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Along the German Autobahnen (prewar picture)

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PUBLIC ROADS ADMINISTRATION

FEDERAL WORKS AGENCY

E. A. STROMBERG, Editor

Concrete Pavements on the German Autobahnen

by F. H. JACKSON, Principal Engineer of Tests
and
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Division of Physical Research
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The inspection upon which this paper is based was prompted by a desire to reconcile conflicting reports which have come out of Germany during the last 3 years regarding the performance of concrete pavements on the autobahnen as compared with the performance of similar pavements in the United States. The survey was made during the summer of 1947 and covered approximately 1,000 miles of four-lane divided pavement in the British and American zones of occupation. In this report, the present condition of the German pavements is discussed from the standpoint of both structural performance and quality of concrete per se. All of the structural defects that usually develop in concrete pavements in the United States were found but, aside from frequently observed transverse cracking, defects such as joint spalling and faulting, settlement, etc., were not serious except in the Frankfurt area. The comparative freedom of the German roads from structural defects is believed to be due primarily to the comparatively small amount of heavy truck traffic using these roads, now and in the past, and to the comparatively mild climate.

The soils of Germany vary from cohesionless sands to plastic silty clays and clays. Most of the silty clays were of such nature as to require careful moisture control for adequate compaction. Such soils would be subject to frost heave under adverse drainage conditions, and pumping at joints would occur on them if free water entered expansion joints or cracks and if a sufficient number of heavy loads passed over the pavement. The practice of placing a layer of granular material under the pavement was, no doubt, a contributing factor in the prevention of pumping in such cases.

The concrete, almost without exception, was of excellent quality. Scaling was confined almost entirely to the München-Salzburg route. Disintegration was practically nonexistent. Outstanding was the absence of the heavy layer of surface mortar frequently found on pavements in the United States. The excellent quality of the concrete is believed due to (1) the high quality of aggregates, (2) the low water-cement ratio, (3) thorough consolidation by tamping and vibration of a very dry mixture with a maximum aggregate size of about 1 inch, (4) thorough curing, and (5) the comparatively mild climate. The effect of the cement is not clear—German cements were definitely inferior as judged by modern American standards, but whether they were actually inferior remains to be seen.

As a result of their observations the authors recommend the initiation of a comprehensive program of research on each of the following subjects:

1. Study of the possibility of insuring greater uniformity in pavement concrete by reducing the maximum size of coarse aggregate.
2. Development of more effective methods of compacting pavement concrete by mechanical means.
3. Study of the effects of variations in chemical composition of cements and the methods of manufacturing cements on the properties of concrete. In this work the authors would go considerably outside the range in composition and fineness now being studied under the "Long-time study of cement performance in concrete." Work of this nature should be carried out by the manufacturers and could be accomplished by an extension of the present program of the long-time study.

THIS report presents the results of an inspection of concrete pavements on the German motor-road system made by the authors during the summer of 1947 under instructions from the Commissioner of Public Roads. The inspection was made with the object of determining first, in what respects, if any, the German pavements are superior to ours; and second, what features of German design and construction practice, if any, can be adopted with profit by American engineers.

The report is in four parts: Part I is a general description of the system, including a section on the soils of Germany, a section on climate, and a section on traffic; part II, a description of the methods of design and construction which were employed; part III, comments on the present condition of the pavements with respect to (1) structural de-

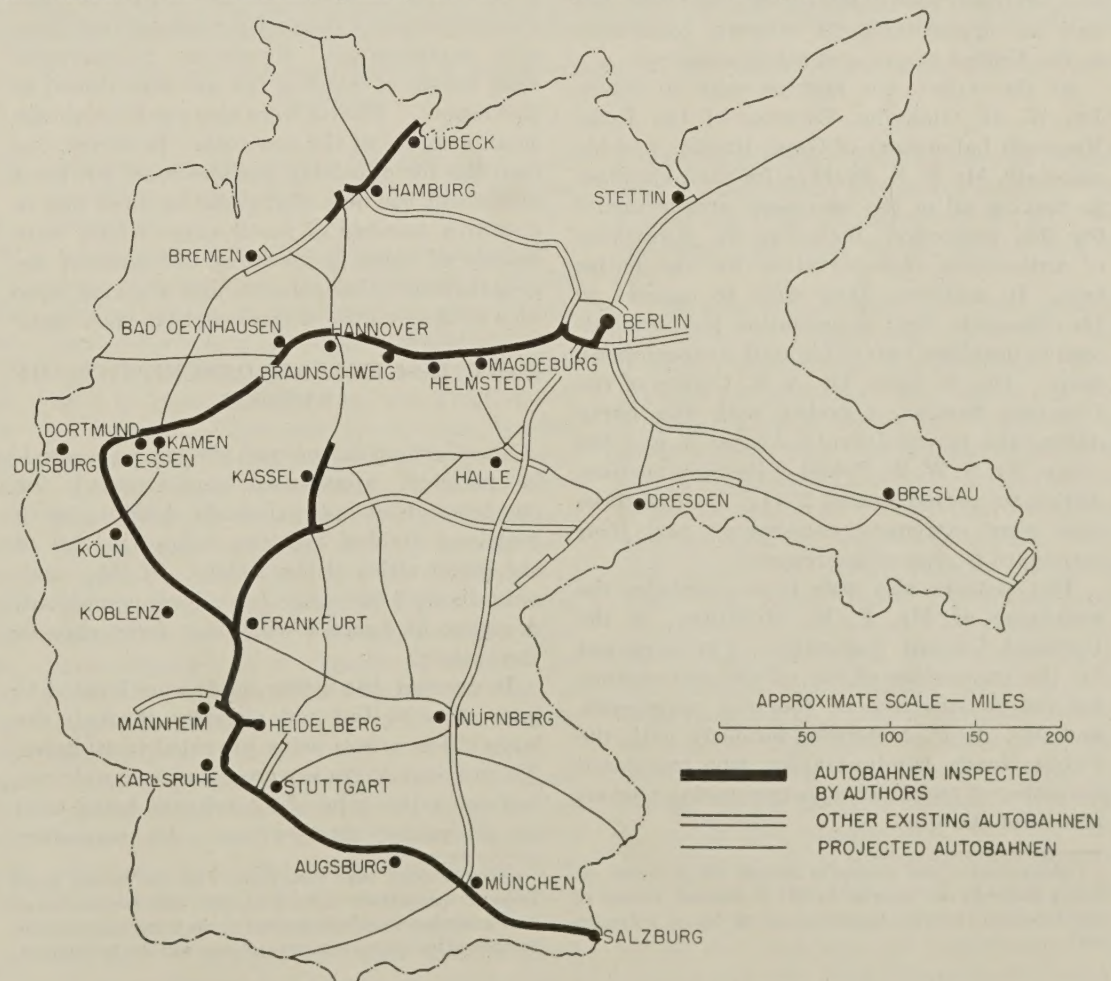


Figure 1.—The German autobahn system.

fects and (2) quality of concrete per se; and part IV, a summary of these comments, together with a number of recommendations based thereon.

During the 3 years that have elapsed since the invasion of Germany, the system of high-speed motor roads created by Hitler has been the subject of considerable comment by returning members of our armed forces as well as by civilian engineers. Much of this comment has been altogether favorable, not only with respect to the lay-out and design of the system as a whole but also as regards the excellent condition of the concrete pavements themselves. Typical of this reaction is a statement by M. A. Swayze, Director of Research of the Lone Star Cement Corporation, who spent about 3 months in Germany in 1945. In a report presented at the 42nd annual meeting of the American Concrete Institute,¹ he said in part: "In all my experience I have never seen concrete highways in better shape after at least 7 years' service than in the German autobahnen * * * Their quality is a tremendous challenge to American engineers to do likewise." On the other hand there has been some comment to the effect that these pavements, although generally in good condition, are not exceptional when compared with similar pavements in this country in that virtually all of the defects that characterize our roads are present in more or less degree in the German pavements. A statement to this effect was recently made, in an interview with the authors, by an outstanding German engineer who played a rather important role in the construction of these roads, and since that time has had an opportunity to observe conditions in the United States and other countries.

At the outset, the authors wish to thank Dr. W. H. Glanville, Director of the Road Research Laboratory of Great Britain, and his associate, Mr. F. N. Sparkes, for their kindness in making all of the necessary arrangements for this inspection, including the furnishing of automobile transportation for the entire trip. In addition, they wish to express to Dr. Glanville their appreciation for his kindness in detailing two of his staff to accompany them. One of these, Dr. A. R. Collins of the Concrete Section, traveled with the party during the period July 14–August 2, and the other, Mr. H. W. W. Pollitt of the Soils Section, during the period August 2–11. Both of these men were extremely cooperative and their assistance is greatly appreciated.

The authors also wish to acknowledge the assistance of Mr. F. R. McMillan, of the Portland Cement Association, who arranged for the translation of the official instructions for the construction of concrete pavements, and Mr. A. P. Anderson, formerly with the Public Roads Administration, who translated a number of technical reports covering various phases of the work.

¹ *Observations of war damage to concrete and to cement industry properties in Germany*, by M. A. Swayze; *Journal of the American Concrete Institute*, vol. 18, No. 6, February 1947.

Of the 4,000 miles of four-lane divided highway planned by Hitler, about 2,500 miles had been completed at the outbreak of World War II, as shown on the outline map of Germany in figure 1. On this map the 1,000 miles of roads traveled at least once by the inspecting team are shown as heavy solid lines. Many of the completed roads are in the Russian Zone and were inaccessible—in fact, with the exception of a very short stretch in the French Zone between Köln and Frankfurt, all of the roads inspected by the team lie within the British and American Zones. In addition, the party traveled over the 75 miles in the Russian Zone from Helmstedt to the Berlin ring road, but the cars were not allowed to stop along this route.

The inspection party arrived at Bad Oeynhausen, headquarters for the British Zone, on July 15 and left Frankfurt at the conclusion of the inspection on August 11. Of the 28 days spent in Germany, 10 were taken up in obtaining the necessary clearances, in making other arrangements, and in interviews, leaving 18 days for the actual road inspections. During this 18-day period, the entire 1,000 miles of motor road shown by the heavy solid lines in figure 1 were covered at least once; many sections twice; and several sections, three or more times. Driving speeds averaged 30 to 35 miles per hour, with frequent stops to check soil conditions, quality of concrete, structural failures, and the like. Stops were for periods ranging from a few minutes to an hour and were, in general, made whenever indications pointed to a marked change in soil conditions or in the condition of the pavement itself. The character of the soil profile was checked at many points by means of test holes and outcroppings. Numerous photographs were taken, of which a few are reproduced in this report. Efforts were also made to obtain small samples of the concrete. However, the facilities for obtaining samples were not good and about the best that could be done was to obtain a number of small chips which were mainly of value in revealing the types of aggregate used in the concrete and in giving some idea of the quality of the matrix in each case.²

PART I.—GENERAL DESCRIPTION OF SYSTEM

The German motor-road system, as originally planned, apparently contemplated the construction of approximately 4,000 miles of four-lane divided highway connecting all of the larger cities of the Reich. Of this total, only about 2,500 miles have been completed. A glance at figure 1 will show many gaps in the system.

In general, the motor roads were located to pass close to but not actually through the large cities, access being provided in all cases. No crossings at grade were permitted, a cloverleaf or similar type of interchange being used at all major intersections. All secondary

² Arrangements have been made with the British Road Research Laboratory to have concrete core samples taken from a number of selected sections on the motor-road system. Samples of the subgrade below each core will also be obtained.

roads were carried either over or under the main highway without provision for access. There are numerous bridges for this purpose on the system.

It is not proposed in this report to describe in detail the methods used by the Germans in locating the motor roads or the standards of grade and alinement that were followed. These features have been discussed quite adequately in reports of previous inspections made by British teams.³ It will be of interest, however, to call attention very briefly to some of the principal standards.

The German motor-road system was designed for speeds up to 100 miles per hour in flat country and up to 80 miles per hour in mountainous regions. Grades varied from a maximum of 4 percent in rolling country to 6 percent in hilly sections and 8 percent in the mountains. Minimum sight distances ranged from 500 to 900 feet, and the minimum radii of horizontal curves from 2,000 to 6,000 feet, depending upon the terrain.⁴ The Germans apparently gave a great deal of thought to esthetic consideration, both as regards the original location and the landscaping of these roads. In general, the motor roads were located so as to take full advantage of scenic possibilities. Cut and fill slopes were rounded and were blended in with the natural terrain so as to give a very pleasing appearance. This, together with adequate roadside planting, gave an appearance of age and permanence which is quite surprising when one realizes that the average age is only about 10 years. Typical views are shown in figure 2.

Construction of the motor roads was begun in 1934 and extended through 1940. By far the greatest mileage, however, at least so far as the roads in western Germany are concerned, was built during the 3-year period 1936–38. This applies also to the 1,000 or so miles of pavement inspected by the authors. According to data furnished by the British Road Research Laboratory,⁵ the approximate mileages by year of construction for this portion of the system are as follows:

Year built:	Miles
1935.....	70
1936.....	270
1937.....	240
1938.....	325
1939.....	0
1940.....	100
Total.....	1,005

³ *Report of an inspection of the German road system, 1945*, B. I. O. S. final report No. 918, by H. S. Keep, A. H. D. Markwick, and A. R. Collins, for the British Intelligence Objectives Subcommittee. H. M. Stationery Office, London, May 1946.

⁴ The indicated sight distances, nonpassing minimums on divided highways, are low as compared with American standards. The A. A. S. H. O. standard uses sight distances of 475 and 600 feet for design speeds of 60 and 70 miles per hour, respectively. While not included in the standards, corresponding values for design speeds of 80 and 90 miles per hour would be 775 and 1,000 feet, respectively.

⁵ According to other records, a short section of road between Frankfurt and Darmstadt, included here as 1935 construction, was actually built in 1934.

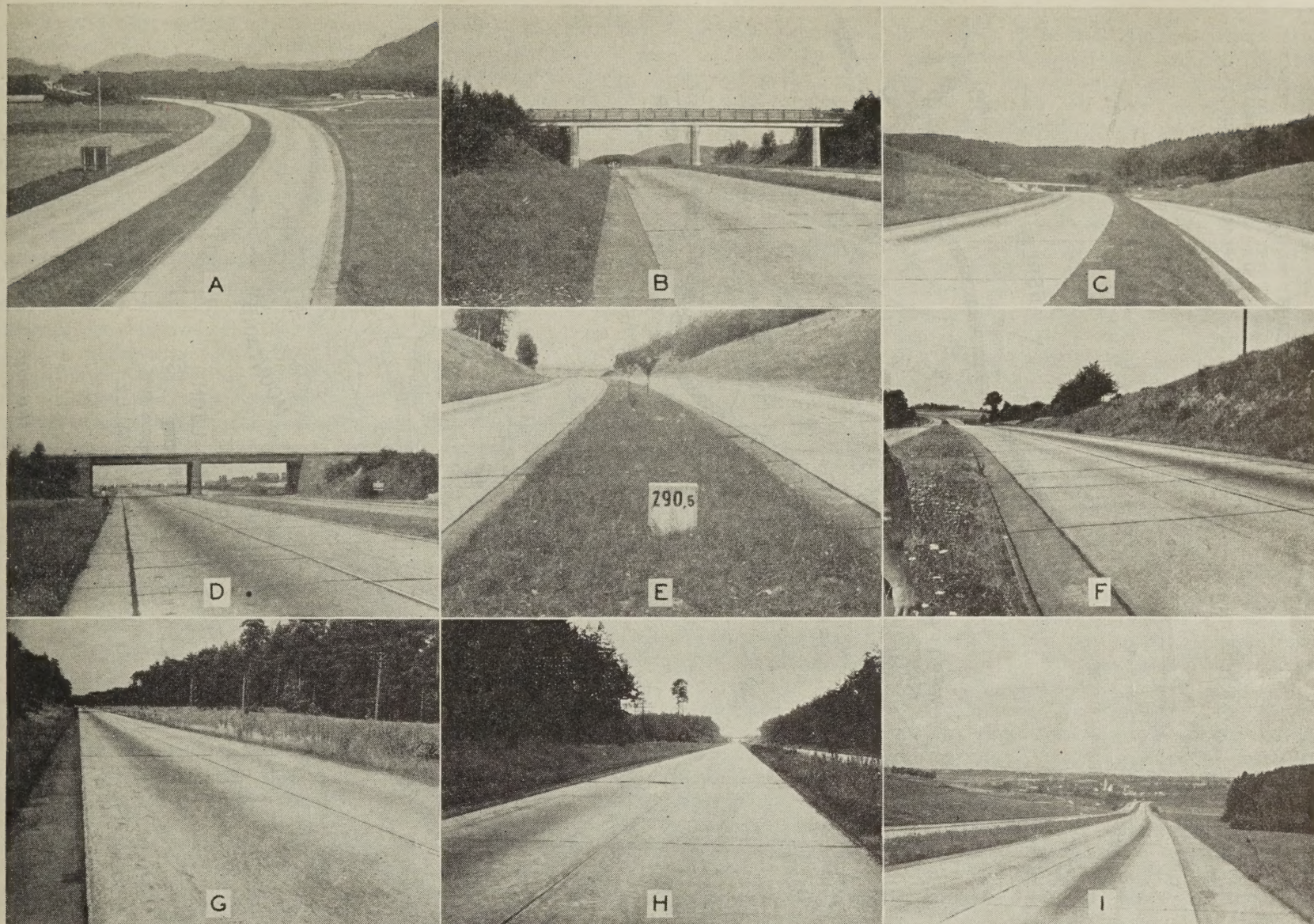


Figure 2.—Typical views of the autobahnen: (A-C) München-Salzburg; (D-F) Hannover-Ruhr; (G, H) Frankfurt-Stuttgart; (I) Stuttgart-München.

