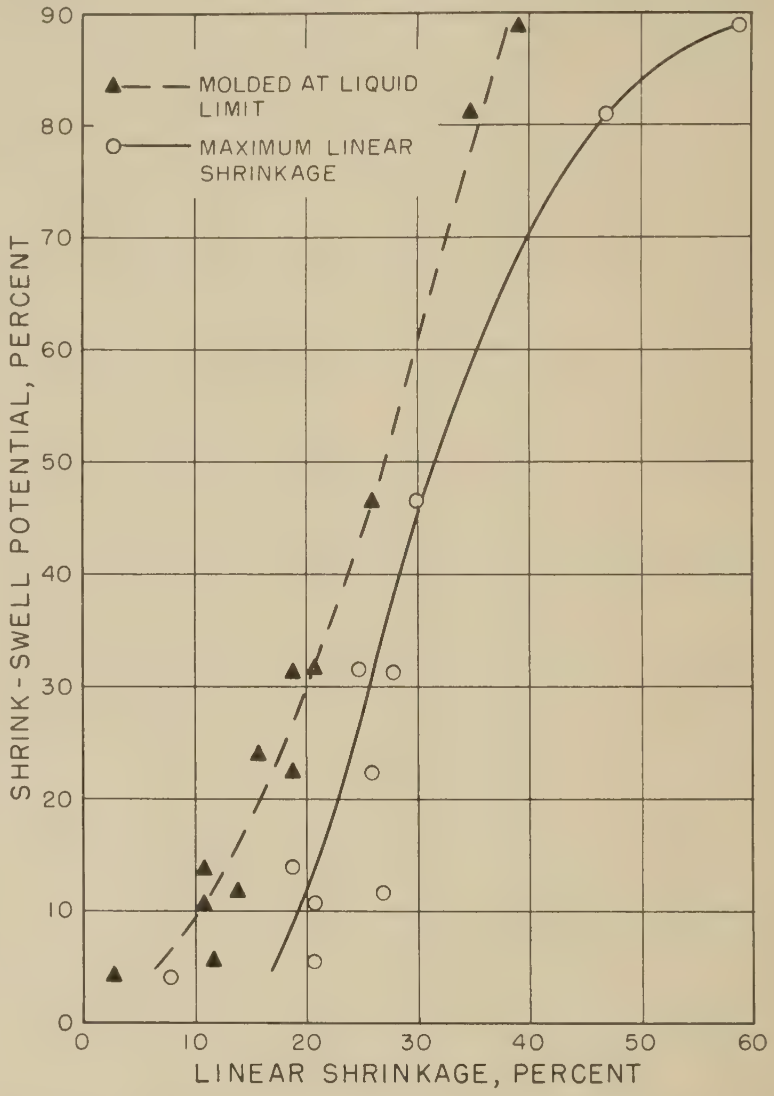


Figure 14.—Relation of linear shrinkage results obtained from two test procedures to shrink-swell potential.

Suggested Test Method for Determining the Linear Shrinkage of Soils

Definition

The linear shrinkage of a soil is the change in length of a bar of soil determined in accordance with the procedure outlined in the following paragraphs.

Apparatus

Apparatus needed for the test is:

- Linear shrinkage mold—a Teflon mold 20 cm. long and having a semicircular cross section 2.54 cm. in diameter.
- Commercial petroleum jelly.
- Distilled water.
- Evaporating dish, about 4½ inches in diameter.
- Balance, 500-gram capacity, sensitive to 0.1 gram.
- Spatula, having a blade about 3 inches long and about three-fourths inch wide.
- Drying oven, 70° C. ± 5° C.
- Scale, length 30 cm. graduated to one-half mm.

Sample

A sample of air-dry soil weighing about grams shall be taken from the thorou mixed part of the material passing the N (420-micron) sieve.

Procedure

The soil sample shall be placed in evaporating dish and thoroughly mixed 45 to 60 cc. of distilled water by altern and repeatedly stirring, kneading, and g with a spatula. Additions of water ill be made in increments of 3 to 8 cc. til the soil is at or slightly above its liquid it (see AASHO T 89–60). Each increment water shall be thoroughly mixed with the il, as previously described, before another rement of water is added.

The mixture shall be placed in a linear inkage mold that has been previously icated with 0.30 gram petroleum jelly. er firmly pressing the mixture into the ld with the spatula, excess material shall removed by trimming the mold with the aight edge of the spatula.

The mold containing the mixture shall be ced in an oven at 70° C.±5° C. for 16 hours until constant weight has been obtained.

Note 1: When the oven must be set at)° C. for other soil tests, linear shrinkage ecimens may be dried at this temperature. However, the higher temperature will cause more cracking of the specimen and slightly lower test values.

The soil specimen shall be removed from the oven, allowed to cool, and then the length of both its top and bottom measured.

Note 2: For broken specimens, the lengths of the individual pieces should be marked and accumulated on a strip of paper; the total length can be determined directly by measuring the end points on the strip.

Calculation

The linear shrinkage of the specimen shall be expressed as follows:

Linear shrinkage (in percent)=

$$\left[\frac{\text{Mold length}-(\text{top}+\text{bottom length of dried specimen})/2}{\text{mold length}}\right]\times 100$$

REFERENCES

(1) *Shrink-Swell Potential of Soils,* by George W. Ring III, Public Roads, a journal of highway research, vol. 33, No. 6, Feb. 1966, pp. 97–105.

