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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

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THE APPLICATION OF RANDOM SAMPLING TO FISCAL STUDIES

A DISCUSSION OF THE PROBLEMS INVOLVED IN DETERMINING HIGHWAY EXPENDITURES BY THE SEVERAL UNITS OF GOVERNMENT

BY THE DIVISION OF CONTROL, PUBLIC ROADS ADMINISTRATION

Reported by THOMAS M. C. MARTIN, Assistant Highway Engineer-Economist

Data on highway income and expenditures by all units of government are needed in planning future highway development. The multiplicity of local units —municipalities, townships, counties, school districts, etc.—makes collecting complete information from each a sizeable and expensive undertaking.

An investigation was made of the feasibility of applying sampling methods to the collection of local financial statistics. The procedures followed, formulas used, and results obtained, are reported herein. A graphical means is given of appraising various sample sizes in terms of their probable reliability.

ADEQUATE INFORMATION about highway finance in all units of government is a prerequisite to the orderly development of a comprehensive, forward-looking highway program. The amounts of money raised locally by the lesser governmental units, the amounts received by them as grants-in-aid from higher units of government, as well as the use made of these funds, are all essential planning data. These facts must be known if highway needs and revenues are to be intelligently proportioned to the other needs and corresponding revenue sources of the State.

The difficulty of obtaining adequate data on highway finances appears to vary inversely with the size of the governmental unit. Ordinarily little trouble is encountered in ascertaining information relative to State revenues and expenditures, and with some exceptions the fiscal operations of the counties in the United States are readily obtainable. These, however, are by no means all of the units of government that engage in highway activities. There are a large number of local units, both rural and urban, data for which are essential to a complete picture of highway operations. Moreover, in many States the gross amounts involved in financing these local roads and streets are as large as the amounts handled by the State highway departments. Frequently, accurate information regarding the receipts and expenditures of these smaller units of government is not readily available, and special studies are necessary to obtain proper information.

LOCAL ROAD FINANCE DATA VALUABLE

Knowledge of local road finances has been of particular value during the past decade in connection with a noticeable trend toward the assumption of greater responsibility for county and local roads by certain States. The taking over of the North Carolina county road system by the State on July 1, 1931, was preceded by a thorough study of the financial status of the county roads.[1]

Similarly, when the 5-year program of county and township road consolidation was initiated in Michigan in 1931, a comprehensive study was made of all Michigan roads.[2]

Accurate information upon which to base an estimate of the probable costs of such programs of consolidation is seldom available when legislative deliberations are in progress. Consequently, when the question suddenly arises, it can be determined only approximately whether or not the resources of the State, usually limited to highway-user revenues, will be adequate for the increased burden. The question of whether a State is financially able to assume the proposed additional responsibilities without seeking new sources of money is a very important matter. It usually happens that the governmental units previously responsible financed their work with a combination of State subventions and receipts from local property taxation.

It is likewise essential to know the mileage of roads owned by local units and the standards to which they were built and are maintained. These facts are necessary to gage properly the annual financial requirements arising from the added responsibilities. The need for such information becomes evident in still other ways, particularly when proposals are made to allocate large sums of highway-user or other State revenues to local units of government for highway or nonhighway purposes either as single lump-sum payments or as continuing annual subventions. The wisdom of enacting such proposals into law can receive a more thorough consideration and fuller debate when complete and accurate data concerning the needs and resources of all units are readily available. Dissipation of State funds into channels where the public does not receive a proportionate return on the funds it has contributed can best be prevented by making all of the facts available.

The several States and the Public Roads Administration have been engaged in studies of this problem for more than 20 years. Detailed reports of State highway mileage, receipts, and expenditures, have been prepared annually since 1921. The gathering of corresponding information for the local units of government was commenced as early as 1912, made more complete in 1917, and has been done annually since 1921. The relative incompleteness of the local statistics has long been recognized, and constant attempts have been made to improve the reporting system.

The difficulty in obtaining accurate data from units of government other than the State arises in a considerable measure from the large number of governmental units involved and the incomplete records kept by many of these spending agencies even for their own purposes.

[1] North Carolina County Road and Finance Survey, PUBLIC ROADS, vol. 11, No. 12, February 1931. (Report of a cooperative investigation by the North Carolina State Highway Commission, the North Carolina State Tax Commission, and the United States Bureau of Public Roads.)

[2] A Survey of Highway Transportation in Michigan, PUBLIC ROADS, vol. 13, No. 12, February 1933.
The Michigan Financial Survey, PUBLIC ROADS, vol. 14, No. 4, June 1933.

Table 1 shows the number of units in each of the States in 1939, as prepared by the Illinois Tax Commission. In the collection of State highway data the Public Roads Administration needs to concern itself only with the 48 States and the District of Columbia, but local rural road data must be obtained from approximately 20,000 units. Most of the counties and, in the States in which they exist, the towns and townships, carry on highway activities. In addition to these units, the sixth column of table 1 lists many road districts which at least until very recently carried on road functions similar to those of the townships.