Soils

The inspection in a limited time of an extended mileage of highways for the purpose of comparing pavement performance with subgrade characteristics requires a regional understanding of soil conditions, especially if the terrain consists of a complex arrangement of geological parent materials with a wide range in relief or climate such as exists in western Germany. In the United States, engineering soil maps prepared from air photographs, geologic maps, or agronomic maps have been useful in developing such a regional understanding. Therefore, prior to leaving the United States, reconnaissance engineering soil maps similar to those shown in figure 3⁶ were made for the terrain in the vicinity of the autobahnen. These strip maps were prepared from geologic, physiographic, and soil information found in available technical publications. They show the major physiographic regions and the general characteristics of the soils in the areas traversed by the autobahnen. Factual data on soils were limited so that the soil information was largely interpreted from data

⁶ These soil maps were prepared by F. R. Olmstead, Senior Soil Specialist of the Public Roads Administration.

showing the distribution of geologic parent materials, the relief, and the climate of Germany.

The predicted soil information on the strip maps was used during the inspection for the general location of significant soil changes and was useful in evaluating the regional engineering soil problems that probably were considered in locating, designing, and building the autobahnen. Observations made at frequent intervals during the inspection indicated that the information shown in the strip maps was satisfactory for the appraisal of soil conditions in the area traversed by the autobahnen. During the field inspection, the boundaries of the significant soil areas shown on the strip maps were found to be quite accurate. They were checked as frequently as possible by examining soil profiles exposed in cuts, in excavations near the road, or by digging shallow pits.

The inspection revealed that with the exception of a section in which warping was observed there was very little relation between the type of soil and the pavement condition. Defects such as faulting, cracking, and spalling were found about equally in areas where either cohesive or cohesionless soils predominated. This condition is probably the result of placing

a layer of granular material over all cohesive soils. The primary purpose of the use of such a layer was to avoid damage to the pavement resulting from frost action on the subgrade soil.

The soils of Germany vary from cohesionless sands to plastic silty clays and clays. Northern Germany is in a glaciated area and the soils are light silty clays, sands, and sand-gravels. Around Berlin and Hamburg, and as far west as Bad Oeynhausen, granular materials are plentiful and were generously used in the autobahnen. From Bad Oeynhausen west to Duisburg, sands and slightly heavier silty clays are about equally distributed. Along the Rhine Valley from Duisburg to Frankfurt are heavier clay soils derived from the metamorphic rocks which predominate in the uplands south of Köln. In several locations in this area granular material could not be found by digging at the edges of the pavement, indicating that scarcity may have limited its use for base courses. This region is somewhat rugged and wooded. From Frankfurt south to Karlsruhe the road follows the Rhine River Valley and crosses the flood plains of several major streams. In this area sands and sand-gravels predominate. The soils in the area from Karlsruhe to Augsburg

are derived from sedimentary rocks and, except for the mileage through the gravel terraces of river beds, are plastic silty clays, often mixed with rock fragments.

From München to Salzburg the road passes through a glaciated region and a considerable mileage passes over an old lake bed bordering on the Chiem See (lake). The soils are poorly drained plastic silty clays. Between the Chiem See and Salzburg the road is located along the foothills of the Bavarian Alps and passes over terminal morainic areas. The drainage is poorly established and, in general, the subgrade conditions are poor.

Most of the silty clays examined on the system would require careful moisture control for adequate compaction and would be subject to frost heave under adverse drainage conditions. Pumping at joints in concrete pavements would occur on these soils if free water entered expansion joints or cracks and if a sufficient number of heavy loads passed over the pavement.

The Berlin-Magdeburg section of the autobahnen is located through the area once covered by a continental ice sheet. The glacial till and outwash areas have been modified by subsequent erosion to produce a variety of glacial land forms having different soil textures and drainage conditions. Most of the soils in the region are granular. The area has been separated into three physiographic regions—the Berlin Plain, the Fläming Plain, and the Elbe River Valley.

The Berlin Plain is a flat to undulating lowland with many canals and swamps having intervening sandy areas derived from glacial outwashes, terraces, or sand dunes. The Fläming Plain is gently rolling, with broad low ridges separated by wide flat valleys. Many of the valleys are poorly drained wet meadows, in which the accumulation of peat bogs is frequent. In the Elbe Valley the soils are better drained than in the Fläming Plain, and along the east edge of the river there are many sand dunes.

All of the soil profiles observed in road cuts and excavations in the Berlin-Magdeburg section seemed to indicate that the soils were generally sands. In some locations the sands appeared to be covered with a silty clay surface soil. Since this area is in the Russian Zone, where stopping was not permitted, inspections were made from an automobile traveling at approximately 30 miles per hour.

The section from Magdeburg to Hannover lies in the Leine-Saale Lowlands. The terrain is flat to rolling, with altitudes ranging from 100 to 200 feet. In general, the soils are closely related to the glacial land forms. The granular soils in the region are associated with outwash plains and terraces and the clay soils with the morainic areas. During the glacial period, the Leine and Elbe Rivers deposited vast amounts of alluvium in rather wide flood plains. These were subsequently dissected by erosion to form the present terrain. The better-drained granular soils occur on terraces, and the poorly drained soils on the more recent flood plains. The poorly drained soils appear to be sandy, but drainage is retarded by the development of an iron-

oxide hardpan. Peat bogs and swamps are quite frequent in areas underlain by this hardpan formation. In some areas the loamy surface soils are probably derived from wind-blown silts of glacial origin.

An examination of an embankment under repair between Helmstedt and Braunschweig disclosed that the fill was composed of a sand-clay-gravel mixture of good quality, with a layer of very fine white sand immediately under the pavement.

Observations made between Braunschweig and Hannover indicated that the soils were predominately sands and sandy gravels. East of Braunschweig the surface soil was silt to silty loam with indications of underlying sand and gravel.

Geologic and soil information of the area around Hamburg were not available to the authors and a preliminary soil map could not be prepared. The autobahn from Hamburg to Hannover has not been built, and time would not permit inspection of the entire road from Hamburg to Bremen. However, an examination was made of the section from Hamburg to the Hamburg-Hannover-Bremen junction and of approximately 10 miles on the Bremen road. Examination was also made of the Hamburg-Lübeck section.

South of Hamburg the subgrade under a section of pavement which had been broken by a bomb was examined. The soil under the pavement was a fine sand containing a very small amount of binder. The section was on a fill approximately 30 feet in height, and the entire embankment was composed of granular material.

At another location in this area, an examination of the soil profile in a shallow excavation showed 6 inches of fine silt on the surface underlain by fine sand. There was evidence that the sand was coarser at greater depths.

In the fields at one location peat was being dug for fuel. On the south part of the section from Hamburg to Lübeck, the soil, exposed in shallow excavations adjacent to the road, was very sandy. On the north end of this section the soils were silty clays underlain by sands.

Discussion of the construction of the autobahnen with German engineers indicated that the peat-bog areas were carefully avoided in locating the entire system. A few short sections are located over such areas, and the embankments over these locations were built by superimposing the fill material and displacing the muck soil with dynamite charges.

The section from Hannover to Wiedenbrück passes through the Osnabrück-Weser Lowlands and a portion of the Ems River Valley. This area is underlain by beds of limestones, greensands, marls, and clays. The more resistant limestones and greensands occur in low-lying ridges, and the softer marls and clays are found in the valleys of the Weser and Werre Rivers. The Ems River Valley is underlain by these softer rocks but during the glacial period, when the Ems served as one of the glacial drainage outlets, the valley floor was covered with a thick layer of alluvium. The soils occurring in this area, with the exception of those found in the valley of the Ems, are mostly clays derived from the

weathering of the underlying calcareous rocks. The granular soils found between the Ems River and the east valley wall occur on an old glacial terrace. In more protected areas, there may be soils derived from remnants of the old glacial drift or deposits of wind-blown silt.

The soil profiles exposed in the fields at most of the points examined between Bad Oeynhausien and Hannover, and to approximately 25 miles west of Bad Oeynhausien, consisted of a gray, silty-clay surface soil (6 to 12 inches) over a more plastic, yellowish silty-clay subsoil.

The practice of hauling granular materials relatively long distances for use in fill construction is illustrated by an observation made at a location about 1 mile east of the Bad Oeynhausien interchange. The embankment at this point was approximately 12 feet high and was composed of cohesionless sand which, it was estimated by visual inspection, would pass a No. 10 sieve and be retained on a No. 100. The sand was covered with 10 inches of friable silt surface soil and a very heavy tough sod. The slope of the fill was approximately 1½:1. A local resident informed the party that the sand for the fill was brought from a pit about 4 miles from the road.

Approximately 30 miles east of Bad Oeynhausien, where the soil profile was exposed in a limestone quarry, the surface soil consisted of a silty clay and the underlying layer was a plastic clay.

At a location 1.5 miles west of the Bad Oeynhausien intersection, an examination was made where the road passes through a sidehill cut. In a pit dug in the median strip, top soil was found to a depth of 10 inches—slightly below the bottom of the pavement. Below this depth was a mixture of sand and stone particles for a depth that could not be determined due to hardness of the ground. An 18-inch pit dug in the embankment contained silty-clay soil. There was no evidence that the granular material extended the full width of the grade. At another location in cut section, 3 inches of cinders were found below the bottom of the pavement over a silty-clay soil mixed with rock particles.

Evidence of substantial sand layers under the pavement and for the full width of the roadway was found at most of the locations examined in this section.

Observations made in the southwest 15 miles of the Hannover-Wiedenbrück section disclose the predominance of sandy top soil underlain by cohesionless sand.

In the section from Wiedenbrück to Dortmund the soils are clays and silty clays derived from the underlying calcareous sedimentary rocks of the low plateau of northern Germany. Except in places where the sedimentary rocks have been exposed by the erosion of the overlying glacial drift, the soils in the section from Dortmund to Duisburg are granular.

Between Duisburg and Köln, granular and fine-grained soils occur in alternate sections of approximately 10 to 20 miles length. The granular materials are located in the Rhine Plain and the silty-clay to clay soils where the road crosses the Sauerland. The Sauer-

land is a rolling sandstone and shale plateau sloping westward and cut by many narrow, steep valleys. The soils derived from these sedimentary rocks usually are silty clays and clays, and are likely to be shallow in depth.

At a location approximately 3 miles west of Kamen, the soils found in an embankment slope, at the toe of the slope of the embankment, and in the shoulder near the pavement edge to a depth of 18 inches, were gray, friable, silty clays and yellow, silty clays. No evidence of a granular layer under the pavement was found by digging shallow pits in the shoulder at this location. It is possible that the granular material was not placed for the full width of the paved shoulder strip.

At another location, approximately 10 miles south of the Ruhr River, a 6-inch layer of crushed shale-like rock (maximum size 1/2 inch) was found beneath the pavement slab. The subgrade soil was a yellowish, mottled, plastic clay. The clay had intruded into the stone subbase and the pavement at this location was rough and the joints had the appearance of being warped.

In some other locations examined, evidence of the use of granular material over plastic clays could not be found by digging near the edge of the paved shoulder strip.

The section from Köln to a point west of Frankfurt, on the River Main, passes through the lower Rhine Plain and the Westerwald. The Westerwald is a folded, contorted mass of schist, quartzite, and other Devonian rocks. The soils derived from these rocks are clays and sandy clays. The soils in the 20 miles immediately south of Köln or to the boundary of the Westerwald were found to be granular. The soils in the remaining mileage of this section were silty clays or clays. No extremely plastic soils were found in this section.

From Frankfurt to Karlsruhe the road follows the terraces of the Rhine Rift Valley. The soils in this valley are usually granular but may be covered with a shallow layer of silty-clay soil. From the Enz River (near Karlsruhe) to Stuttgart is an area of calcareous soft rocks or marls. The soils in this area are clays or clays mixed with stone fragments.

Observations made between Frankfurt and Heidelberg indicated 5- to 10-mile sections of cohesionless sand covered with sandy surface soil, interspersed with sections of clay and

silty clay similar in length. In the areas where the clay soil predominated, the fills were built of cohesionless sand.

From Karlsruhe to Stuttgart, silty-clay and clay soils predominate. In some locations the clay soils seemed to be quite plastic. Granular material could not be uncovered by the digging of shallow pits in the embankment slopes in this area. Pits dug at the edge of the pavement in the median strips disclosed a layer of either fine sand or sand-gravel under the pavement for a minimum depth of 12 inches.

From Stuttgart to München the road traverses the Swabian Upland, the Donau-Lech Valley and the Alpine Foreland. The Swabian Upland is flat to gently rolling and is dissected by streams with steep valley walls. The parent material in this area is limestone and the soils are chiefly clays. The Donau-Lech Valley and the lower Alpine Foreland are glaciated areas. The soils vary from fine-grained, poorly drained alluvium and clay soils to sands and sand-gravels. There are terraces of granular material along the Lech River. Loess soils cover the glacial terrain in sheltered locations.

Between München and Salzburg the road traverses the München Plain and the Upper Alpine Foreland. The München Plain is a granular outwash developed from water sorting of granular material contained in the terminal moraines that flank the Bavarian Alps to the south of the area. The Upper Alpine Foreland is a glaciated area made up of terminal moraines, drumlins, and an old lake bed. The soils in this section are fine sands, silts, and silty clays mixed with gravel and stone. Between the Chiem See and Salzburg the road passes along the foot of the Bavarian Alps, cutting through limestone outcrops and passing over terminal moraines.

The soils in this area are clays or alluvial silts and the land is swampy in many places. The drainage is poor and there are evidences of distress in the pavement at some locations. The view of the Alps from this section of road is good and it is probable that, in choosing the location, the scenic value outweighed the poor soil and drainage conditions. A location further north would have passed through more suitable terrain.

The autobahn from Kassel to Frankfurt

passes through the Hesse Uplands, the West Hesse Hills, the Wetterau Corridor, the Westerwald, and the lower plain of the Main River. The Bunter sandstone and tertiary clays and silts outcrop in the area around Kassel. The road is flanked on the west by the West Hesse Hills and on the east by the Hesse Uplands, which are complex geologically since they were formed by the erosion of faulted and folded sandstones and limestones and the subsequent deposition of loess. The road enters the Wetterau Corridor at a point west of a basaltic mass called the Vogelsberg Mountains which rise to 2,500 feet and which have been dissected by radial stream gorges to depths of more than 600 feet. Many of the stream valleys are filled with peat. The ruggedness of the terrain necessitated the construction of many high embankments which observations indicated were built of cohesionless sand.

The soils in the Wetterau Corridor are heavy clays derived from the weathering of basalt. The soils in the Westerwald and the Main River Plain are tertiary clays and wind-blown loess. The clays were derived from the weathering of the shales and schists of the Westerwald. The loess has weathered to a more plastic silty clay in some areas.

Climate

Western Germany has a milder climate than would be expected from its latitude because it is exposed to mild westerly winds. There is little difference between the north and south sections since in the latter the higher elevations counterbalance the southern location. The mean annual temperature of Hamburg is 48° F., of Leipzig, 47° F., and of München, 45° F. The average annual rainfall varies from 16 inches in the plains to 47 inches on the southeast uplands. July is normally the rainiest month and January or February, the driest. The snowfall occurs during the months of January, February, and March, and in that period snowfall occurs about 30 days in the northern lowlands and 30 to 50 days in the southeastern highlands.

It would be difficult to select an area in the United States having an extensive mileage of concrete pavement in which the soil and climatic conditions are exactly the same as those found in Germany. The area including southern Michigan, Ohio, Indiana, Kentucky, western Pennsylvania, and West Virginia has similar geologic formations, soil conditions, and a fairly comparable climate. The temperature and rainfall data for six cities in the selected area of the United States and for four cities in Germany, as supplied by the United States Weather Bureau, are shown in table 1. A comparison of these data shows that the average temperatures for January are approximately the same in both areas, that the average July temperatures in the United States cities are approximately 10 degrees higher, and that the range from maximum to minimum is greater in the United States. The rainfall in Indianapolis, Columbus, or Pittsburgh is comparable to that of München, but for the rest of Germany it is lower than that of the middlewestern area of the United States.