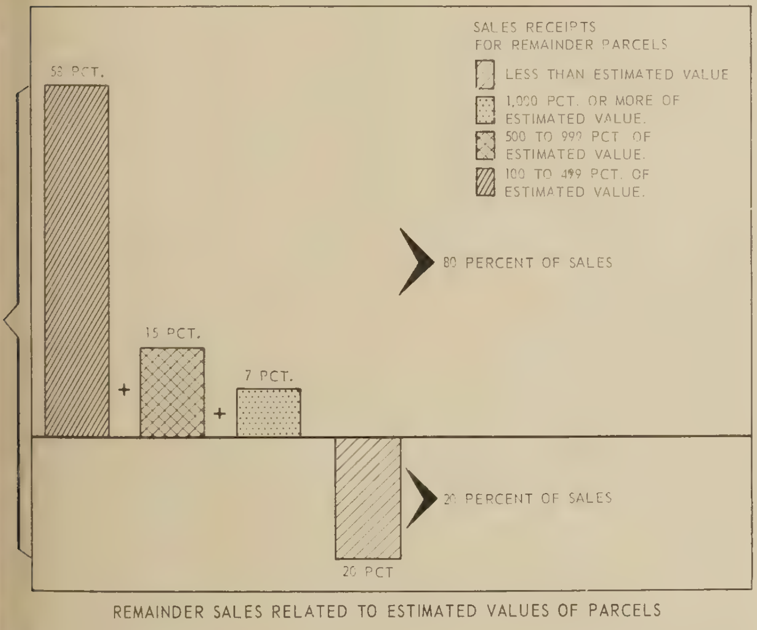
(2) *Determining the Shrinkage Factors of Soils,* AASHO Designation: T 92–60, Standard Specifications for Highway Materials and Methods of Sampling and Testing, Part II. Methods of Sampling and Testing, The American Association of State Highway Officials, 1961, pp. 295–298.

(3) *Method of Determining the Shrinkage Factors of Soils,* THD-68, revised Item 5, Linear Shrinkage (Bar Method), by Texas Highway Department, Soil Testing Procedures, Materials and Test Division, 1953.

(4) *Developing a Procedure for Rapid Field Soil Testing and Classification,* by G. Hondros, Commonwealth Engineer, vol. 45, No. 4, Nov. 1, 1957, pp. 50–54.

Errata

Data shown in two figures in the article, *Severance Case Studies—idging the Gap Between Findings and Their Application*, by Floyd I. iel, which appeared in the April 1967 issue of Public Roads, a rnal of highway research, vol. 34, No. 7, were in error. Corrected figures 4 and 8 are reprinted here. Please make the substitution for figure 4 on page 147, and for figure 8 on page 149, of the April magazine.



gure 4.—Remainder sales and estimated values, based on 2,262 narrative reports on severance studies.

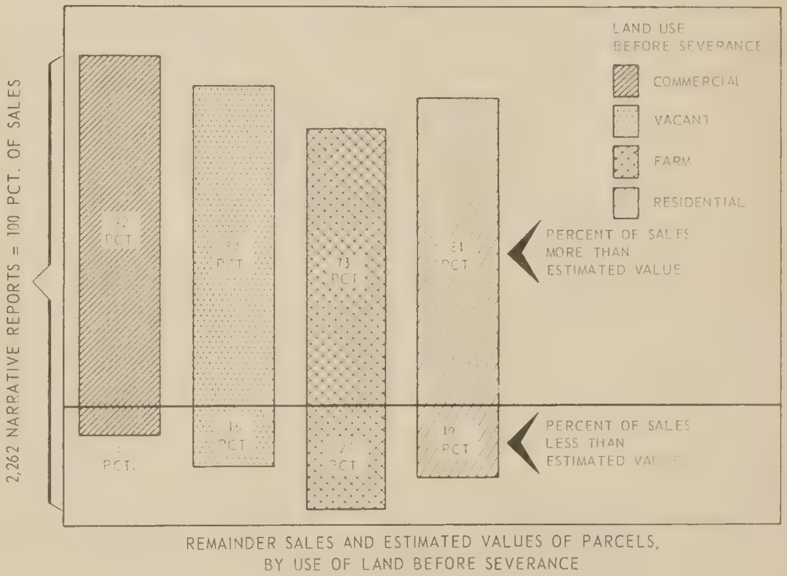


Figure 8.—Effect of land use before severance on remainder sales and estimated values, based on 2,262 narrative reports on severance studies.