TABLE 1.—*Taxing units in the United States, 1939*

State	Counties	Incorporated places	Towns and townships	School districts	All others	Total [1]
Alabama	67	296		112		476
Arizona	14	33		416	22	486
Arkansas	75	389		3,062	834	4,361
California	57	282		2,957	265	3,562
Colorado	62	237		2,051		2,351
Connecticut	8	40	173	26	114	362
Delaware	3	52		15		71
Florida	67	289		893		1,250
Georgia	159	593				753
Idaho	44	150		826		1,021
Illinois	102	1,134	[2] 1,625	12,115	123	15,100
Indiana	92	544	1,017	163	5	1,822
Iowa	99	917	1,602	4,873		7,492
Kansas	105	580	1,550	8,772	65	11,073
Kentucky	120	369		263	14	767
Louisiana	64	210		66	161	502
Maine	16	20	494	512		1,043
Maryland	23	137		24	20	205
Massachusetts	13	39	312		63	428
Michigan	83	477	1,267	6,550		8,378
Minnesota	87	726	1,902	7,692	1	10,409
Mississippi	82	305		5,796	756	6,940
Missouri	114	773	345	8,957		10,190
Montana	56	116		2,437		2,610
Nebraska	93	529	477	7,098		8,198
Nevada	17	16		293	19	346
New Hampshire	10	11	224	241		487
New Jersey	21	331	238	551	17	1,159
New Mexico	31	63		1,100		1,195
New York	57	615	927	7,913	96	9,609
North Carolina	100	386		169	139	795
North Dakota	53	333	1,470	2,271	37	4,165
Ohio	88	869	1,337	1,756		4,051
Oklahoma	77	463		4,697		5,238
Oregon	36	192		2,114	93	2,436
Pennsylvania	67	986	1,577	2,582	66	5,279
Rhode Island		7	32		54	94
South Carolina	46	265		39	10	361
South Dakota	64	311	1,136	3,437		4,949
Tennessee	95	233		95		424
Texas	254	580		6,000	271	7,106
Utah	29	197		40		267
Vermont	14	110	246	272		643
Virginia	100	215		100	2	418
Washington	39	221	73	1,491	371	2,196
West Virginia	55	202		55		313
Wisconsin	71	525	1,280	7,390	6	9,273
Wyoming	23	82		385		491
Total	3,052	16,450	19,304	118,667	3,624	161,145

[1] Includes one for State.
[2] Includes road districts in commission counties.

The total number of taxing counties in the United States, excluding those in States where the counties exercise no highway functions, is 2,666,[3] or an average of 72 counties per State in each of the 37 States involved. Although this figure is influenced slightly by the large number of counties in Georgia and Texas, the median number of counties is 67 or only slightly lower than the arithmetic mean. While the States must therefore deal with an average of approximately 50 percent more counties than the number of States with which the Public Roads Administration is concerned, the problem of the States is still more complicated. The 48 States have 16,450 incorporated places or a mean of 343, with the median State possessing a total of 286 such incorporated places. In addition 20 States have a mean of 851 towns and townships engaged in road work,[4] with a median of 710.

SAMPLING OBVIATES LARGE EXPENDITURES IN OBTAINING LOCAL DATA

Some information on the highway activities of the rural units (counties, towns, townships, and road districts) is available for the years since 1921, with the exception of 1922. More exhaustive surveys were conducted in 1921, 1926, and 1931 and the data for these years are believed to be more reliable than those for the intervening years when lack of funds and personnel made thorough studies impracticable.

The most exhaustive studies ever undertaken in this field were those conducted by the State-wide highway planning surveys which commenced in 1935. In these studies financial data were collected covering the receipts and expenditures for 1 fiscal year of all taxing agencies for all public purposes. This was done in order that a proper relationship could be established between highway and other governmental activities.

With the completion of these 1-year studies the problem again arises of collecting regularly adequate annual financial data relative to highways. The task has even increased recently, since it is becoming evident that the most urgent traffic problem requiring current solution is the provision of arterial approaches to urban centers both large and small. Authorities are generally agreed that the weakest links in the existing highway system are usually these roads in and near cities. It is on these roads that most traffic congestion occurs; consequently, it is there that the chief threat to the efficiency of highway transportation arises. It is also a fact that since these highways as a class are the most expensive of all to build, the financing of them is one of the most difficult and important tasks of the immediate future. Consequently, data from more than 16,000 incorporated places should also be collected annually if a complete representation of the street and highway problem is to be obtained.