Table 1.—Comparison of climatological data for Germany and midwestern United States

City	Temperature					Annual average rainfall Inches
	Average		Variation ¹			
	January	July	Maximum	Minimum	Range	
Germany:	° F.	° F.	° F.	° F.	° F.	
Berlin.....	30	64	99	-15	114	23
Hamburg.....	32	63	92	-6	98	29
Frankfurt.....	32	66	100	-7	107	24
München.....	28	64	97	-14	111	36
Average.....	30.5	64.2	97	-10.5	107.5	28
United States:						
Indianapolis, Ind.....	28	75	107	-22	129	38
Columbus, Ohio.....	31	75	106	-20	126	34
Pittsburgh, Pa.....	32	74	103	-20	123	35
Charleston, W. Va.....	38	77	108	-17	125	46
Frankfort, Ky.....	36	78	111	-17	128	43
Lansing, Mich.....	23	71	102	-26	128	31
Average.....	31.3	75	106.2	-20.3	126.5	37.8

¹ Over a period of 35 to 75 years.

Data as to the penetration of frost in Germany were not available from the weather records. It was learned from a German engineer, now working in England, that the frost penetration varied from a few to 70 or 80 inches.

Traffic

The traffic on the autobahnen observed during the inspection was very light both as to the number of vehicles and the probable axle loads. For example, in traveling during the middle of a Sunday forenoon from Helmstedt to the outer Berlin circle, a distance of 70 miles, no vehicles were seen in a period of 45 minutes. Six light trucks were counted in the 70 miles. In many places in this area and other sections of the autobahnen, the pavement had not been discolored by the oil streak parallel to the center line which usually results from the passage of a large number of heavy motor vehicles. The appearance of the pavement indicated that the greatest volume of traffic on the system was from Frankfurt to Kassel, south of Frankfurt toward Heidelberg, and from Frankfurt to Hannover via the Ruhr Valley. Due to the very light traffic on the system, now and in the past, it is difficult to make any comparison of the pavements with those in the United States laid under similar climatic conditions and on similar soils. It is likely that the volume of traffic on most of the primary roads in the United States is considerably greater than was observed at any location on the autobahnen.

PART II.—DESIGN AND CONSTRUCTION

Earthwork and Subgrade

In the location and planning of the autobahnen, detailed surveys were made to evaluate soil and drainage conditions. Whenever possible, bad subsoils were bypassed. Where such areas could not be avoided, detailed investigation was carried out to determine the magnitude of the settlement of soft undersoils due to loading, the time required for such settlements to occur, and the most economical height and slope of embankments. When it was necessary to place high embankments over yielding subsoils, the completion of the earthwork was planned so as to permit the major portion of the settlement to take place before the concrete slab was placed. The course of the settlement was accelerated through lateral displacement of the subsoil by overloading or by blasting.

In the design of the earthwork, danger from frost damage was given special attention. Soils which contained more than 3 percent of particles smaller than 0.02 millimeter were classified as subject to damage by frost either by heaving or by softening. It was recognized that water must be able to reach the frost zone by capillarity before frost damage could occur, and if the ground-water surface was at a greater distance below the frost zone than the capillary rise in the soil, no frost damage would result provided there was no entrance of water from the shoulders or side slopes. Efforts were made to avoid frost damage by the following means:

1. Placing the concrete slab a sufficient height above ground water to avoid saturation by capillarity, or lowering the ground-water table by drainage.
2. Replacement of soils susceptible to frost damage with soils not susceptible to such damage.
3. Use of layers of coarse granular material or of impervious strata such as bituminous concrete slabs, to prevent capillary rise of ground water.

The following statement, from a publication entitled "Guiding Principles for Roadway Slabs" issued for the use of engineers engaged in the design and construction of the autobahnen, illustrates the importance placed on investigation of possible frost damage: "The prevention of frost damage must in every single case be exhaustively investigated according to terrain and change in soil, and may not be handled in a routine manner."

In areas where ground water was found to flow toward the road, it was intercepted by longitudinal drains. Drains were also used to lower the ground-water elevation under the roadway when such a procedure was considered effective and economical.

In a great majority of the mileage, the concrete slab was placed on embankment. In flat terrain the grade line was raised 3 feet or more above the natural ground level, and in cuts the roadway was excavated below grade and backfilled with selected material. This procedure provided good drainage and in the open plains simplified snow removal.

Cohesionless sand was used in fills wherever it was available. Sand was hauled long distances to avoid the disturbance of farm lands that would result from taking borrow with a shorter haul. When sand was not available for the entire embankment, it was used in the top 8 to 32 inches. A thickness of 8 inches was most common. The sand was placed the full width of the grade in fill sections.

Where sand for the entire fill was not available, relatively thin layers of granular material were placed at several elevations and to the full width of the embankment.

Embankment slopes were made steep (usually 1½:1 or steeper) to reduce to a minimum any encroachment on farm land, and embankment slopes were protected against the entrance of moisture by sodding immediately after completion. In some instances a full-width layer of granular material was placed in the bottom of embankments composed of cohesive soil, to aid in the drainage.

Cohesionless sands were compacted with a rammer called a "jumping frog." The device



Figure 4.—"Jumping frog" used to compact cohesionless sand.

was actuated by explosions of gasoline in a cylinder and the explosions were controlled by a manually operated spark. A photograph of one of these machines taken by F. R. McMillan is shown in figure 4. The weights of the "frogs" for embankments varied from ½ to 1 ton and for the area under the paved shoulder a machine weighing 800 pounds was used.

Cohesive soils (silty clays and clays) were compacted with smooth-face and sheepsfoot rollers. The thickness of each layer of soil before rolling was approximately 18 inches. A field laboratory was maintained on each project where cohesive soil was used, and the moisture and density of the soil were carefully controlled. Very little effort was made to compact sand or cohesive soil back of bridge abutments and, as a result, there are many settlements in the approaches to bridges. That such settlements were anticipated is evidenced by the fact that stone setts were placed adjacent to many bridges so that fill settlement could be corrected conveniently. The setts were arranged in a fanlike pattern similar to that employed in the case of Durax block in this country.

Shoulders

The use of paved shoulder strips, described in detail in the next section of the report, has been effective in keeping the traffic on the pavement. As a result, there has been no rutting or destruction of the sodded area of the shoulders. The sod in all cases observed was very thick and tough and even though there has been very little mowing or other care given to the shoulders or embankment slopes, they were in good condition and looked reasonably neat. Typical views of the shoulder strips are shown in figure 2.

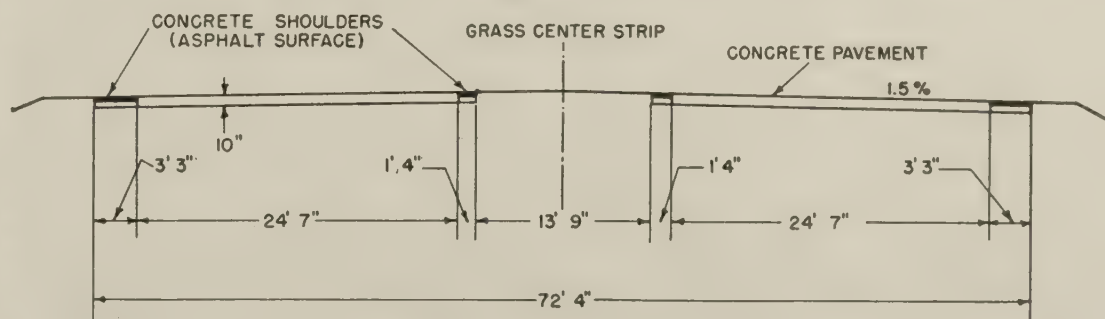


Figure 5.—Typical cross section of four-lane autobahn.



Figure 6.—Where dark-colored pavement was used, the shoulder strips were of plain concrete for contrast.

Pavement Design

With few exceptions, the entire system was designed to provide for a four-lane divided highway, with opposing traffic separated by a grass center strip about 13 feet wide. A cross section of a typical design is shown in figure 5. In this figure and throughout the text, the metric dimensions used by the German engineers have been converted to their approximate English equivalents. Pavements were uniformly 24 feet 7 inches wide, with a longitudinal center joint. Construction, in general, was full width in one operation, with a longitudinal joint of the weakened plane type. Lane-at-a-time construction was occasionally used, the longitudinal joint between the slabs in this case probably being of the butt type, although information on this detail is lacking. Light steel tie bars were used across the longitudinal joint to prevent separation of the pavement slabs due to settlement or other causes.

All pavement slabs were of uniform depth, 8 or 10 inches in thickness, although heavier sections were used occasionally. A unique feature of the design was the hard-surfaced shoulder strips. These were usually 1 foot 4 inches wide on the inside shoulder and 3 feet 3 inches wide on the outside. On later work, however, the outside width was increased to 7 feet 4 inches, apparently with the idea of providing more adequately for emergency parking. The shoulder strips were usually about 10 inches in depth, the lower 7 inches usually being of concrete and the upper 3 inches of asphalt. The purpose of the asphaltic surface was to provide a contrast in color and thus define the outside edge of the traffic lane. In cases where dark-colored concrete or asphalt was used in the pavement proper, plain concrete was used in the top course of the paved shoulder strip (fig. 6).

The concrete base for the shoulder strip was constructed before the pavement slab and was utilized as a base upon which to mount the rails carrying the heavy construction equipment used in the mixing, placing, and finishing operations. The asphalt top was placed to the elevation of the finished roadway after the completion of the pavement proper.

So far as the design of the slab itself is concerned, practice appears to have been patterned largely after that followed in this country, except that the Germans did not use

the thickened-edge section. Practice as regards transverse expansion-joint spacing varied widely. For nonreinforced sections the joint spacing was generally indicated as 26 feet, whereas for reinforced sections spacings varied from a minimum of 39 feet to a maximum of 98 feet. Observations at the time of the inspection indicated an average expansion-joint spacing of from 40 to 60 feet. It was also quite common practice to systematically increase the joint spacing from slab to slab in increments of 3 feet or so, with periodic return to the first spacing, apparently in an effort to avoid setting up rhythmic vibrations in cars traveling at constant speed. Weakened-plane contraction joints were frequently installed, particularly in the case of the longer expansion-joint spacings.

Load transfer was provided by means of conventional round steel dowels, 14 inches long and $\frac{7}{8}$ inch in diameter, with a sleeve over one end. One typical design for an 8-inch section shows the dowels spaced 12 inches apart at the center of the slab with somewhat closer spacing near the edge. In this design the expansion-joint opening is shown as $\frac{3}{4}$ inch at the top, with a wood filler $\frac{1}{2}$ inch in thickness.

Practice as regards the use of reinforcing steel also varied. It was apparently not used at all in 1935 but was employed quite extensively in 1936, mostly in the form of a light mesh weighing from 42 to 51 pounds per 100 square feet. Steel mesh was usually placed after screeding the lower course of concrete and before placing the upper or wearing course. This placed the steel at about one-third the depth of the slab, or about 3 inches from the top. Use was also made of corner- and edge-bar reinforcing during this period. The use of reinforcing was discontinued in 1937 and thereafter, due to a shortage of steel.

Materials

Mr. M. A. Swayze's trip to Germany in 1945 was made for the specific purpose of studying the German cement industry. With Mr. G. G. J. Davis, representing the portland cement industry of Great Britain, he visited some 28 cement plants, all located within the American and British Zones. The results of their survey have been reported in considerable detail elsewhere,⁷ but are summarized by Mr. Swayze in his report on war damage in Germany, previously cited, as follows: "The great majority of the plants visited were making a product which was inferior in practically every way to the cements manufactured in the United States or in Great Britain. Their control of raw mix proportions was generally lax, and fine grinding of these materials which we believe essential for good quality was nowhere near that of American practice. The burning operation is much lighter than ours, even in plants with modern kilns, and free

⁷ *German cement industry*, F. I. A. T. final report No. 519, by M. A. Swayze and G. G. J. Davis, for the Field Information Agency, Technical, United States Group Control Council for Germany; published by the British Intelligence Objectives Subcommittee. H. M. Stationery Office, London, November 20, 1945.

lime contents as high as 2.5 percent are not uncommon."

Mr. Swayze's appraisal was, of course, based on observations of conditions in 1945. To just what extent economic conditions, particularly an acute fuel shortage, may have tended to lower postwar manufacturing standards as compared with those in effect during the thirties, cannot be stated definitely. Furthermore, the cements used in the construction of the motor roads were supposed to meet special requirements for flexural strength, drying shrinkage, and fineness which were not imposed in the case of the ordinary product. Even so, it seems probable that even these specially selected cements were quite inferior, when judged by our standards, to modern American cements. For example, Swayze's survey indicated that, entirely aside from the matter of fuel economy, the methods for controlling uniformity of product, such as facilities for storing and handling clinker, proportioning materials including the gypsum, etc., were very crude as compared to modern American practice. This conclusion would, of course, apply as well to the prewar German cement as to the postwar product.

The autobahn cements were undoubtedly somewhat coarser than our present-day American cements. The Germans required that between 5 and 25 percent be retained on a sieve having 178 openings per inch. Compare this with the modern American cement, which rarely will have more than 3 or 4 percent retained on the standard No. 200 sieve (200 openings per inch). Particle-size distribution in the typical autobahn cement was also probably quite different. According to Swayze, practically all finish grinding in German cement mills is by open circuit as compared with our modern closed-circuit operation using air separators. In open-circuit grinding, the final cement product contains the various sizes just as produced in one continuous passage through the grinding mills. In closed-circuit grinding, the various sizes are classified by means of air separators and the coarser particles are returned to the mills for further grinding.

It is probable also that the German cements were not as hard burned as our modern cements. Apparently the only requirement for soundness was a 2-hour boiling test. The autoclave test has never been used. According to the best technical thought in this country, an autoclave test is absolutely essential for the proper control of soundness. Many failures in the United States, particularly in the southeast, have been attributed to the absence of an autoclave requirement in specifications, and it will be interesting to note when discussing the present condition of the autobahnen that such a requirement has never been used by the Germans.

According to Prof. Otto Graf,⁸ cements for concrete road construction were selected from the commercially available products, on the basis of the following characteristics:

⁸ *Choice of cements for concrete road construction and some questions regarding same*, by Otto Graf; *Zement*, vol. 25, No. 33, Aug. 13, 1936.

1. High flexural strength.
2. Moderately high (adequate) compressive strength.
3. Low drying shrinkage.
4. Moderately coarse grind.

In addition, as mentioned above, all cements were required to pass a 2-hour boiling test for soundness. This was a field control test for uniformity during manufacture, the other tests being used primarily as prequalification tests to determine the acceptability of a particular source.

Apparently no attempt has been made in Germany to classify cements by chemical composition in a manner similar to that employed in the five-type specification recognized by the American Society for Testing Materials. So far as could be determined, any cement that would qualify on the basis of the physical tests outlined above would be acceptable as to chemical composition on much the same basis as the Type I cement in the A. S. T. M. specifications, with one exception—the Germans placed an upper limit of 3 percent on magnesia instead of the 5 percent used in most American specifications. As a matter of information it should also be noted that the alkalis (sodium and potassium oxides) in German cements are said to run quite low as compared to many American cements.

In addition to straight portland cement, the Germans recognized two types of portland-blast-furnace slag cement: Eisenportland, containing about 70 percent portland cement and 30 percent blast-furnace slag, and Hochofen cement, containing about 15 percent portland cement and 85 percent blast-furnace slag. Because of its slow hardening characteristics, the Hochofen cement was not used to any extent in road work. The Eisenportland, however, was used quite extensively, particularly in northern Germany. The manufacture of this type was, of course, limited to those plants sufficiently close to the blast furnaces to make the utilization of the slag economically justifiable. The availability of almost unlimited quantities of slag, together with the great saving in fuel, made the manufacture of Eisenportland very attractive; and it is understood that in the neighborhood of one-third of the entire mileage of motor roads contains this type of cement.

The instructions governing the selection, processing, and control of aggregates for concrete pavement construction were such as to insure excellent results from the standpoint both of the quality of the hardened concrete and the uniformity of the consistency of the mixture as it was placed. Aggregate quality was insured through the use of compressive strength and wear tests in the case of crushed rock and wear tests in the case of gravel. Hard rocks, such as granite, basalt, and quartzite, were preferred. Durability was judged apparently on the basis of experience, as the record does not indicate that laboratory tests for soundness were used. Blast-furnace slag was permitted but the inspections did not reveal that it had been used to any extent. The requirements for blast-furnace slag were stated to be about the same as for crushed

rock. Considerable attention was paid to the matter of impurities, and freedom from flat and elongated particles was insured by requiring that the breadth of an individual particle should be not less than 65 percent of its length and that its thickness should be not less than 25 percent of its length.

In the matter of separated sizes, German practice differed markedly from that employed in this country in that it was apparently the universal practice to require that the sand or fine aggregate be furnished in at least two fractions, 0–0.12 inch and 0.12–0.28 inch. To insure better control of grading, the material below 0.12 inch was sometimes still further separated into two or three sizes. It is obvious that with so many separations, very close control of the total fine aggregate grading could be obtained.

For the larger sizes, the separations were usually 0.28–0.59 inch and 0.59–1.18 inches. These were the size separations for the top course. In two-course construction, a somewhat larger (1.58–1.97 inches) maximum size was used for the bottom course.

Most of the pavements on the autobahnen were constructed in two courses. This was necessary not only because of the difficulty of properly compacting a single course of very dry concrete 8 to 10 inches in depth, but also because this type of construction made it possible to use a somewhat inferior type of aggregate in the lower course—a matter of considerable economic importance in some instances.