Design Use of the Capacity Manual

BY THE OFFICE OF
ENGINEERING AND OPERATIONS
BUREAU OF PUBLIC ROADS

Reported by [1] DONALD W. LOUTZENHEISER, Chief,
Highway Standards and Design Division,
WILLIAM P. WALKER, Chief, Geometric Standards Branch,
and DONALD B. LEWIS, Highway Engineer

The authors of the report presented here believe that the highway designer should make use of the Highway Capacity Manual because it makes possible a greater knowledge and understanding of all factors related to highway types, speeds, volumes, and operations than any other source. Use of the Manual is necessary for certain analyses, particularly for ramps and intersections at grade, but it provides broad ranges of values only, and does not furnish the designer with a specific value for use in actual design. However, the essential working data for most design analyses are available in condensed, usable form in the AASHO Policy on Geometric Highway Design (Blue Book). The Blue Book also gives national policy guidance on all main factors, except the new intersection data. For design use, the authors suggest the use of a peak hour factor of 0.85 for most cities, and a load factor of 0.3. The factors provide the missing guidance for intersection design. The authors advise that the AASHO Blue Book be used as the source for policy guidance and as a major tool for carrying out capacity analysis within the areas covered and that the Manual be used to supply supplemental information for design analyses.

Introduction

WHEN THE SWEDISH astronomer, Anders Celsius, presented the Centigrade scale for temperature measurement to the Swedish Academy of Science some two and a quarter centuries ago, he gave to the scientific world a device marked by simplicity, utility, and lasting endurance. Celsius arrived at his centigrade scale by dividing the temperature range for the liquid state of water under standard conditions into 100 equal parts. But, Celsius did not concern himself with the biologic effects that water at different temperatures would have on the human body, nor did he discover, deduce, or decree that water at 60° C. would destroy human tissue, or that a water temperature of 20° to 30° was most comfortable for body immersion, or that a human is not likely to survive longer than 2 hours if immersed to his chin in water at 5° above freezing. Neither is he credited with any discovery of the effect that variations from the standard condition, such as admixtures or changes in atmospheric pressure, might have on the boiling point or freezing point of water. These findings were left for scientists in other fields of interest.

In like manner, the Highway Capacity Committee of the Highway Research Board has given the highway engineering profession a valuable instrument, the *Highway Capacity Manual, 1965* (hereafter referred to as *Manual*), which gives a scale of measurement for highway operating conditions and deals with the effects of variations from standard conditions. The *Manual* scale used to measure the operating conditions that exist with different traffic loads is as simple as the Celsius centigrade scale. The range in possible highway operating conditions is divided into five, unequal quantitative parts and the parts are ranked from superior, at negligible traffic volume, to intolerable, at full capacity load. These quantitative parts are called levels of service and are lettered A through E, from good to bad. The Highway Capacity Committee attempted to give an adjective-type description of each level of service and avoided any recommendation as to how good the service provided by the highways should be. This was done in recognition of the fact that the level of service is governed as much by economics as by the desires of highway users. Therefore, the *Manual* is a valuable research report but it is not a national policy or guide that can be used directly for design.

The *Manual* must be supplemented by design guidelines before it can be applied in practice. The *AASHO Policy on Geometric Design of Rural Highways, 1965* (referred to as Blue Book) provides most of the needed guidance. The authors of the report presented in this article advise that this Blue Book is the essential handbook for the highway designer and the *Manual* is a necessary academic supplement. However, as discussed later, the Blue Book does not contain all the needed data for design of ramps and intersections at grade. In the interest of simplicity, brevity, and practicality, the Blue Book has compromised with precision in many places where the AASHO Committee determined that the accuracy of the result would be within the limits of accuracy of the estimated traffic data upon which the design would be based.

Manual factors and procedures suggest accuracy of details in capacity analyses that may be beyond the practical need for design; nothing is gained in stressing such details that are inconsistent with the reliability of highway section data or the practical use of analyses conclusions. Design data from the Blue Book may be applied to computational procedures in the *Manual* to determine geometric dimensions that satisfy operational requirements for a given volume of traffic or to determine the level of service that an existing highway will afford any given volume of traffic. Examination of the material from each book will show the reasonableness of the design data and procedures.

Explanation of Terms

The *Manual* has been written with some terms that are new or somewhat different than those designers have been using. The single word *capacity* refers to the upper limit of possible capacity, as previously known. It also is defined as the maximum number of vehicles that can reasonably be expected to pass over a given section of a lane or a roadway in one direction, or in both directions for a 2-lane highway, during a given time period under prevailing roadway and traffic conditions.

Level of service denotes any one of an infinite number of different combinations of operating conditions that may occur on a given lane or roadway when it accommodates different traffic volumes. Each service level is a measure of the effects of operating factors such as speed, traveltime, freedom to maneuver, safety, operating costs, and so on. For uninterrupted flow, that is, where traffic movement is not interrupted by stop signs or signals, the levels of service are defined by operating speed related to the volume-to-capacity ratio. Application of this volume-

---

[1] Presented at the 46th annual meeting of the Highway Research Board, Washington, D.C., Jan. 1967.

capacity ratio to the 2,000 vehicles per ur (v.p.h.) lane capacity for multilane hways gives service volume limits of aver- lane volumes:

erating speed and service volume for each evel of service for multilane highways ther than freeways

Level of service	*Operating speed, m.p.h.*	*Average lane volume, number of cars*
A	70–60	0– 600
B	60–55	600–1, 000
C	55–45	1, 000–1, 500
D	45–35	1, 500–1, 800
E	35–30	1, 800–2, 000

As generally used, a *service volume* is the ximum number of vehicles that can pass er a given section of lane or roadway, one ection on multilane highways or both direc- ns on 2-lane highways, during a specified ie period while operating conditions are intained at a specified level of service. *vice volume*, generally an hourly volume, he limiting volume for that level of service, l corresponds to the *design capacity* for determined level of service.