Composition of Mix and Proportioning

It was the general practice to design all paving mixes in the laboratory prior to construction, using the same cement and aggregates to be furnished later to the job. The basis of the design was the development of a mix that would (1) have sufficient workability for placing and finishing with the equipment to be used on the work, and (2) result in concrete conforming to the following requirements for strength at 28 days, using a cement content of

not less than 1.35 barrels per cubic yard and not more than 1.57 barrels per cubic yard:

Flexural Strength:	<i>Lbs. per sq. in.</i>
Minimum.....	540
Average.....	640
Compressive Strength:	
Minimum.....	4,700
Average.....	5,700

It was further provided that the strengths at 7 days should not be less than 70 percent of the above values. It was the practice also to check the strength design by means of tests of compression and flexure specimens made during construction and by means of tests on cores made at the age of approximately 60 days.

As previously stated, aggregates were supplied in a number of separated sizes. Instructions for the design of mixes required that the available sizes be so proportioned that the combined grading curve would fall within the limits shown by lines *K* and *D* in figure 7. The instructions also stated that the exact position of the combined grading curve between these limits would depend upon the nature of the aggregate (that is, the curve would be lower for gravel than for crushed stone) and also upon the efficiency of the placing equipment. In general, a curve below line *J* but above line *D* would be permitted only in cases where compacting machines of proved efficiency were to be employed.

The amount of mixing water apparently was not specified directly but was indicated as the minimum required for the chosen method of placing. For the zero-slump concrete which seems to have been used normally, a water-cement ratio between 0.35 and 0.50, by weight, was specified. Apparently a somewhat higher water-cement ratio was permitted when the concrete contained crushed stone than when gravel was used.

Paving Operations

In contrast to design practices, the construction methods used by the Germans were

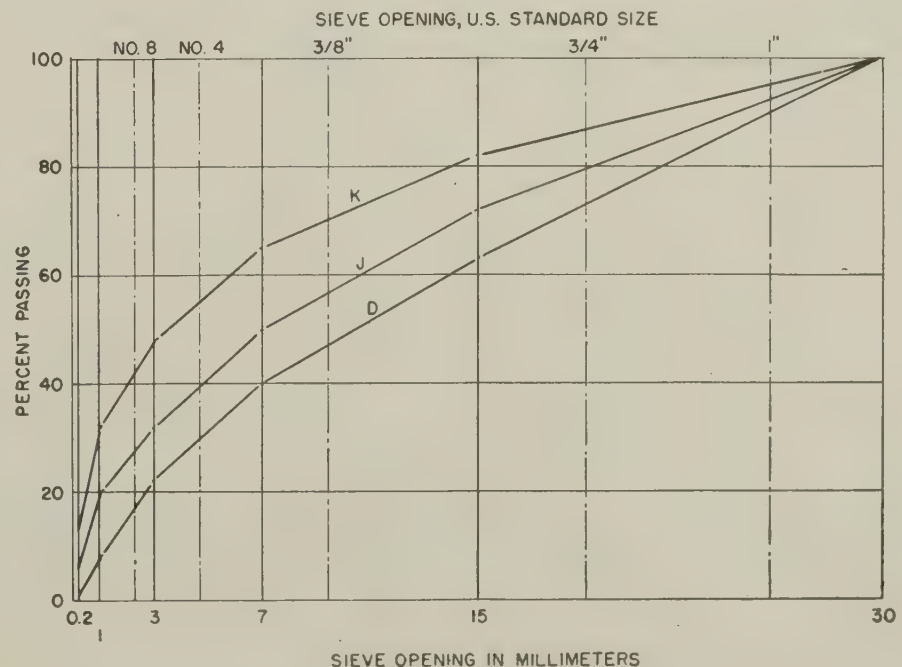


Figure 7.—Grading curves for proportioning combined aggregate.

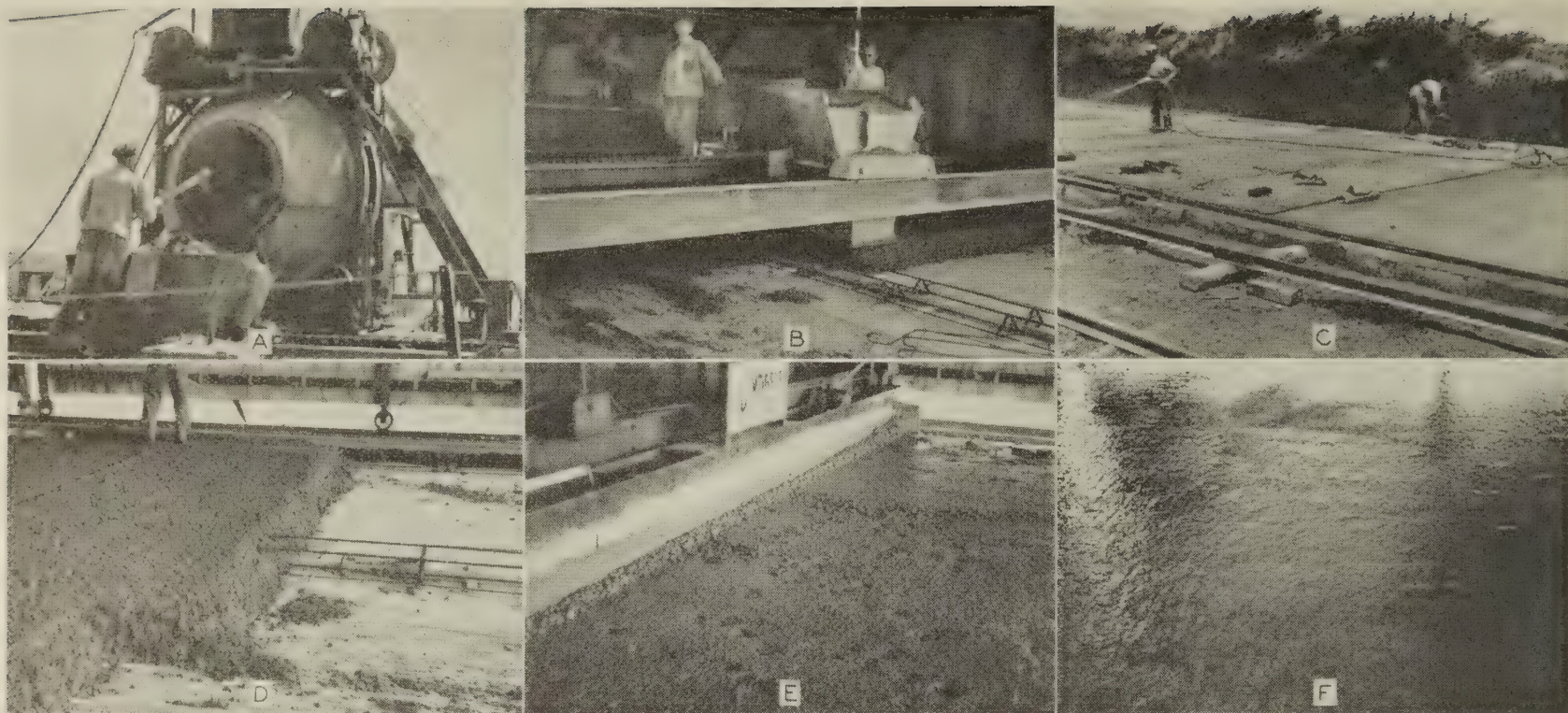


Figure 8.—Construction on the autobahn south of Stuttgart: (A) Dumping concrete into spreader hopper; (B) spreading concrete; (C) watering curing mats. Texture of the concrete is shown (D) after spreading, (E) as the Vögele vibrator began a second pass, and (F) after the final finishing.

totally different from those employed in this country. With minor exceptions it was the German practice to use trains of heavy traveling equipment in connection with the various paving operations on the autobahnen. This equipment, spanning the entire 25-foot-wide pavement slab, was carried on steel rails which, in turn, were mounted on the concrete shoulder strips previously mentioned. Separate mixers, spreaders, and finishers were used for each course. Mixers traveled on the rails and not on the subgrade and were serviced

by industrial railways running on the center strip. Proportioning was at a central plant, each size of aggregate being weighed separately. The mixer was sometimes mounted separately and sometimes on the same frame as the spreader. Spreaders were of the vertical-discharge hopper type. After receiving a load of concrete they were discharged by moving the hopper slowly from one side to the other as the entire machine moved forward.

It was the general practice to use a very dry (zero-slump) mix in pavement construc-

tion. In this respect the Germans followed British practice rather than that favored generally in this country. Such mixtures are, of course, entirely unsuited for use with an ordinary power-driven screed, which exerts comparatively little compactive effort but merely smooths the surface after the concrete has been distributed by the spreader. German engineers have developed several types of compaction equipment. In fact, considerable research appears to have been conducted on the problem, the results of which are

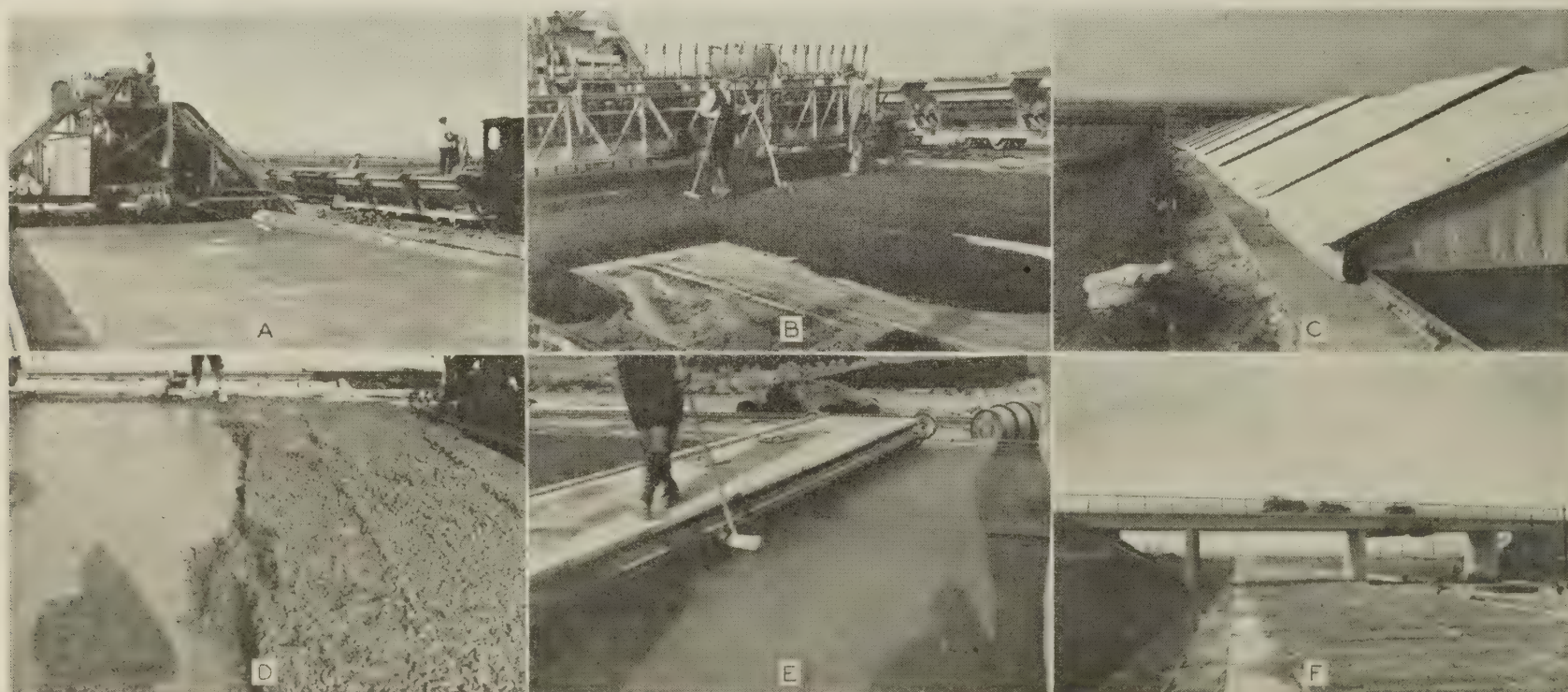


Figure 9.—Construction on the autobahn north of Kassel: (A) Mixer spanning subgrade, with industrial railway alongside; (B) Dinger tamper in operation; (C) sunshades used for preliminary curing. Texture of the concrete is shown (D) before and after tamping, and (E) after brooming. (F) Woven-reed curing mats were used.

discussed in detail in a report by Schade originally issued in 1940.⁹

There were two general types of finishers used on the motor roads, the Vögele and the Dingler. The Vögele was a three-element machine, the first and third elements being reciprocating screeds of the usual type. Mounted between the screeds was a heavy plate which could be operated either as a vibrator or a tamper. Vibrating frequencies were originally quite low (150–250 per minute) but were later increased to as high as 3,600 per minute. It was sometimes operated alternately as a tamper and as a vibrator in successive passes, and frequently several passes were required to obtain the desired finish. The Dingler machine was a very much heavier piece of equipment. It consisted of a series of tamping hammers, with 4- by 10-inch faces, weighing 55 to 120 pounds each and mounted in a row just behind the front screed. The height of fall was about 6 inches and they were operated at about 70 blows per minute. These tamping hammers were apparently only used for compacting the lower course. The final finishing was by means of a combined screed and tamper operating at about 600 strokes per minute and several passes were usually required to produce the required finish.¹⁰

Another unique feature was the practice of conducting the placing, finishing, and preliminary curing operations under sunshades. These were mounted on light wooden frames spanning the entire pavement and carried on light steel rails mounted outside and parallel to the concrete shoulder strips. In this way the concrete was protected from sun and wind from the time it was placed until it had hardened sufficiently to receive the final curing mats. The frames used for protecting the concrete during placing were sufficiently high to permit the equipment to pass under them and to accommodate the men working on the final finish. The frames used for the preliminary curing were low and were completely enclosed.

During the summer of 1936 Mr. F. R. McMillan, of the Portland Cement Association, visited Germany and took a number of photographs of the various paving operations in use that year. The authors have his very kind permission to include in this report several of these photographs, which illustrate in a striking manner the marked difference between the construction methods used by the Germans and those in use in this country.

Figure 8 illustrates several operations on sections of the autobahnen just south of Stuttgart. The pictures were taken on July 23, 1936, and this particular section was inspected by the authors on July 28, 1947, just 11 years later. Figure 8A shows the mixer discharging a batch of concrete into the

spreading hopper which then moves forward under the sunshade and discharges as shown in figure 8B. The spreader was moving from right to left when the photograph was taken. Note the very dry concrete, shown in greater detail in figure 8D. Figure 8E shows the Vögele finisher starting its second pass and figure 8F the final finish which, in this case, was only secured after several passes of the machine. Figure 8C is another view of the final surface. It also shows the woven-reed curing mats which were left in place for 3 weeks and were kept continuously wet during that period. This view also shows the light rail used to carry the sunshade frames. In his notes accompanying the photographs, Mr. McMillan states that the usual rate of progress, using machinery of the type de-

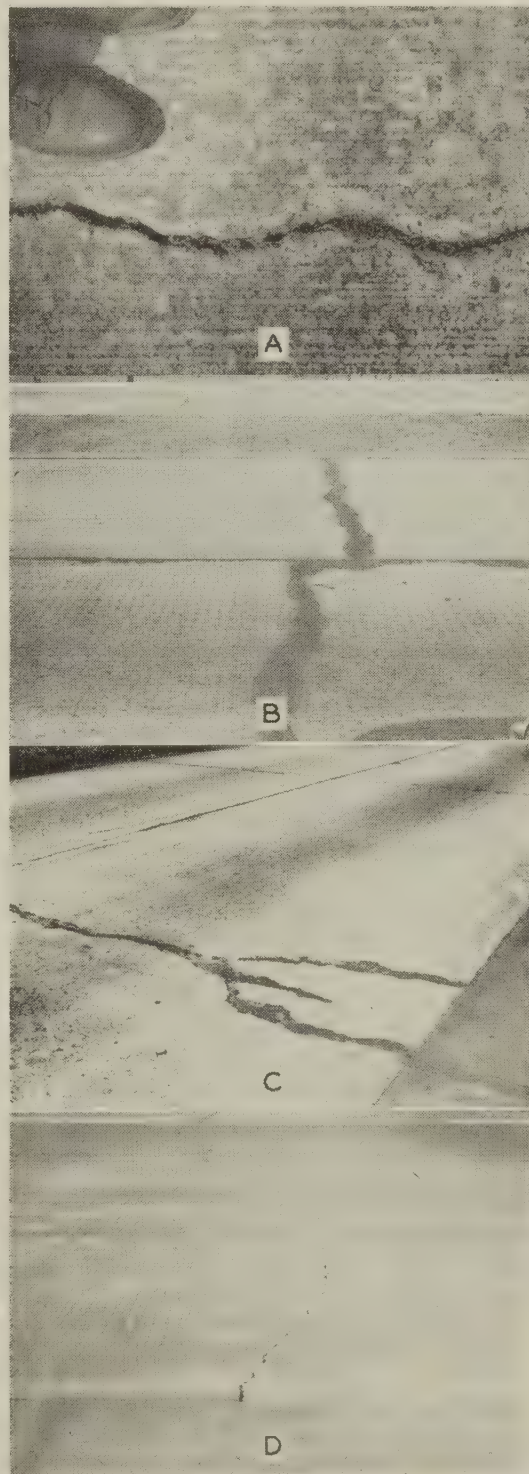


Figure 10.—Transverse cracking was common throughout the system.

scribed, was about 1,000 feet in 16 hours for full-width construction of 24 feet 7 inches.

Figure 9 illustrates operations on a section about 40 miles north of Kassel on which the Dingler machine was being used. Figure 9A shows the concrete mixer spanning the subgrade with the industrial railway on the right. Figure 9B shows the tamping machine in action compacting the lower course. Note again the dry consistency of the concrete. Also note that this particular operation was not protected by sunshades. Figure 9D illustrates the appearance of the concrete in the surface course before and after compaction by the tamping screed. Figure 9E shows the finished surface being given a light brooming and figure 9C shows the low sunshades used during the preliminary curing. Figure 9F is a general view of the pavement with the curing mats in place.

PART III.—PRESENT CONDITION

Structural Defects

The structural defects that usually develop in concrete roads in the United States were found in the German pavements.

Transverse cracking was common and was not confined to any particular part of the system. There seemed to be no relation between transverse cracking and terrain or soil conditions. In most instances the cracks were closed and were not discernible at speeds normally traveled by automobiles. In some cases the cracks were open and the expansion joints had closed. This condition was noted on a section just north of Göttingen, near Kassel. Transverse cracking seems to be most pronounced on the route from Hannover to Frankfurt through the Ruhr Valley, from Frankfurt to Kassel, and from Frankfurt toward Heidelberg. The cracking shown in figure 10 is typical of that observed.

Spalling at transverse and longitudinal joints was found principally in the area around Frankfurt. At the transverse joints this appeared to be due in most instances to the infiltration of sand and other incompressible debris into the joint and the subsequent overstress of the concrete upon expansion of the pavement. Figure 11 illustrates the spalling at joints in the vicinity of Frankfurt.

Faulting at joints and at transverse cracks, in most instances less than $\frac{1}{4}$ inch, was observed in areas where there was evidence of a considerable amount of traffic. At a location approximately 50 miles west of Kamen faulting of joints in the outside lanes of the pavement was found to be $\frac{1}{2}$ to $\frac{3}{4}$ inch. Faulting in the outside lane in the Ruhr-Frankfurt area is shown in figure 12A. The photograph was taken with the camera facing in the direction of the traffic and it should be noted that the expansion joints in the left lane, which had not faulted, are visible while those in the right lane had faulted sufficiently to be obscured.

In the München-Salzburg section there were a few small corner breaks. With few exceptions, these were the only corner breaks found in the entire mileage inspected. A

⁹ *The mechanical finishing of concrete in road construction, with special reference to vibration. A new way of increasing output*, by R. Schade, 1940; translated from the German by L. Baumgartel; Library Communication No. 70, Department of Scientific and Industrial Research, British Road Research Laboratory, February 1947.

¹⁰ Further details regarding construction equipment may be found in *Concrete road construction on the Reichsautobahnen*, by F. G. Turner, for the British Ministry of Transport. H. M. Stationery Office, London, 1939.



Figure 11.—Spalling at joints occurred principally in the vicinity of Frankfurt.

small corner break which has been patched is shown in figure 13A.

Some faulting and some spalling were observed at the longitudinal center joints. In many instances the longitudinal center joint was crooked. The longitudinal joints shown in figures 13C and D are typical of the condition frequently observed.

The riding surface of the pavement over a considerable portion of the mileage inspected was poor as compared with standards in the United States. For example, in the section from Berlin to Braunschweig the pavement was rough and there were many small areas where there was evidence that bush hammering had been resorted to in an effort to produce a better riding surface. On the other hand, there were many long stretches of smooth-riding pavement. Here again, there appeared to be no relation between the degree of smoothness and soil conditions, terrain, or other factors.

Mud Pumping and Warping

Evidence of mud pumping at joints in the concrete pavement was not found on the autobahnen. The absence of pumping over the areas where fine-grain soil subgrades predominate may be attributed to the passage of relatively few heavy loads and to the use of

a layer of granular material such as sand or sand-gravel under the pavement. The sealing of the expansion joints and cracks to prevent the entrance of surface water was inadequate.

Warping was observed in a short section 31 miles south of Köln. The soil in this area is a silty clay and would be susceptible to sufficient volume change with an increase in moisture content from a dry compact state to cause warping. The layer of cohesionless granular material usually used under the pavement on the autobahnen would not be effective in preventing warping. Observation of this section was made during a light rainstorm and the water falling on the pavement was running away from the joints, indicating that they were high. This condition is illustrated in figure 12B.

The asphaltic surfaces of the shoulder strips were generally in good condition. Occasional local failures were noted.

Quality of the Concrete

So far as could be determined from an examination of the surface of the pavements and from chip samples taken from the corners or edges of the slabs, the quality of the concrete, with few exceptions, appeared to be excellent. The concrete had a good ring under the

hammer and the difficulty encountered in breaking off the chip samples revealed, in general, a high degree of strength and toughness. As a further check on quality, a number of the chip samples were examined under the microscope in an effort to determine something of the character of the matrix, the presence and nature of voids, etc. Thin-section examinations were also made of many of the igneous aggregate types found in the samples. These data are summarized in table 2.

As will be noted, the examination of the chip samples revealed the concrete generally to be of excellent quality. There was no evidence of excess water. On the contrary, with the exception of sample No. 7, which was of bridge concrete, such voids as were noted were quite large and evidently of air origin, indicating that mixtures of very dry consistency had been used. This is borne out by the construction record to which reference has already been made. The bond between matrix and aggregate particles was in practically all cases sufficiently strong to fracture the aggregate particles themselves, again revealing good strength and probably a high degree of durability.

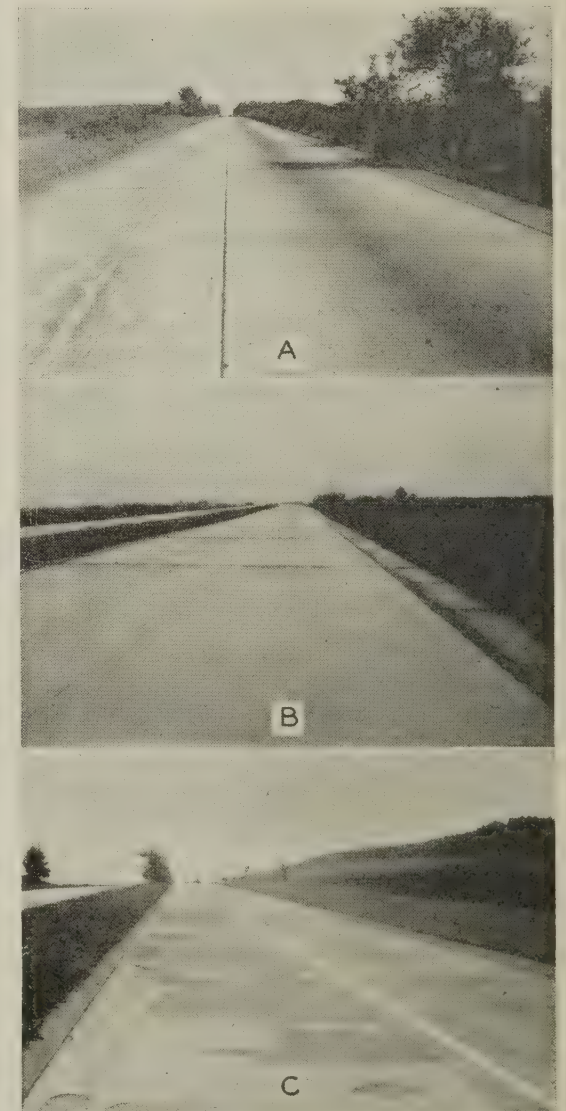


Figure 12.—(A) Joint faulting in the left lane (Ruhr-Frankfurt). (B) Warping resulted in high joints (Ruhr-Frankfurt). (C) Surface scale was seen only on the München-Salzburg route.

In the microscopic examination a magnification of 20 diameters was used, with a change to 50 diameters when some special detail seemed to indicate the desirability of the increased power. In none of the specimens was there any evidence of water gain beneath the lower surfaces of the aggregate particles.

Surface Texture

A characteristic feature of the pavement surface is the general absence of the rather heavy mortar top which we frequently leave on our roads and which so often scales off, leaving the surface rough and unsightly. An examination of the surface indicated, in most instances, the use of a mix that contained just enough paste to fill the voids in the aggregate and leave a small excess for finishing. Surfaces were dense and compact, with the coarse aggregate either just below the top or actually exposed in some degree. Typical illustrations of variations in the surface texture are shown in figure 14. Figure 14A is a view of a surface on which the screed marks are still showing. The corrugations are quite shallow and are about 1 inch apart. This particular photograph was taken just east of the Bad Oeynhaus interchange on the road to Hannover. Other sections showed the same condition, particularly on the inner lanes which, of course, carry less traffic. Figure 14B illustrates a fine-grained sand finish texture which was also noted in several places. The photograph was taken on the same road as figure 14A and about 5 miles east of the interchange.

Figures 14D-F illustrate varying degrees of coarse aggregate exposure. Figure 14D indicates the type of surface about 7 miles south of Frankfurt on the road to Stuttgart; figure 14E a section of the Hamburg-Lübeck road; and figure 14F, a section between Hannover and Berlin at kilometer post 194. Note in the last-mentioned illustration that coarse aggregate up to 1 inch in size, which is close to the maximum size used in the top course, is exposed. Many of the exposed aggregate surfaces are smooth, with a terrazzolike texture, probably the result of slight

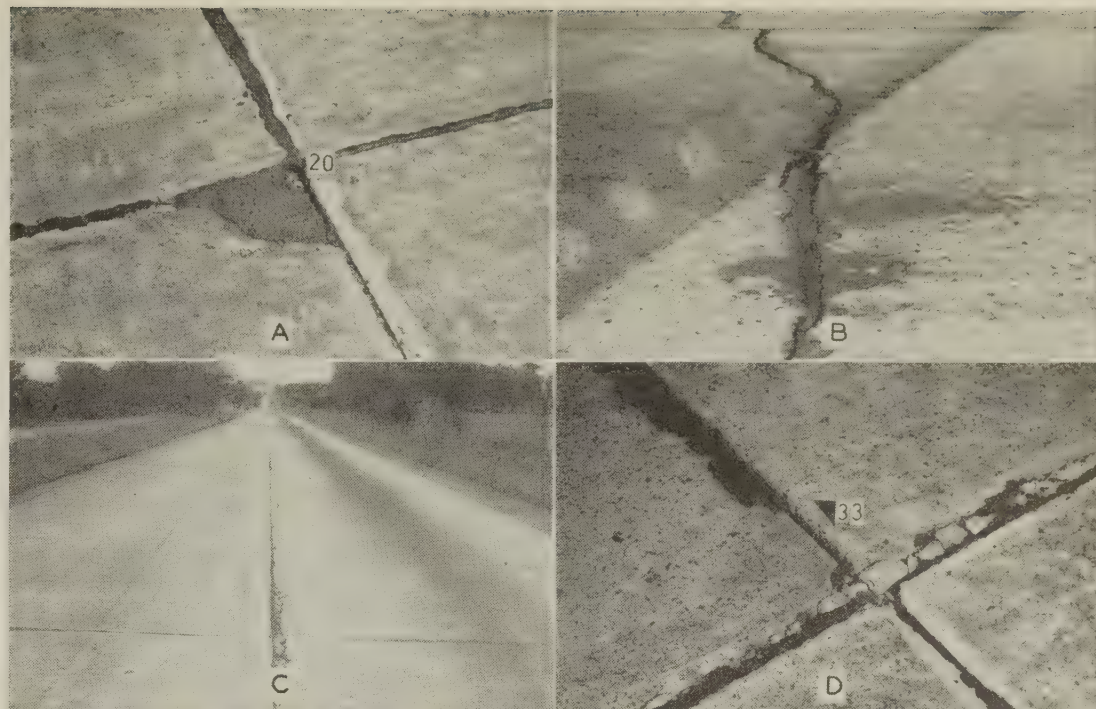


Figure 13.—(A) A patched corner break (Frankfurt-Stuttgart). (B) A large random crack (Hannover-Ruhr). (C, D) Typical faulting of longitudinal joints (Ruhr-Frankfurt).

surface wear. In many places this type of surface was noted in the center of the traffic lanes (particularly the outside lane) with the original thin mortar surface showing at the extreme edges. In other sections where the coarse aggregate was exposed, particles of the hard igneous rock protruded slightly above the mortar, producing a slightly rough surface. This condition may have resulted from the scaling off of the light surface mortar or it may have been the result of wear—it is difficult to determine the exact cause. However, it should be emphasized that the slightly rough surface is not objectionable from the standpoint of rideability. Neither should the exposure of the aggregate have any detrimental effect on the ultimate durability of the concrete, since the quality of both aggregate and concrete was excellent.

There was evidence of surface grinding in several places. This was undertaken, evidently, to correct irregularities resulting from the difficulty of finishing the very dry concrete generally used.

A detail of the surface texture of black-colored concrete is shown in figure 14C.

Scale and Disintegration

With the exception of one section in southern Germany, the entire 1,000 miles of pavement covered by this inspection were found to be practically free from scale, either of the surface or of the progressive type. Only on certain stretches of the München-Salzburg section in Bavaria was surface scaling noted in any appreciable amount. Figure 12C is a general view of the pavement showing about the worst condition observed. This picture was taken

Table 2.—Results of microscopic examination of chip samples

Sample No.	Location	Aggregate types		Comment concerning—		
		Coarse	Fine	Aggregates	Matrix	Fracture
1	Hamburg-Bremen.....	Olivine basalt.....	Quartz, with sandstone, quartzite, and chalcedony.	Sound. No weathered particles.	Mass crystalline. Excellent bond on aggregate surfaces.	Through aggregate particles.
2	Frankfurt-Stuttgart, 20 miles south of Frankfurt.	Basalt.....	Quartz, quartzite with some chalcedony and limestone.do.....	Mass crystalline and well bonded to aggregate.	Do.
3	Frankfurt-Stuttgart, 34 miles south of Heidelberg.	Rhyolite.....	Quartz, feldspar, and chalcedony.do.....	Mass crystalline and well bonded to aggregate. Excellent bond. No limy deposit in air pockets.	Do.
4	Stuttgart-München, 15 miles south of Stuttgart.	Limestone and syenite.	Quartz, limestone, chalcedonic chert, granite, feldspar.	Sound and strong. Break surfaces of yellowish tint. Possibly some weathering.	Mass crystalline and well bonded to aggregate. No limy deposit in air voids.	Through aggregates.
5	München-Salzburg, 20 miles west of Austrian border.	Altered granite.....	Sandstone, quartz, granite, limestone, feldspar.	Sound.....	Mass crystalline. Mortar very dense.	Through nearly all aggregate particles.
6	Hannover-Ruhr, 30 miles west of Bad Oeynhaus.	Basalt.....	Quartz, quartzite, plus other unknown fragments.	Sound, unweathered materials.	Mass crystalline.....	Through aggregate particles.
7	Ruhr-Frankfurt (bridge).....	Basalt, feldspathic sandstone.	Quartz, syenite, feldspar, feldspathic sandstone.	Sound. No evidence of weathering.	Mass crystalline. Water voids show no evidence of limy deposit. Strong and dense in appearance.	
8	Ruhr-Frankfurt, 28 miles south of Leichingen.	Basalt.....	Quartz, granite, feldspar.....	Sound. One weathered particle.	Mass crystalline with few air voids. No evidence of limy deposit. Dense, strong concrete.	
9	Frankfurt-Stuttgart, 20 miles south of Frankfurt.	Granodiorite, feldspathic sandstone.	Quartz, quartzite, feldspar.....	Sound.....	Light gray; mass crystalline. Few air voids. No limy deposit.	Few fractured aggregate particles, indicating lower bond strength than in other samples.