Average highway speed is the weighted erage of the design speeds within a highway tion. *Operating speed* is the highest overall ed at which a driver can travel on a given hway under favorable weather conditions l under prevailing traffic conditions, with- exceeding the safe speed used for design. the Blue Book, the data are shown in ms of the relation of design speeds to erage running speeds and in the *Manual*, relations are in terms of average highway eds and operating speeds; however, the erall data are essentially the same. Within limits of practical design, operating speed, used in the *Manual*, is the same as average nning speed plus 4 to 8 m.p.h., as used in Blue Book.

Peak hour factor (PHF) is the ratio of the lume of traffic occurring during the peak ur to the maximum rate of flow during a rt of the peak hour, usually 5 minutes for eration on freeways and 15 minutes on ersections. In the *Manual* this PHF is plicable to urban freeways at service els C and D and to intersections, but not rural freeways. A low PHF indicates a arp peaking characteristic within the hour. ie upper or theoretical unity value of PHF never attained in practice. Common HF values are about 0.8 to 0.9. *Load tor* at signal controlled intersections is the io of the total number of green signal ervals that are fully utilized by traffic proaching the intersection from one direc- n during the peak hour to the total number green intervals within the hour. Common d factor values within the design range 0.2 to 0.6.

Capacity values in the *Manual* are estab- hed in terms of maximum numbers of pas- ger cars per hour under ideal conditions. derive the values for highway capacity ere conditions are not ideal, adjustment tors are applied to the upper limit in a ies of downward corrections. Adjustment tors include lane width, lateral clearance, cks and grades, buses and grades, alinement, peak hour factor, traffic interruptions, passing sight distance, and load factor. However, of these, some factors are not applicable to all classes of highways. In the following discussion, only the maximum passenger car per hour values are cited. The fact that the adjustment factors need to be applied is not repeated because the application of adjustment factors always is the basic procedure. The same is true of the design capacity values cited from the Blue Book.

Uninterrupted Flow

Procedures are given in the *Manual* for the calculation of uninterrupted flow capacities on different types of highways: (1) Freeways and other expressways; (2) multilane highways having no access control; and (3) 2-lane highways.

Freeways and other expressways

Freeways and other expressways are controlled-access highways and have 4 or more lanes. Maximum service volumes—the design capacities—given in the *Manual* for the three levels of service within the framework of normal design, are listed in table 1. The average service volume per lane varies substantially in relation to the number of lanes. Depending on the PHF, the volumes of vehicles during any 1 hour may be considerably less than the maximum volumes quoted in table 1 for the levels of service. The *Manual* recommends that, for levels C and D, the maximum service volumes should not be exceeded during the peak 5 or 6 minutes of the hour. Also, the *Manual* notes a variation in service volume in relation to the alinement characteristics of the highway, expressed in terms of the average highway speed. The Blue Book covers the same variation in service volume by relating the average running speed to the design speed; this is a less precise relationship but one more convenient to use in design, where the average highway speed cannot be readily determined.

Because of the three major variables, number of lanes, peak hour factor, and average highway speed, service volumes may vary over a significant range up to the maximum values given in table 1. It is fortunate that in most of today's freeway design work, these three variables fluctuate within a rather narrow range because the designer probably would not have the necessary information on peak hour factor and average highway speed to make the precise calculations described in the *Manual*. Rounded values for design capacities of freeway lanes, which will fit most situations without appreciable error, are given in the Blue Book.

The average lane value of 1,000 vehicles per hour is applicable to rural freeways in level and rolling terrain according to the Blue Book. This value is within the range of level of service B as described in the *Manual* for 4-, 6-, and 8-lane freeways with 70 m.p.h. design characteristics. On such freeways the service volume will permit an operating speed of 55–60 m.p.h., corresponding to an average running speed of 45–50 m.p.h. The average lane value of 1,200 vehicles per lane per hour as given in the Blue Book is applicable for freeways approaching urban areas and those in mountainous terrain. With this volume of traffic most freeways, regardless of number of lanes, will provide level of service C, unless the design speed is below 60 m.p.h. and then level D will be achieved. The average lane value of 1,500 vehicles per lane per hour in the Blue Book is applicable for freeways in urban areas. This volume will result in level of service D in most instances. If the design speed is below 50 m.p.h. and the alinement is continuously winding, operation would fall slightly below level D. If the design speed were 70 m.p.h., an 8-lane freeway would provide a service slightly better than level D, but a 4- or 6-lane freeway would provide only level D. Some years ago it was concluded that freeways designed on the basis of these hourly values of traffic per lane would yield satisfactory service at reasonable cost for construction; experience has confirmed this conclusion.

Table 1.—Maximum service volumes for 3 levels of service within the normal design range

Level of service	Average maximum service volumes, 1-way		
	2-lane	3-lane	4-lane
	v.p.h.	*v.p.h.*	*v.p.h.*
B	2, 000	3, 500	5, 000
C [1]	3, 000 (2, 300)	4, 800 (3, 700)	6, 600 (5, 100)
D [1]	3, 600 (2, 800)	5, 400 (4, 150)	7, 200 (5, 600)

[1] Numbers in parentheses show the lower limit of service volumes under ideal conditions as governed by a peak hour factor of 0.77.

Multilane Highways Without Access Control

The maximum service volumes given in the *Manual* for multilane highways without access control are 1,000 vehicles per lane per hour for service level B, 1,500 vehicles for C, and 1,800 vehicles for D. These maximum service volumes are the same as the average volumes per lane given in the *Manual* for 4-lane freeways for the same levels of service. For levels C and D the operating speed is said to be about 5 m.p.h. lower than that for freeways having the same service volumes. For freeway operation the *Manual* data for levels C and D give different service volumes for different numbers of lanes and for fluctuations within the hour, that is, the peak hour factor. However, for multilane highways without control of access, the service volumes per lane are the same regardless of the number of lanes and there are no peak hour factor adjustments.

In the Blue Book the design capacities for rural multilane highways and rural freeways are the same except that rural multilane highway capacities are reduced by the interference from cross traffic and roadsides. The maximum design capacity of 1,000 to 1,200 vehicles per lane on rural multilane highways where there is little or no interference may be within level of service B or C, the same as freeways depending on the average running speed. The maximum design capacities of 700 to 900 vehicles per lane with moderate interference and 500 to 700 vehicles with considerable interference reflect the effect of unsignalized intersections at grade and roadside interference.

2-lane highways

The 2-lane highways have a restriction not existing on multilane highways, which is the result of the need for traffic occupying the opposing lane when passing. Therefore the percent of passing sight distance available along the route must be determined. Service volumes or design capacities are the total for both directions regardless of the distribution of traffic. The maximum service volume given in the *Manual* for level of service B is 480 to 900 vehicles in both lanes; for level C, 1,080 to 1,400 vehicles; and for level D, 1,600 to 1,700 vehicles. The range of service volumes reflects the effect of available passing sight distance, the higher volumes each time being applicable where sight distance is not restricted.

Maximum service volumes

The maximum service volumes for the levels of service given in the *Manual* correspond approximately with the maximum design capacities for the average running speeds given in the Blue Book. The maximum design capacity for an average running speed of 45 to 50 m.p.h. is 480 to 900 vehicles; for 40 to 45 m.p.h., 680 to 1,150 vehicles; and for 35 to 40 m.p.h., 1,110 to 1,500 vehicles. The indicated range of design capacities reflects the effect of available passing sight distance that is always applicable where there are no sight distance restrictions. The maximum design capacity for the 45 to 50 m.p.h. running speed is the same as the maximum service volumes of level B given in the *Manual*. This average running speed and corresponding design capacity is recommended for application in the design of most main rural 2-lane, 2-way highways in level and in rolling terrain. The maximum design capacity for the 40 to 45 m.p.h. average running speed corresponds approximately to level C. This average running speed is recommended for application in the design of 2-lane highways approaching urban areas and, wherever feasible, for 2-lane highways in mountainous terrain. The maximum design capacity for the 35 to 40 m.p.h. average running speed corresponds approximately to level D. This average running speed is recommended for application in the design of 2-lane rural highways in mountainous terrain where design for higher level of service is not feasible. The reduction of these design capacities by the several adjustment factors that are applicable to the service volumes in the *Manual* has been largely accomplished in a series of tables (tables II–8 through II–10) of the Blue Book.

Intersections at Grade

The method used in the 1965 *Manual* for determining the service volumes, as now termed, for approaches to intersections is essentially the same as used in the 1950 *Manual*. The source data for the most recent *Manual*, however, have been based upon much more research. New adjustments have been added, particularly the peak hour factor—fluctuation within the hour—, the load factor—loaded cycles within the hour—, and metropolitan area size. Level of service is defined in terms of selected load factors, that is, the larger the number of loaded cycles within the design hour, the poorer the level of service.

The Blue Book does not furnish capacity design values for intersections, but refers the reader to the *Manual*. The procedures and factors shown in the *Manual* should be used directly for design. But if design is to be on a uniform basis, the user of the *Manual* must have some guidance in selecting the peak hour factor and the load factor values. Studies should be made to determine local values for these factors for the type of intersection being designed. Where such data are not available general use of a peak hour factor in the range of 0.80 to 0.90 and a load factor of 0.3 is recommended. In cities of 1 million or more the PHF should be about 0.90 and for cities of less than 100,000 about 0.80. For cities within the range of 100,000 to 1 milllon a rounded value of 0.85 is suggested but the user is at liberty to make a more exact interpolation if he chooses. The load factor of 0.3 is that of level of service C, which should be a justifiable realm of design. With these guides, intersection capacity analysis follows the same general procedures and details as have been used in the past. Many designers found that the intersection capacity charts published in *PUBLIC ROADS, A Journal of Highway Research*, February 1951, made the analysis simple and assuredly complete. A similar set of revised charts is expected to be published in the August 1967 issue of this magazine.