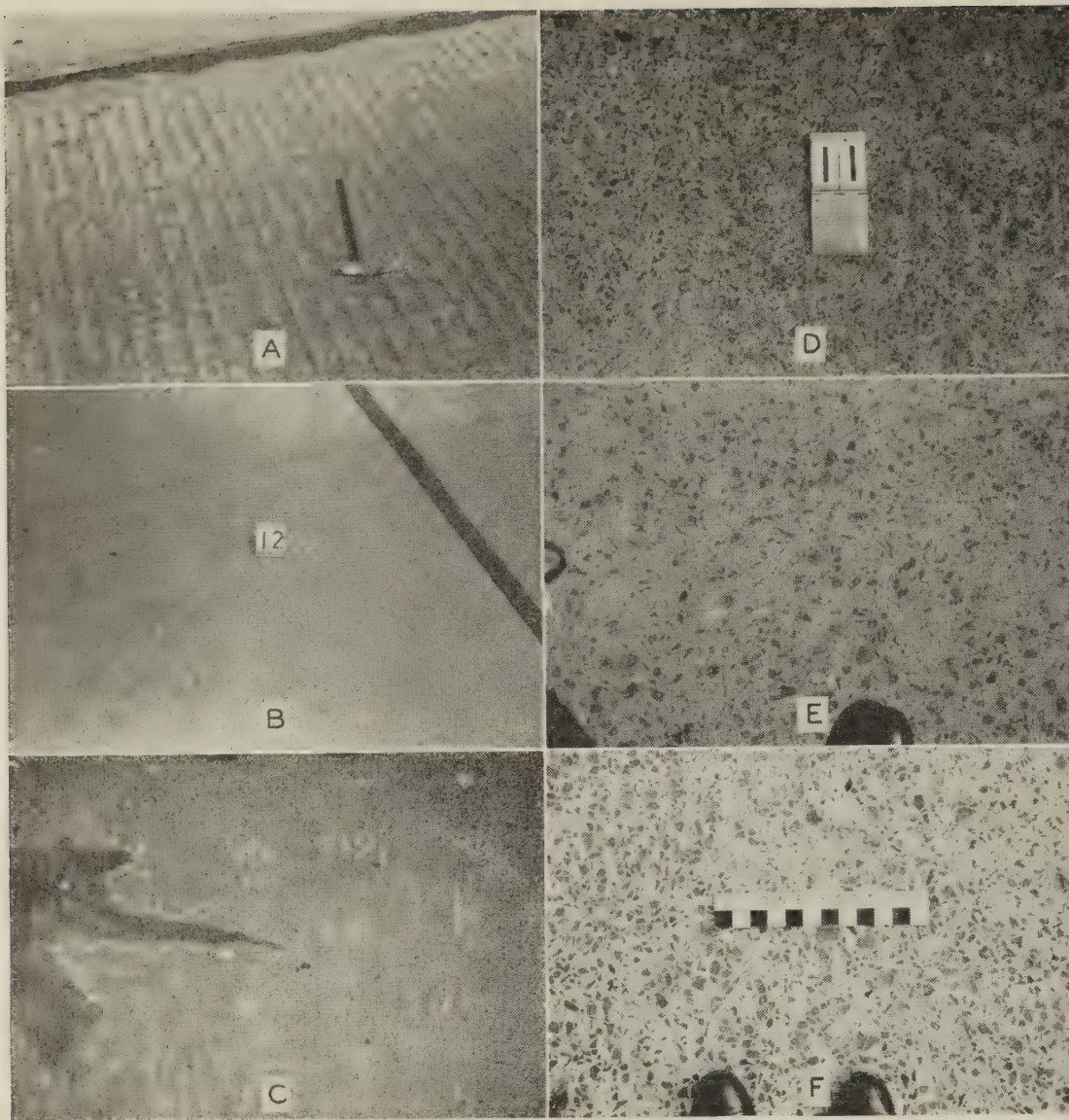


Figure 14.—(A) In some places screed marks still appear as shallow corrugations. (B) A fine-grained sand finish. (C) Black colored concrete. (D-F) Typical appearance of varying degrees of coarse aggregate exposure.

about 20 miles west of the Austrian border at Salzburg. Figures 15A and C are details showing the type of scaling that has developed on this section. It is apparently of the surface type only; that is, the concrete below the scaled areas appeared to be sound. Figures 15A and C indicate a rather heavy layer of surface mortar at this point and this fact, coupled with the possibility that the winters in this region, due to its high elevation, may be somewhat more severe than elsewhere in western Germany, may account for the scaling. However, in this connection it should be noted that, according to weather records as shown in table 1, the average January temperature at München was only slightly lower and the annual rainfall only slightly higher than for the more northern cities.

It has been stated that the distress which developed on the München-Salzburg section at a relatively early period was due to the use of inferior aggregate. A chip sample (No. 5 in table 2) taken at the approximate location shown in figure 15A did not, however, reveal the presence of an appreciable amount of inferior material, although the petrographer did indicate that the coarse aggregate in this sample was an "altered granite." Just how

much the generally poor subgrade conditions in this area, or other factors, may have contributed to the scaling is not known.

Only isolated scaled areas, all of the surface type, were noted elsewhere on the system. Figure 15D is a detail of a local scaled area on the Frankfurt-Stuttgart section, a few miles south of Frankfurt. From the standpoint of general condition this is one of the poorest sections on the autobahnen. As previously noted, considerable spalling at joints (see fig. 11) has also developed on this section. Figure 15B is a detail showing a thin surface scale on the Hannover-Ruhr section, about 30 miles east of the Bad Oeynhausen interchange. This pavement, as well as many other portions of the system, had evidently been sprayed with bituminous material at some time, probably for camouflage purposes. The scale here is not deep enough to affect the riding quality of the surface.

Incipient disintegration of the concrete, as revealed by the formation of so-called D lines along the edges of the pavement and along joints, was found at only one place on the system; a point on the München-Salzburg section where the worst scaling was found. The D lines in this case were found along the

outside (north) lane and were very fine. The condition is shown in figures 15E and F. In the United States, cracks of this type have come to be associated with the beginning of disintegration due to accelerated weathering.

The general freedom of the motor roads from scaling and other troubles associated with severe weathering may be attributed to the following factors, the listing not necessarily being in order of importance:

1. The general excellent quality of the aggregates.
2. The low water-cement ratio (usually 0.45 or less, by weight).
3. The practice of designing mixtures and compacting concrete in such a manner as to avoid the formation of a heavy surface layer of mortar.
4. The comparatively mild winters.
5. The fact that chloride salts were not used for ice removal.
6. Thorough curing.

The possible influence of all of these factors is fairly obvious. The role of the cement is not so apparent. According to Swayze's report, the cements used in the autobahnen were distinctly inferior to those manufactured in this country. However, the concrete is good and from this fact we can conclude at least that it is possible to make good concrete with the German cements provided the other conditions are right. Whether these cements would perform any better than the American cements under conditions that prevail in the northern part of the United States is another matter. An intensive research program would be necessary to answer this question.

PART IV.—SUMMARY AND RECOMMENDATIONS

As mentioned in the body of this report, the comparatively light traffic on the German motor roads, particularly as regards the movement of heavily loaded vehicles, as well as the comparatively mild climate, make comparisons with the performance of concrete pavements in this country difficult. This is especially true if we attempt to compare the German primary roads with the heavy-duty concrete pavements of the industrial States of the north and middle west. Many of these roads undoubtedly carry a far heavier volume of truck traffic than any of the motor roads in Germany. The question therefore is: How would the German roads perform under similar climatic and traffic conditions? It is, of course, impossible to say definitely. However, it is the authors' opinion that these two factors—the low volume of heavy truck traffic and the relatively mild climate—probably account for the comparative freedom of the German motor roads from structural defects and that, under comparable climatic and traffic conditions, the structural performance of the German roads would be about the same as the average concrete pavement in this country. This opinion is borne out by the fact that structural failures such as joint spalling, faulting, etc., were found to be more numerous and, in general, more serious on sections of the system where the indications pointed to the heaviest traffic.

In this opinion, the authors also recognize the beneficial effect of using a layer of granular material under the pavement, particularly with respect to the control of mud pumping at joints, a condition which was not found anywhere on the system.

The quality of the German concrete was excellent. Although the design strengths were not unusually high as compared to standards in use in the United States, the indications were that a high degree of uniformity was obtained. This may possibly be due to the care used in proportioning mixes, as, for example, the use of several sizes of aggregate and the practice of limiting the maximum size in the top course to about 1 inch. Reference was also made to the general absence of the relatively heavy mortar top which we see so often on our roads, and to the fact that this might have reduced the tendency to scale. It is believed that the principle of consolidating the pavement slab by vibration is good although, in the authors' opinion, the Germans may have gone too far in their effort to use excessively dry concrete. This is indicated by the rather rough surface with occasionally noted evidence of grinding off of high spots. However, the principle of using as dry a mix and as low-sanded a mix as can be consolidated properly by the effective use of vibratory equipment is good and, in the authors' opinion, should be encouraged in this country.

The uniformly excellent durability of the concrete is probably also due, in part at least, to the excellent quality of the aggregates generally used. This is also a factor to which we might give more consideration in the United States.

The probable influence of long continued and thorough curing also should not be ignored. Whether the elaborate precautions for protecting the fresh concrete from sun or wind were always necessary is problematical. However, the practice of thorough water curing, although old-fashioned in terms of modern American practice, is still, in the opinion of the authors, the preferred method.

The survey revealed nothing regarding cement except that, regardless of the quality of the cement used, the quality of the concrete was excellent; and also the fact that concrete containing Eisenportland (portland-blast-furnace slag cement) was, on the average, as good as concrete containing straight portland cement. Although it was impossible to determine on just what portions of the autobahnen the Eisenportland was used, it was stated to have been used on about one-third of the total mileage. This fact would seem

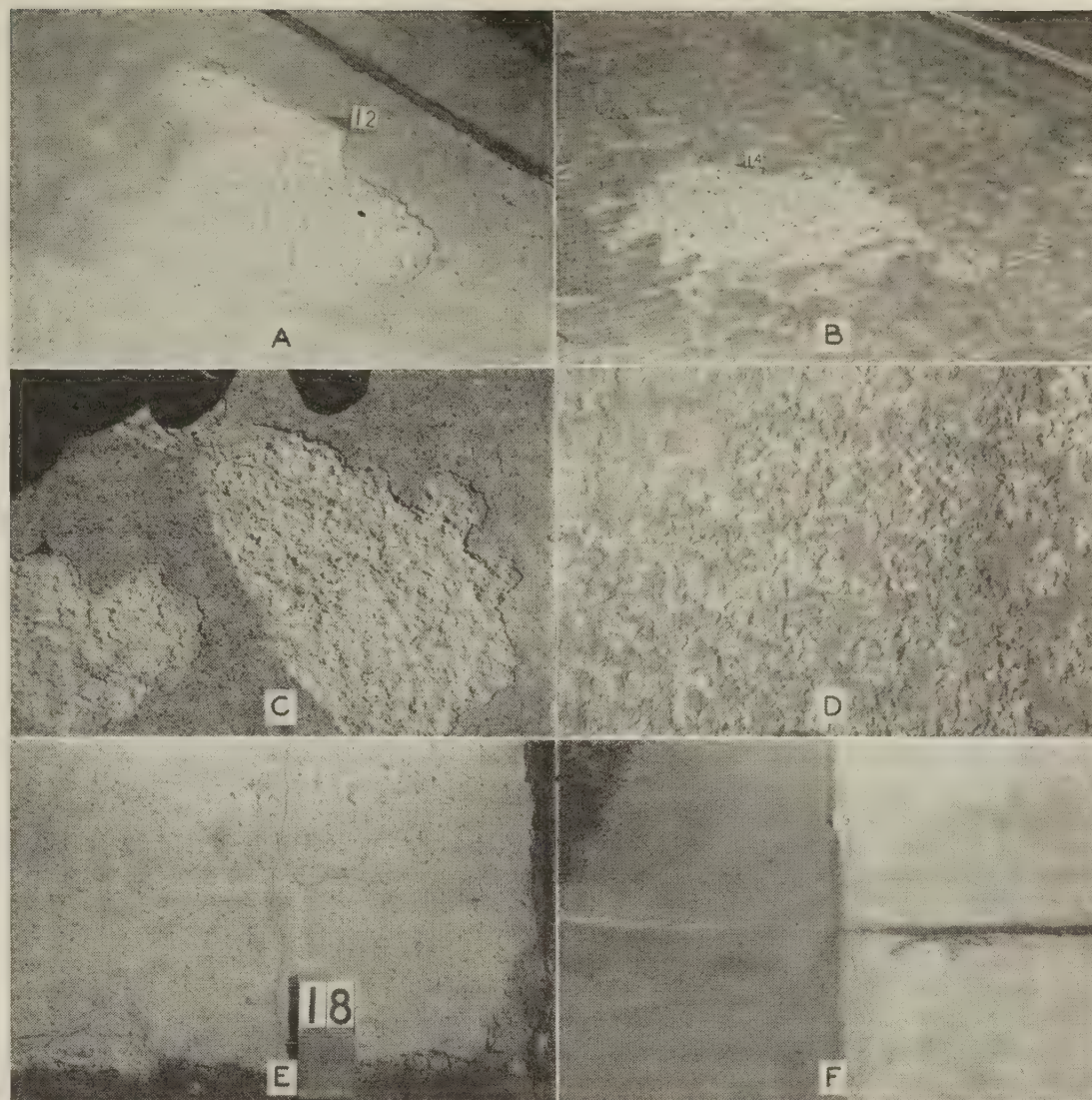


Figure 15.—(A-D) Surface scale occurred commonly on the München-Salzburg route but only isolated cases were observed elsewhere. (E, F) Incipient disintegration, revealed by D lines, was found only at one place on the system.

to warrant the statement that its performance was satisfactory.

Recommendations

As the result of their examination of concrete pavements on the German motor-road system the authors recommend that steps be taken to initiate a comprehensive program of research on each of the following subjects:

1. Study of the possibility of insuring greater uniformity in pavement concrete by reducing the maximum size of the coarse aggregate.

2. Development of more effective methods of compacting concrete in pavements by mechanical means, such as vibration, tamping, etc.

3. Study of the effects of variations in the chemical composition of cements and the methods of manufacturing cements on the properties of concrete. In this work the authors would go considerably outside the range in composition and fineness now being studied under the program of the "Long-time study of cement performance in concrete."¹¹ Work of this nature should be carried out by the manufacturers and might well be accomplished by an extension of the present program of the long-time study to cover these variables.

¹¹ For a description of this project, see *Progress in the long-time study of cement performance in concrete*, by F. R. McMillan; Proceedings of the American Concrete Institute, vol. 13, No. 5, April 1942.

Analysis of Rectangular Reinforced Concrete Sections Subjected to Direct Stress and Bending in Two Directions

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The problem of determining the unit stresses in a reinforced concrete section subjected to direct stress and bending in two directions often occurs in the analysis of continuous frames, and a solution that simplifies the work and removes some of the drudgery is indeed welcome. The method presented in this article is an improvement over procedures now being used since it eliminates one of the variables—the slope of the neutral axis of the section—at an early stage. The slope is determined within a close degree of correctness, with fairly simple calculations, by first considering the entire area of the cross section as effective.

ALTHOUGH a wealth of material has been presented to the engineering profession on the subject of direct stress and bending in reinforced concrete sections, most of the data presently available are limited to the special case of the general problem; i. e., direct stress plus bending in only one direction. The more general problem, direct stress plus bending in two directions along axes 90 degrees apart, has been treated in general terms only by Cross and Morgan,¹ and more specifically by Saville.³ The standard specifications of the American Association of State Highway Officials⁴ gives the general equation for determining the position of the neutral axis for the general case, but this equation is not very helpful without a knowledge of how to apply it.

The problem of determining the unit stresses in a reinforced concrete section subjected to bending and direct stress in two directions occurs rather frequently in practice. Everyday examples of this problem, among many others, are building columns in ordinary beam-and-girder construction with bending induced by girders in two directions framing into the

columns, open-framed bridge bents and abutments in continuous construction with bending induced by longitudinal girders and transverse cross beams, and the single column or umbrella-type bridge pier with bending induced by longitudinal girders and the transverse cross beam at the top of the pier. While the analysis of members of this type is not particularly difficult, it is rather long and involved. The purpose of this article is to demonstrate the method outlined by Cross by applying it to a specific example, and to show the general applicability of the method. Suggestions for helping the designer to make more reasonable assumptions in the first trial and thus reduce the number of solutions required to bring the assumed and actual positions of the neutral axis to sufficiently close agreement are also given. The reader is referred to Cross's works for the derivation of the formulae employed.

NOTATION

The notation used in this article is for the most part standard and is as follows:

a is the area, in square inches, of a portion of the section. In the case of the steel reinforcing, it is the transformed area of the steel in terms of the concrete.

A is the total effective area of the section, including the transformed area of all reinforcement.

X and Y axes are the coordinate axes chosen to compute the properties of the section. These axes may pass through any desired point, but should be chosen so as to simplify the calculations of the section properties. In general, these X and Y axes are not through the center of gravity of the effective cross section.

x and y are the coordinates of any point referred to the X and Y axes arbitrarily chosen to compute the section properties, as defined above.

ax and ay are the products of the elementary areas and the coordinates of the center of gravity of those areas referred to the X and Y axes. These products are really the statical moments of the elementary areas about the X and Y axes.

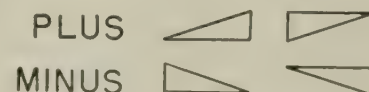
$I_{yy} = ax^2 + i_{yy}$. This value is the moment of inertia in inches⁴ of the effective area of the

cross section (including the transformed area of the reinforcement) about the Y axis. It is computed by summing the moments of inertia of the several elementary areas obtained by the familiar transfer axis theorem, i. e., the product of the elementary area and the coordinate x to its center of gravity squared, plus the moment of inertia of the area about the y axis through its own centroid.