Weaving Areas

The level of service afforded by a weaving section is a function of the number of lanes and of the length of the weaving area. The relation of length to level of service and service volume is shown in figure 7.4 of the *Manual*. The needed number of lanes is determined by an equation. The curves on the graph that illustrate the relationship of needed length of weaving section to the volume of weaving traffic are numbered for correlation to the level of service for different highway conditions. This correlation is shown in table 7.3 of the *Manual*. In determining the number of lanes needed for weaving sections, an equation is used in which the smaller of two weaving volumes is multiplied by a K factor. In the *Manual* the K factors are shown on the weaving chart. In the equation the service volume value (SV) is the service volume per lane used in determining the number of la
for the approach roadway.

Weaving values obtained from two cha
in the Blue Book (figs. IX–16 and IX–
agree with those from the *Manual* for the t
levels of service covered by the Blue Bo
But, the information is presented somewh
differently. The 40 m.p.h. curve in fig
IX–16 corresponds to curve III in the *Manu*
the 30 m.p.h. curve in figure IX–17 c
responds to curve IV in the *Manual*. In
Blue Book, K factors for use in the equat
for determining the number of lanes nee
for weaving sections are obtained from s
plemental charts.

The C value used in the formula in
Blue Book for determining the width of
weaving section is the design capacity
lane of the approach roadway. C values
1,000, 1,200, and 1,500 with appropri
adjustments are given in the Blue Book
rural, suburban, and urban respectively,
described in connection with freeways.
C value or design capacity per lane of
approach roadway, may also be used as serv
volume (SV) if the *Manual* equation is u
and the results obtained by the two meth
are the same.

Ramps

The capacity of either terminal or
capacity of the cross section of the ramp
control ramp capacities. Usually, ramp
pacity is controlled by the terminal a
Ramp terminals fall into three broad ca
gories, at-grade intersections, merging ar
and diverging areas. At-grade intersecti
may be solved by the procedures alrea
discussed. The *Manual* gives two procedu
for determining the capacity of merging
diverging areas, one for determining levels
service A, B, and C, the other for levels
and E. Usually design should provide
level C or better, therefore use of the f
procedure for design is appropriate.
maximum service volumes for lane 1, includ
ramp traffic and through traffic, used for le
of service B is 1,200 vehicles merging
1,300 vehicles diverging; and for level
service C is 1,700 vehicles merging and 1,
vehicles diverging. The volume of thro
traffic in lane 1 depends on a number
variables and may be determined by
series of nomographs (figs. 8.3 to 8.19)
equations included in the *Manual*.

The Blue Book recommends use of
equations and nomographs in the *Man*
It specifies the maximum volumes in lan
for use in design: 1,200 vehicles merging
1,300 vehicles diverging for rural highw
and 1,500 vehicles merging and 1,600 vehi
diverging for urban highways. These m
mum volumes correspond to the level
service B for rural areas and are within le
C for urban areas. The volumes as gi
in the *Manual* and Blue Book differ slig
in that the volumes given in the *Ma*
include 5 percent trucks on an assumed le
grade and the volumes given in the Blue B
are in terms of equivalent passenger car

NEW PUBLICATIONS

Highway Statistics, 1965

The 21st issue of the annual compilation of stistical and analytical tabular matter taining to Federal-aid for highways, *hway Statistics, 1965*, has recently been ued by the Bureau of Public Roads, leral Highway Administration, U.S. partment of Transportation. This pubtion may be ordered from the Superindent of Documents, U.S. Government nting Office, Washington, D.C. 20402, for $00 a copy, prepaid.

ighway Statistics, 1965, a 182-page pubtion, presents information, primarily in ular form, on motor fuel, motor vehicles, ver licensing, highway-user taxation, State l local highway financing, road and street eage, and Federal-aid for highways.

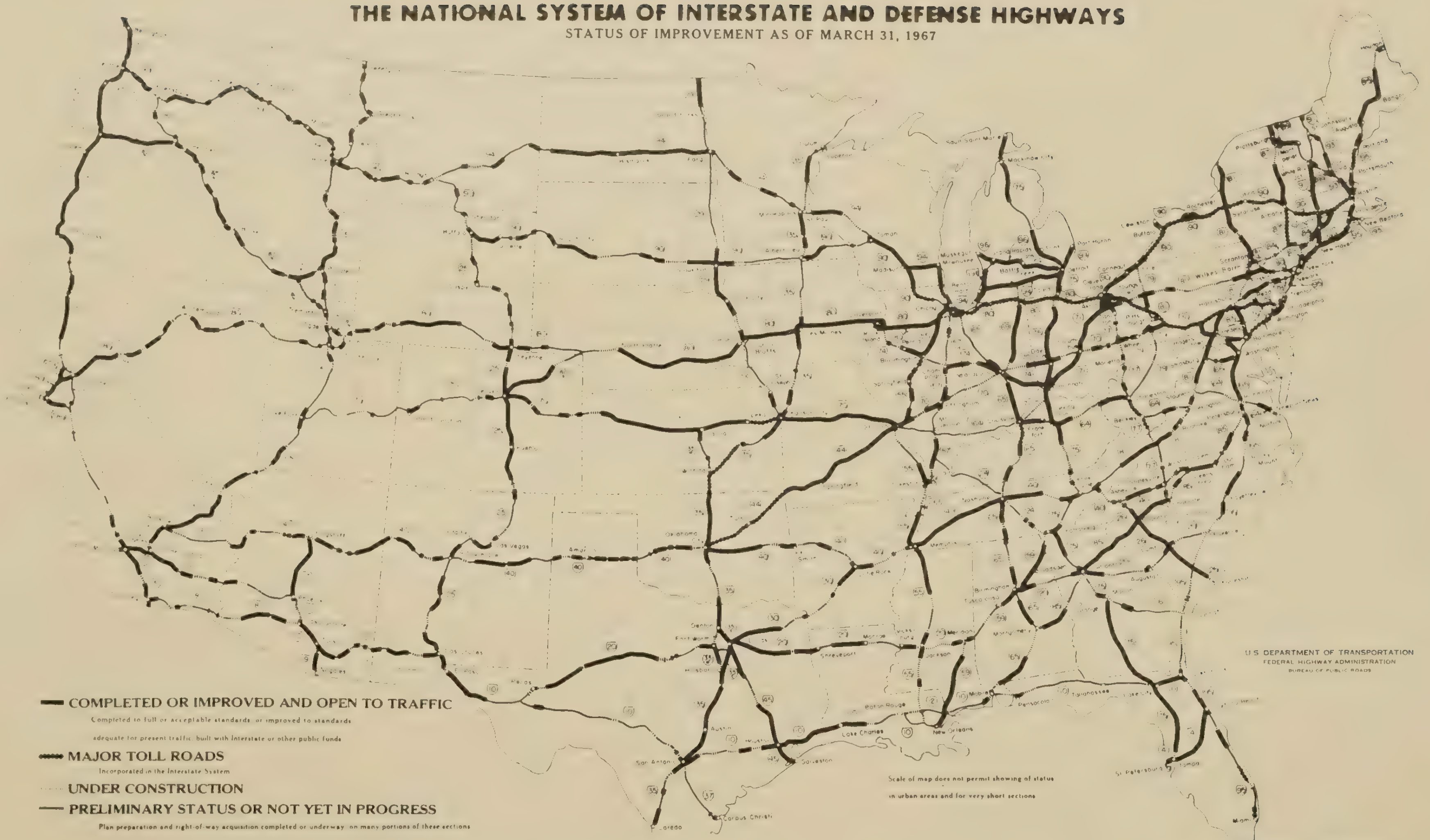
Highway Statistics, Summary to 1965

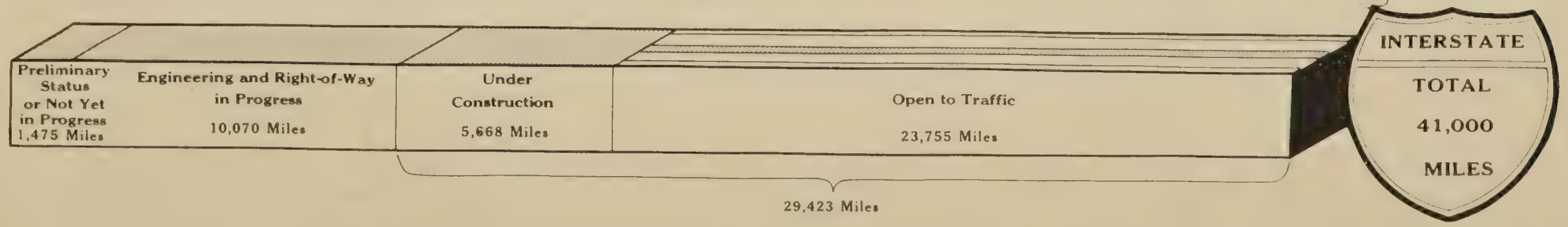
Highway Statistics, Summary to 1965, an historical summary of information on highways, their use, and financing has recently been published by the Bureau of Public Roads, Federal Highway Administration, U.S. Department of Transportation, and is available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402. The 178-page publication is a comprehensive statistical review of highway development in the United States through 1965, and it includes all information previously presented in the Highway Statistics summaries for 1945 and 1955, as well as information published in the annual issues of *Highway Statistics* through 1964.

Highway Statistics, Summary to 1965, contains statistical and analytical tables on four major subject areas. The data on motor fuel include analysis of motor-fuel consumption, tax rates, and tax receipts. The section on motor vehicles includes tables on motor-vehicle registration and operator's licenses, their fee schedules, and the revenues received therefrom and from motor-carrier taxes; this section also includes travel data. The highway finance section covers the disposition of highway user imposts, receipts, and expenditures for highways and highway debt. Because of the interest in the subject, data for toll facilities are presented separately. For the first time, the summary includes the local highway finance and related data; these data were previously given only in the annual bulletins. The mileage section includes reports on road and street mileage existing and the mileage constructed each year, classified by system and type. The section on Federal-aid includes tables on Federal excise taxes and on Federal-aid funds, construction, and system mileage.