$I_{xx} = ay^2 + i_{xx}$. This value is the moment of inertia in inches⁴ of the effective area of the cross section (including the transformed area of the reinforcement) about the X axis. It is computed by summing the moments of inertia of the several elementary areas obtained by the transfer axis theorem, i. e., the product of the elementary area and the coordinate y to its center of gravity squared, plus the moment of inertia of the area about the x axis through its own centroid.

$I_{xy} = axy + i_{xy}$. This value is the product of inertia in inches⁴ of the effective area of the cross section (including the transformed area of the reinforcement) about the X and Y axes. It is computed by summing the products of inertia of the several elementary areas obtained by multiplying the elementary area by the coordinates x and y of its center of gravity, plus the product of inertia of the elementary area about the x and y axes through its own centroid. In dealing with rectangular sections, or irregular sections bounded by straight lines, the elementary area will be either a rectangle or a triangle or a combination of both. For any rectangular section, $i_{xy} = 0$. For any triangular area about x and y axes through its center of gravity and parallel to

the base and altitude of the triangle, $i_{xy} = \frac{b^2h^2}{72}$, where b and h represent the base and altitude, respectively, of the triangle. The product of inertia of a triangle will be positive in sign if its hypotenuse slopes downward to the left and negative in sign if its hypotenuse slopes downward to the right, as indicated in the sketch below.



¹ *The column analogy*, by Hardy Cross; University of Illinois Engineering Experiment Station Bulletin No. 215; October 1930.

² *Continuous frames of reinforced concrete*, by Hardy Cross and N. D. Morgan; John Wiley & Sons; 1932.

³ *Analyzing non-homogeneous sections subjected to bending and direct stress*, by W. G. S. Saville; Civil Engineering, vol. 10, No. 3, March 1940; p. 170.

⁴ *Standard specifications for highway bridges (1944)*; American Association of State Highway Officials.

$M_{yy}=Px$ and $M_{xx}=Py$. These values are the applied moments, in inch kips, about the X and Y axes. They are computed as the product of the direct load P and the coordinate x or y of its point of application.

X_o and Y_o axes are the coordinate axes parallel to the X and Y axes first chosen to compute the properties of the section but passing through the center of gravity of the total effective area A .

$x_o, y_o, I_{x_o x_o}, I_{y_o y_o}, M_{x_o x_o}, M_{y_o y_o}, I_{x_o y_o}$. These values are the section properties and the applied moments referred to the X_o and Y_o axes.

The following values are the section properties about the X_o and Y_o axes through the center of gravity of the effective area A but corrected for the dissymmetry of the section:

$$I'_y = I_{y_o y_o} - I_{x_o y_o} \frac{I_{x_o y_o}}{I_{x_o x_o}}$$

$$I'_x = I_{x_o x_o} - I_{x_o y_o} \frac{I_{x_o y_o}}{I_{y_o y_o}}$$

$$M'_y = M_{y_o y_o} - M_{x_o x_o} \frac{I_{x_o y_o}}{I_{x_o x_o}}$$

$$M'_x = M_{x_o x_o} - M_{y_o y_o} \frac{I_{x_o y_o}}{I_{y_o y_o}}$$

The equation that follows is that of the neutral axis or line of zero stress referred to axes X_o and Y_o , and coordinates x and y in this equation are also measured from axes X_o and Y_o :

$$\frac{P}{A} + \frac{M'_y}{I'_y} x + \frac{M'_x}{I'_x} y = 0.$$

X_n and Y_n are the distances from the neutral axis to the extreme fibers of the section, measured parallel to the X_o and Y_o axes.

f is the fiber stress at any point and is equal to

$$\frac{M'_x}{I'_x} Y_n \text{ or } \frac{M'_y}{I'_y} X_n$$

f_c and f_s are the unit stresses, in pounds per square inch, in the concrete and steel, respectively.

METHOD OF ANALYSIS

To compute the unit stresses in a rectangular reinforced concrete section subjected to bending moments in two directions and direct stress the designer should proceed as follows (it is assumed that a trial section complete with reinforcement has been tentatively selected for analysis and this section drawn accurately to scale):

1. Estimate the position of the neutral axis of the section. This estimate of the position of the neutral axis requires judgment and experience since it involves drawing a line on the sketch of the section with position and slope unknown. Moreover, since the succeeding calculations are rather lengthy, an inordinate amount of time may be spent in obtaining sufficiently close agreement between the assumed and the computed positions of the neutral axis unless the position first selected

is approximately correct. It has been found that one variable, the slope of the actual neutral axis, may be determined quite accurately by first considering the entire area of the cross section as effective (which is equivalent to assuming that the concrete takes its full share of the tension induced by the bending moments) and computing the position of the neutral axis on this basis. The slope of the final neutral axis will be found to deviate only slightly from the slope of this fictitious neutral axis.

It is recommended, then, that the first step in the solution consist of computing the location of this fictitious neutral axis, considering the entire transformed area of the section effective, and plotting it upon the sketch. The first trial neutral axis for the cracked section should then be drawn parallel to this fictitious neutral axis and on the compression side of the section from it. Thus one variable has been eliminated. Also, if the stresses on the tension side of the section when computed considering the concrete to take tension are moderate, say 150 p. s. i. or less, it will be unnecessary to proceed with the more exact calculation. The remaining variable, the distance from the fictitious neutral axis of the entire section to the assumed trial neutral axis of the cracked section, will depend upon the eccentricity of the direct load, and must be estimated based upon the amount of this eccentricity.

2. The trial neutral axis of the cracked section having been selected, the designer proceeds to the calculation of the section properties about some convenient pair of axes. These quantities should preferably be tabulated, using some form similar to that shown later on in this article. After all of the section properties have been computed, tabulated, and totaled, they must then, in general, be corrected for two factors: First, they must be reduced to the X_o and Y_o axes passing through the centroid of the cracked section and parallel to the arbitrarily chosen X and Y axes used for computation purposes; and second, they must be corrected for the dissymmetry of the section.

3. The position of the centroid of the cracked section is found by dividing the sum of the statical moments of the elementary areas about the X and Y axes by the area A , the sum of the elementary areas. Thus $\sum \frac{ax}{A}$ and $\sum \frac{ay}{A}$ will give x_c and y_c , the coordinates of the centroid of the cracked section measured from the X and Y axes. The corrections to the quantities I_{yy} , I_{xx} , and I_{xy} are respectively Ax_c^2 , Ay_c^2 , and $Ax_c y_c$, and are always subtracted algebraically from the summations about the axes X and Y . Likewise, the corrections to be applied to M_{yy} and M_{xx} are, respectively, Px_c and Py_c and are also always subtracted algebraically. The values after these corrections have been made represent the section properties and applied moments about the X_o and Y_o axes and are the quantities $I_{y_o y_o}$, $I_{x_o x_o}$, $I_{x_o y_o}$, $M_{y_o y_o}$, and $M_{x_o x_o}$.

4. The second and last series of corrections

to be applied to the quantities enumerated in the last sentence of the preceding paragraph are the corrections for the dissymmetry of the section. The corrections to be applied to $I_{y_o y_o}$ and $I_{x_o x_o}$ are

$$I_{x_o y_o} \frac{I_{x_o y_o}}{I_{x_o x_o}} \text{ and } I_{x_o y_o} \frac{I_{x_o y_o}}{I_{y_o y_o}},$$

respectively, and are always subtracted algebraically from $I_{y_o y_o}$ and $I_{x_o x_o}$, giving the final quantities I'_y and I'_x . The corrections to be applied to $M_{y_o y_o}$ and $M_{x_o x_o}$ are

$$I_{x_o y_o} \frac{M_{x_o x_o}}{I_{x_o x_o}} \text{ and } I_{x_o y_o} \frac{M_{y_o y_o}}{I_{y_o y_o}},$$

respectively, and are subtracted algebraically from $M_{y_o y_o}$ and $M_{x_o x_o}$, giving the final quantities M'_y and M'_x .

5. The equation of the neutral axis is then

$$\frac{P}{A} + \frac{M'_y}{I'_y} x + \frac{M'_x}{I'_x} y = 0.$$

After substituting the numerical values in this equation and simplifying, the intercepts of the neutral axis on the sides of the section are obtained by placing values of x and y , the coordinates of the sides of the section measured from the X_o and Y_o axes in the equation, and computing the resulting values. These values are then compared with the actual values of the assumed position of the neutral axis. If the agreement is reasonably close, say within approximately 1 inch, no further trials will be necessary. If, however, the computed and actual intercepts of the neutral axis on the sides of the section differ by more than about 1 inch, then a new trial neutral axis should be selected and the entire process thus far described repeated.

6. When the designer has a solution wherein the computed and assumed positions of the neutral axis agree within the limits of error desired, all the data will be available for computing the actual stresses in the steel and concrete. The general equation for the stress f is:

$$f = \frac{M'_x}{I'_x} Y_n \text{ or } f = \frac{M'_y}{I'_y} X_n.$$

If the sketch has been accurately drawn to scale, the values of Y_n or X_n may be scaled. If computed values are desired, they may be found by solving the geometric figures composing the sketch, or they may be computed by substituting the proper values into the general equation for the neutral axis. X_n and Y_n may be the distances, measured parallel to the X_o and Y_o axes, to any point where the stresses are desired. However, the maximum stress values for the concrete and steel are usually the only values in which the designer is interested. Accordingly, the maximum distances X_n and Y_n , when substituted in the equations for stress given above, will give the maximum stresses desired. The stress in the steel is found, of course, by multiplying the stress computed from the equation at the beginning of this paragraph by the value of n for the concrete strength being used.

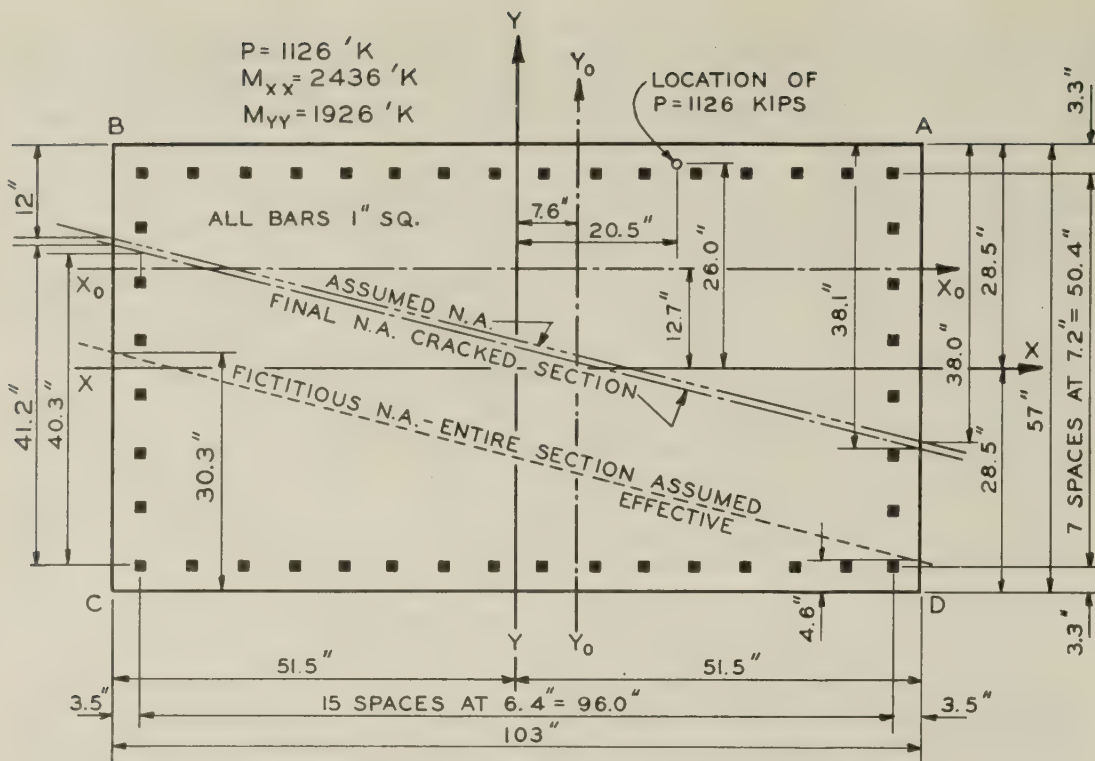


Figure 1.—Scale drawing of section analyzed in example.

APPLICATION TO AN EXAMPLE

In illustration of the analysis process, a specific example has been assumed and followed through, step by step.

Step 1. Make a scale drawing of the section for which the stresses are to be computed, as shown in figure 1. The assumed X and Y axes are included in the drawing. The three neutral axes shown in figure 1 are not drawn as part of the original sketch, nor are axes X_0 and Y_0 , since their positions are determined by the calculations that follow.

Step 2. The values of P , M_{xx} , and M_{yy} , shown in figure 1, are known. From these values the distances x and y from the X and Y axes to the point of application of the load are computed as follows:

$$x = \frac{M_{yy}}{P} = \frac{1,926}{1,126} \times 12 = 20.5 \text{ inches.}$$

$$y = \frac{M_{xx}}{P} = \frac{2,436}{1,126} \times 12 = 26.0 \text{ inches.}$$

Step 3. Compute I_{xx} about the X axis which was chosen through the centroid of the total area:

$$\text{Concrete} = \frac{bh^3}{12} = \frac{103 \times 57^3}{12} = 1,589,600.$$

Steel (44 1-inch squares) = summation of ay^2 (the transformed area of each bar = nA_s , and for $n=10$, each bar is equivalent to $10 \times 1 = 10$ square inches of concrete):

$$\begin{aligned} 32 \times 10 \times 25.2^2 &= 203,200 \\ 4 \times 10 \times 18.0^2 &= 12,900 \\ 4 \times 10 \times 10.8^2 &= 4,700 \\ 4 \times 10 \times 3.6^2 &= 500 \end{aligned}$$

$$\text{Total} = 221,300$$

$$\text{Total } I_{xx} = 1,589,600 + 221,300 = 1,810,900.$$

Step 4. Compute I_{yy} about the Y axis which was chosen through the centroid of the total area:

$$\text{Concrete} = \frac{57 \times 103^3}{12} = 5,190,400.$$

Steel = summation of ay^2 , with $a=10$ for each bar:

$$\begin{aligned} 16 \times 10 \times 48^2 &= 368,600 \\ 4 \times 10 \times 41.6^2 &= 69,200 \\ 4 \times 10 \times 35.2^2 &= 49,600 \\ 4 \times 10 \times 28.8^2 &= 33,200 \\ 4 \times 10 \times 22.4^2 &= 20,100 \\ 4 \times 10 \times 16.0^2 &= 10,200 \\ 4 \times 10 \times 9.6^2 &= 3,700 \\ 4 \times 10 \times 3.2^2 &= 400 \end{aligned}$$

$$\text{Total} = 555,000$$

$$\text{Total } I_{yy} = 5,190,400 + 555,000 = 5,745,400.$$

Step 5. Compute total area:

$$A = 103 \times 57 + 44 \times 10 = 6,311 \text{ square inches.}$$

Step 6. Compute

$$\begin{aligned} f &= \frac{P}{A} \pm \frac{M_{yy}}{I_{yy}} x \pm \frac{M_{xx}}{I_{xx}} y \\ &= \frac{1,126,000}{6,311} \pm \frac{1,926,000 \times 12}{5,745,400} \times \\ &\quad 51.5 \pm \frac{2,436,000 \times 12}{1,810,900} \times 28.5 \\ &= 178 \pm 207 \pm 460. \end{aligned}$$

Then:

$$\begin{aligned} f \text{ at point } A &= +845; \\ f \text{ at point } B &= +431; \\ f \text{ at point } C &= -489; \\ f \text{ at point } D &= -75. \end{aligned}$$

Step 7. Compute the location of the neutral axis:

$$\begin{aligned} \text{Along side } DA \text{ from point } D &= \frac{75}{920} \times 57 = \\ &4.6 \text{ inches.} \end{aligned}$$

$$\begin{aligned} \text{Along side } CB \text{ from point } C &= \frac{489}{920} \times 57 = \\ &30.3 \text{ inches.} \end{aligned}$$

These calculations locate the fictitious neutral axis with the entire section assumed effective. This fictitious neutral axis is drawn on the sketch, as shown in figure 1.

Step 8. An assumed neutral axis for the cracked section is now selected and drawn on the sketch (see fig. 1), parallel to the fictitious neutral axis and on the compression side of the section from it. The section properties of the trial neutral axis as related to the X and Y axes are computed, tabulated, and totaled as shown in the first four lines of figure 2. The entries in the first line are for the steel reinforcement, while those in the second and third lines are for the concrete, considered as a rectangle and a triangle which, combined, include the total area of the cracked section.

It will be observed that ax and ay for the steel are zero because of the symmetry of the section. Values of I_{yy} and I_{xx} for the steel are the same as those computed in steps 3 and 4 for the uncracked section. The values i_{xy} and axy for the steel are zero because of the symmetry of the reinforcement. Values of i_{yy} and i_{xx} for the triangular section of the concrete are moments of inertia of the triangle about its own centroidal axes, and are $\frac{bh^3}{36}$ and $\frac{hb^3}{36}$. The value of i_{xy} for the triangle is computed as $\frac{b^2h^2}{72}$, with the sign determined as previously explained.

Step 9. The computed properties are corrected to the centroid in the fifth line of the tabulation (fig. 2). The values of x_c and y_c are computed as $\sum \frac{ax}{A}$ and $\sum \frac{ay}{A}$, respectively. The X_0 and Y_0 axes can now be drawn on the sketch (fig. 1). The corrections are computed as previously explained, and are subtracted algebraically from the totals in the line above.

Step 10. In the last line of the tabulation (fig. 2), corrections for the dissymmetry of the section are made. These are computed as previously explained, and are subtracted algebraically from the figures in the line above.

Step 11. From the values computed in the tabulation, the equation of the neutral axis, previously given, may be solved as follows:

$$\begin{aligned} \frac{1,126,000}{3,015} + \frac{20,760,000}{2,567,500} x + \frac{16,090,000}{491,000} y &= 0; \\ 373 + 8.09x + 32.7y &= 0; \\ y + 0.248x &= -11.4. \end{aligned}$$

For the assumed trial neutral axis (fig. 1), x equals $+43.9$ or -59.1 and y equals -22.2 or $+3.8$. Substituting the values of x in the equation above,

$$\text{if } x = +43.9, y = -22.3;$$

$$\text{if } x = -59.1, y = +3.3.$$

The computed values of y are in close agreement with the actual values of the assumed position of the neutral axis, and no further trial is necessary.



Part of Section	Area σ (sq. in.)	x (in.)	y (in.)	ax	ay	$I_{yy} = ax^2 + i_{yy}$		$I_{xx} = ay^2 + i_{xx}$		$I_{xy} = axy + i_{xy}$		$M_{yy} = P_x$ (in. kips)	$M_{xx} = P_y$	
Steel (44 1-in. sq., $n=10$)	440	varies	varies	0	0	555,000	0	221,300	0	0	0	23,110	+29,230	
Concrete 	1,236	0	+22.5	0	+27,810	0	1,092,700	625,700	14,800	0	0	—	—	
Concrete 	1,339	+7.17	+7.83	+22,990	+10,490	394,600	789,200	82,200	50,300	+179,900	-99,600	—	—	
Totals	$A = 3,015$	—	—	+22,990	+38,300	949,600 + 1,881,900		929,200 + 65,100		+179,900 - 99,600				
						2,831,500		994,300		+80,300		+23,110	+29,230	
Correct to centroid	$x_c = +7.63$	$y_c = +12.70$	Correction			175,500		486,300		+292,300		+8,590	+14,300	
			Corrected values			2,656,000 $= I_{y_0 y_0}$		508,000 $= I_{x_0 x_0}$		-212,000 $= I_{x_0 y_0}$		+14,520 $= M_{y_0 y_0}$	+14,930 $= M_{x_0 x_0}$	
Correct for dissymmetry of the section				Correction			88,500		17,000		—		-6,240	-1,160
				Corrected values			2,567,500 $= I'_y$		491,000 $= I'_x$		—		+20,760 $= M'_y$	+16,090 $= M'_x$

Figure 2.—Tabulation for analysis of cracked section.

Step 12. The actual stresses may now be computed from the equation previously given:

$$\text{Maximum } f_c = \frac{+16,090,000}{491,000} \times 38.1 = +1,250 \text{ p. s. i. (compression);}$$

$$\text{Maximum } f_t = \frac{+16,090,000}{491,000} \times -40.3 \times 10 = -13,200 \text{ p. s. i. (tension).}$$

These results may be checked by computing the intercepts of the neutral axis on the top and bottom of the section. The method of computation has been suggested previously. Using the values so calculated:

$$f_c = \frac{+20,760,000}{2,567,500} \times +153.5 = +1,240 \text{ p. s. i. (compression);}$$

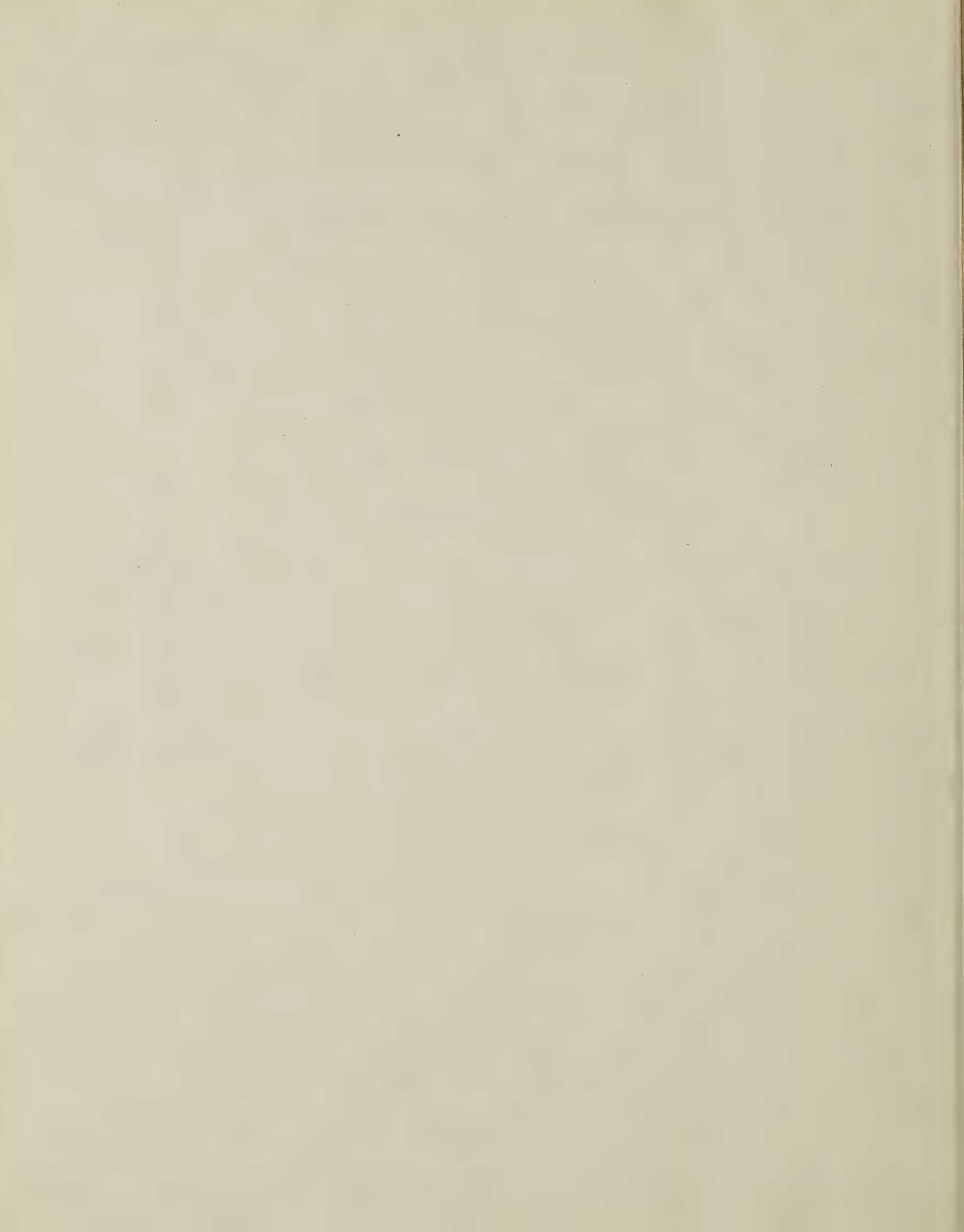
$$f_t = \frac{+20,760,000}{2,567,500} \times -162.4 \times 10 = -13,150 \text{ p. s. i. (tension).}$$

These agree very closely with the values computed above.

The discussion and calculations in this article are based on inch and pound units, but any other consistent set of units could be used. Theoretically, the area of the reinforcement embedded in the concrete on the compression

side of the section should be multiplied by $(n-1)$ instead of n in computing its transformed properties, but this refinement is considered of negligible importance.

The designer who will study the explanations and example that have been presented should have no difficulty in applying the general method to his own particular problems. As suggested by Saville, this method may also be applied to a variety of problems, including the computation of foundation pressures when the foundation is composed of materials having widely different supporting powers, and when overturning moments are applied in more than one direction.



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- The Taxation of Motor Vehicles in 1932. 35 cents.
- Guides to Traffic Safety. 10 cents.
- An Economic and Statistical Analysis of Highway-Construction Expenditures. 15 cents.
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- Tire Wear and Tire Failures on Various Road Surfaces. 10 cents.
- Legal Aspects of Controlling Highway Access. 15 cents.
- House Document No. 379. Interregional Highways. 75 cents.
- Highway Statistics, Summary to 1945. 40 cents.
- Highway Statistics, 1945. 35 cents.
- Highway Statistics, 1946. 50 cents.
- Model Traffic Ordinance. 15 cents.
- Principles of Highway Construction as Applied to Airports, Flight Strips, and Other Landing Areas for Aircraft. \$1.50.

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- No. 296MP . . . Bibliography on Highway Safety.
- No. 1036Y . . . Road Work on Farm Outlets Needs Skill and Right Equipment.
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- Act II.—Uniform Motor-Vehicle Operators' and Chauffeurs' License Act.
- Act III.—Uniform Motor-Vehicle Civil Liability Act.
- Act IV.—Uniform Motor-Vehicle Safety Responsibility Act.
- Act V.—Uniform Act Regulating Traffic on Highways.

STATUS OF FEDERAL-AID HIGHWAY PROGRAM

AS OF MARCH 31, 1948

(Thousand Dollars)

STATE	UNPROGRAMMED BALANCES			ACTIVE PROGRAM						TOTAL					
				PROGRAMMED ONLY			CONSTRUCTION NOT STARTED			CONSTRUCTION UNDER WAY					
	Total Cost	Federal Funds	Miles	Total Cost	Federal Funds	Miles	Total Cost	Federal Funds	Miles	Total Cost	Federal Funds	Miles	Total Cost	Federal Funds	Miles
Alabama	\$10,120	\$6,446	366.2	\$2,626	\$1,350	153.9	\$11,554	\$5,385	340.5	\$26,882	\$13,181	860.6			
Arizona	1,665	2,854	56.0	1,683	1,148	13.5	5,770	3,891	69.6	11,514	7,893	139.1			
Arkansas	1,679	7,054	310.2	6,702	3,486	131.7	12,549	6,621	319.8	32,701	17,161	761.7			
California	1,750	10,065	319.6	6,510	3,318	45.1	65,696	33,772	340.4	96,534	47,155	705.1			
Colorado	7,274	3,496	64.1	6,599	3,896	93.2	10,719	6,273	176.8	22,866	13,665	334.1			
Connecticut	4,074	4,806	32.7	1,672	819	4.4	4,935	2,341	15.9	15,889	7,966	53.0			
Delaware	3,435	1,337	28.7	2,146	1,175	29.7	1,055	531	17.0	5,771	3,043	75.4			
Florida	8,526	3,449	148.9	6,617	1,043	57.1	11,955	5,503	236.4	21,620	9,995	442.4			
Georgia	2,373	15,210	594.3	6,462	3,487	241.9	25,357	13,210	661.8	61,567	31,907	1,501.0			
Idaho	5,275	1,587	167.8	3,883	2,497	122.7	3,418	2,177	62.8	9,795	6,261	353.3			
Illinois	22,237	21,148	520.1	27,543	12,422	465.7	21,762	11,813	307.3	89,381	45,383	1,293.1			
Indiana	9,430	10,886	202.1	7,319	3,843	53.6	16,979	8,703	143.9	45,101	23,432	399.6			
Iowa	6,206	10,160	873.8	11,237	4,452	411.3	14,895	7,386	573.6	45,875	21,998	1,858.7			
Kansas	5,127	8,448	1,570.9	8,290	3,721	824.5	11,074	11,074	901.4	46,110	23,243	3,296.8			
Kentucky	2,722	7,969	199.2	4,723	2,358	70.4	14,121	7,173	219.7	35,008	17,500	489.3			
Louisiana	8,261	9,169	300.9	10,911	5,013	57.2	11,279	5,869	141.8	41,126	20,051	499.9			
Maine	5,362	2,239	31.4	893	434	11.9	5,294	2,921	50.6	10,488	5,594	93.9			
Maryland	1,906	7,248	52.9	3,823	1,849	20.3	4,648	2,351	42.6	23,566	11,448	115.8			
Massachusetts	18,160	6,736	22.6	5,489	3,248	26.8	9,518	4,918	26.7	27,461	14,902	76.1			
Michigan	8,104	11,513	457.5	11,154	5,228	136.1	31,221	14,729	245.0	65,095	31,470	838.6			
Minnesota	18,249	9,239	798.1	7,021	3,193	282.5	18,245	8,836	624.9	43,515	21,268	1,705.5			
Mississippi	3,207	4,730	406.3	11,194	5,391	319.0	16,731	8,430	456.3	37,517	18,551	1,181.6			
Missouri	14,272	14,079	782.9	6,782	3,347	275.1	19,985	10,391	496.7	56,635	27,817	1,554.7			
Montana	11,881	4,514	282.5	4,819	2,853	272.1	5,453	3,327	135.7	17,579	10,694	690.3			
Nbraska	4,730	8,944	592.6	5,822	2,713	310.0	9,231	4,635	437.9	31,928	16,292	1,340.5			
Nevada	977	5,031	280.7	1,233	923	47.1	2,828	2,210	87.3	10,370	8,164	395.1			
New Hampshire	2,817	1,594	22.9	1,460	773	12.3	4,268	2,209	16.4	8,545	4,576	51.6			
New Jersey	3,933	8,363	59.5	5,802	2,898	16.2	20,568	10,393	14.4	42,994	21,654	90.1			
New Mexico	4,726	4,044	236.6	1,597	1,038	26.2	6,649	4,253	192.6	14,535	9,319	456.4			
New York	39,720	42,567	538.6	14,801	6,777	47.1	35,684	15,911	202.0	126,789	65,255	787.7			
North Carolina	5,262	10,684	577.6	7,360	3,490	135.3	23,448	11,670	609.6	51,938	25,844	1,322.5			
North Dakota	4,484	7,617	1,438.2	7,941	4,420	437.9	4,043	2,252	310.4	25,922	14,289	2,186.5			
Ohio	9,563	24,706	265.4	10,845	5,614	128.8	37,880	19,170	94.2	99,279	49,490	488.4			
Oklahoma	11,683	8,266	827.0	5,269	2,717	288.0	4,685	2,418	306.7	24,626	13,401	1,421.7			
Oregon	1,200	4,819	152.1	3,319	1,699	97.1	12,238	6,699	254.9	24,514	13,217	504.1			
Pennsylvania	5,346	23,176	116.7	24,025	10,957	56.9	58,974	30,330	237.8	127,641	64,463	411.4			
Rhode Island	4,312	3,286	19.1	1,999	984	9.1	2,738	1,345	7.3	11,047	5,615	35.5			
South Carolina	2,511	4,864	233.7	2,438	1,178	60.2	11,324	5,484	266.3	23,994	11,526	560.2			
South Dakota	6,483	5,197	812.0	5,930	3,502	522.8	6,778	4,158	406.2	21,647	12,857	1,741.0			
Tennessee	9,027	9,831	252.9	7,238	3,239	153.2	13,330	7,178	263.8	40,364	20,248	669.9			
Texas	10,831	10,014	1,040.3	21,446	10,063	663.3	58,702	30,615	1,499.4	99,866	51,692	3,203.0			
Utah	1,578	4,463	226.3	1,182	868	58.0	4,835	3,578	166.8	12,018	8,909	451.1			
Vermont	1,247	2,227	96.6	688	331	12.8	4,078	2,050	48.1	9,223	4,608	157.5			
Virginia	8,427	7,730	266.4	5,204	2,521	108.7	12,860	6,223	153.6	33,380	16,474	528.7			
Washington	2,585	5,439	286.6	2,820	1,373	58.5	12,150	6,356	166.0	28,758	13,268	491.1			
West Virginia	2,915	6,062	250.8	1,800	931	48.7	7,442	3,898	58.6	21,711	10,891	358.1			
Wisconsin	14,947	10,791	614.8	12,339	4,594	190.2	14,902	5,708	379.4	50,533	21,093	1,184.4			
Wyoming	670	4,174	317.3	2,103	1,364	96.3	7,699	5,052	219.8	16,174	10,590	633.4			
Hawaii	1,132	3,454	36.5	3,913	1,680	14.5	2,234	1,267	9.7	12,835	6,401	60.7			
District of Columbia	815	179	4.4	273	136	1.0	12,353	6,231	4.0	12,983	6,946	5.4			
Puerto Rico	2,039	4,047	46.7	3,156	1,334	19.1	5,904	2,143	29.5	17,769	7,524	95.3			
TOTAL	329,989	417,051	18,156.0	324,051	157,688	7,742.0	762,344	391,045	13,049.9	1,890,979	965,784	38,949.9			