THE NATIONAL SYSTEM OF INTERSTATE AND DEFENSE HIGHWAYS

STATUS OF IMPROVEMENT AS OF MARCH 31, 1967





U.S. GOVERNMENT PRINTING OFFICE: 1967

PUBLICATIONS
of the Bureau of Public Roads

list of the more important articles in Public Roads *and title* *ts for volumes 24–33 are available upon request addressed to* *eau of Public Roads, Washington, D.C. 20235.*

he following publications are sold by the Superintendent of *uments, Government Printing Office, Washington, D.C. 20591.* *ers should be sent direct to the Superintendent of Documents.* *payment is required.*

NUAL REPORTS

ual Reports of the Bureau of Public Roads:
960, 35 cents. 1963, 35 cents. 1964, 35 cents. 1965, 40 cents. 1966, 75 cents. 1966 supplement, 25 cents.
(Other years are now out of print.)

PORTS TO CONGRESS

leral Role in Highway Safety, House Document No. 93 (1959). 0 cents.

hway Cost Allocation Study:
upplementary Report, House Document No. 124 (1965). $1.00.

ximum Desirable Dimensions and Weights of Vehicles Operated n the Federal-Aid Systems, House Document No. 354 (1964). 5 cents.

1965 Interstate System Cost Estimate, House Document No. 2 (1965). 20 cents.

BLICATIONS

Quarter Century of Financing Municipal Highways, 1937–61, 1.00.

idents on Main Rural Highways—Related to Speed, Driver, nd Vehicle (1964). 35 cents.

gregate Gradation for Highways: Simplification, Standardization, and Uniform Application, and A New Graphical Evaluation hart (1962). 25 cents.

erica's Lifelines—Federal Aid for Highways (1966). 20 cents.

librating and Testing a Gravity Model for Any Size Urban rea (1965). $1.00.

pacity Charts for the Hydraulic Design of Highway Culverts (Hydraulic Engineering Circular, No. 10) (1965). 65 cents.

sign Charts for Open-Channel Flow (1961). 70 cents.

sign of Roadside Drainage Channels (1965). 40 cents.

deral-Aid Highway Map (40 x 63 inches) (1965). $1.50.

deral Laws, Regulations, and Other Material Relating to Highvays (1966). $1.50.

eeways to Urban Development, A new concept for joint development (1966). 15 cents.

ghway Bond Financing . . . An Analysis, 1950–62. 35 cents.

ghway Finance 1921–62 (a statistical review by the Office of Planning, Highway Statistics Division) (1964). 15 cents.

ghway Planning Map Manual (1963). $1.00.

PUBLICATIONS—Continued

Highway Planning Technical Reports—Creating, Organizing, and Reporting Highway Needs Studies (1964). 15 cents.

Highway Research and Development Studies, Using Federal-Aid Research and Planning Funds (1966). $1.00.

Highway Statistics (published annually since 1945):
1966, $1.00.
(Other years out of print.)

Highway Statistics, Summary to 1966.

Highway Transportation Criteria in Zoning Law *and* Police Power and Planning Controls for Arterial Streets (1960). 35 cents.

Highways to Beauty (1966). 20 cents.

Highways and Economic and Social Changes (1964). $1.25.

Increasing the Traffic-Carrying Capability of Urban Arterial Streets: The Wisconsin Avenue Study (1962). Out of print. Appendix, 70 cents.

Interstate System Route Log and Finder List (1963). 10 cents.

Labor Compliance Manual for Direct Federal and Federal-Aid Construction, 2d ed. (1965). $1.75.

Landslide Investigations (1961). 30 cents.

Manual for Highway Severance Damage Studies (1961). $1.00.

Manual on Uniform Traffic Control Devices for Streets and Highways (1961). $2.00.
Part V—Traffic Controls for Highway Construction and Maintenance Operations (1963). 25 cents.

Opportunities for Young Engineers in the Bureau of Public Roads (1965). 30 cents.

Presplitting, A Controlled Blasting Technique for Rock Cuts (1967). 30 cents.

Reinforced Concrete Bridge Members (1966). 35 cents.

Reinforced Concrete Pipe Culverts—Criteria for Structural Design and Installation (1963). 30 cents.

Road-User and Property Taxes on Selected Motor Vehicles (1964). 45 cents.

Standard Plans for Highway Bridges (1962):
Vol. III—Timber Bridges. $1.00.
Vol. IV—Typical Continuous Bridges. $1.00.
Vol. V—Typical Pedestrian Bridges. $1.75.

Standard Traffic Control Signs Chart (as defined in the Manual on Uniform Traffic Control Devices for Streets and Highways) 22 x 34, 20 cents—100 for $15.00. 11 x 17, 10 cents—100 for $5.00.

The Identification of Rock Types (revised edition, 1960). 20 cents.

The Role of Economic Studies in Urban Transportation Planning (1965). 45 cents.

Traffic Assignment and Distribution for Small Urban Areas (1965). $1.00.

Traffic Assignment Manual (1964). $1.50.

Traffic Safety Services, Directory of National Organizations (1963). 15 cents.

Transition Curves for Highways (1940). $1.75.

UNITED STATES
GOVERNMENT PRINTING OFFICE
DIVISION OF PUBLIC DOCUMENTS
WASHINGTON, D.C. 20402

OFFICIAL BUSINESS

POSTAGE AND FEES PAID
U.S. GOVERNMENT PRINTING OFFIC

If you do not desire to continue to receive this publication, please CHECK HERE ☐; tear off this label and return it to the above address. Your name will then be removed promptly from the appropriate mailing list.

DOT LIBRARY
00195134