The planning surveys have greatly stimulated the adoption of uniform reporting methods for all levels of government in many States and thus contributed to the solution of part of the problem, i. e., an improvement in the quality and availability of information. There is still the problem of the large number of units and the correspondingly large costs involved in analyzing their reports. And not to be overlooked is the fact that if uniform centralized reporting does not exist, field investigation will normally be required to obtain anything approaching the desired information. An investigation was made, therefore, of the feasibility of applying sampling methods to the collection of local financial statistics with the fiscal survey data of the Wisconsin State-wide Highway Planning Survey as the basis for the inquiry.[5] It is with the conduct and results of this investigation that the present article is chiefly concerned.

[3] The five counties of Rhode Island do not exist as taxing jurisdictions, while the counties in Connecticut, Delaware, Maine, Massachusetts, New Hampshire, North Carolina, Vermont, and West Virginia perform no highway functions. Only three of Virginia's counties engage in highway activities and in Pennsylvania the county road programs are of extremely minor character, with the possible exception of Allegheny County. None of these States has been included, therefore, in this figure.

[4] The townships of Indiana and Michigan for all practical purposes may be considered to have no road functions.

[5] The studies at Madison, Wisconsin, reported here were under the sponsorship of Dr. H. R. Trumbower, Senior Agricultural Transporation Economist, and were made in connection with the cooperative agreement between the University of Wisconsin and the Public Roads Administration.

The data employed in this investigation were gathered by the Wisconsin planning survey during its original operations and were for the calendar year 1935. The fiscal statistics for each Wisconsin town[6] had been recorded on forms which provided for a fairly extensive itemization of revenues and disbursements for all purposes. The separate classifications employed on these forms are not all of equal importance. Included to facilitate tabulation, many of them are not of particular significance in final analyses. The sampling of financial statistics should be restricted to include only items that are relatively stable in their occurrence in the reports of the individual units. Certain categories of both receipts and expenditures are entirely too variable to permit accurate estimation by sampling techniques. This would be true, for example, in the case of the borrowings of Wisconsin towns. In ordinary times these jurisdictions resort to such methods of financing very infrequently, and totals obtained by sample expansion would not be very reliable. Sampling accuracy depends upon a substantial degree of similarity in the individual members of the population or universe being sampled.

IN SAMPLING CERTAIN ELEMENTS REQUIRE SPECIAL TREATMENT

If the individual members of a population are radically different one from another, an accurate description of the characteristics of the group will be possible only through the inclusion of the entire population in the analyses. All that sampling procedures afford, where they are applicable, is a more expeditious and economical means of describing the properties of a large class of things or events by means of the observation of less than the whole number of individuals constituting the group. What is requisite is that there be some well-defined central tendency in the properties of the individual members of a given group. An arithmetic average may be computed for any set of numbers, but for purposes of sample expansion it is important that the individual numbers be fairly closely centered about their average value.

[6] A "town" in Wisconsin is an unincorporated rural unit of government and should not be confused with so-called towns which are in reality incorporated villages or cities.

In the present inquiry it was possible to include but a few of the more important classifications of receipts and disbursements which were provided for on the basic planning survey forms. On the revenue side these included (1) total local revenues, (2) total nonlocal revenues, and (3) total current receipts. On the disbursement side only total net expenditures were considered. All of the foregoing were further subdivided to show amounts collected and expended for highway purposes and for all public functions.

Table 2 presents data on the population distribution and expenditures of Wisconsin governmental units for 1935 from which the relative importance of the fiscal operations of the town units may be judged.

In applying random sampling to local units of government it is necessary to consider the possibility that certain extreme elements of the universe may require special treatment. In Wisconsin the seven towns of Milwaukee County provide such an example. The receipts and expenditures of towns necessarily bear some relation to the population which they include. The average population of the seven Milwaukee County towns is approximately 6,600. This is more than eight times the average population of the remaining 1,273 towns in the State, which is about 800. If there were a sufficient number of these large towns it would be possible to employ stratified sampling. These especially large towns would in effect be treated as a separate universe and independently sampled. A random sample, however, should ordinarily contain no fewer than 30 items. In this instance, consequently, as in most others which would be encountered in sampling data for local units of government, it was considered advisable to obtain data for all of the Milwaukee County towns. The sampling inquiry was extended therefore only to the remaining 1,273 towns.

There are a variety of ways in which a random selection could be effected. The method selected in this instance used the so-called "Tippett's Numbers."[7] This procedure involved the superposition of a new and independent characteristic, that of an ordinal number, upon the individual members of the universe.

[7] L. H. C. Tippett, Random Sampling Numbers, No. XV, Tracts for Computers, Edited by Karl Pearson, Cambridge University Press, London, 1927.

TABLE 2.—*Population distribution and net expenditure data for Wisconsin governmental units, 1935, classified by purpose* [1]

Unit of government	Number of places	Total population	Percentage of total population	Highways		Other public functions				Grand total	
						Education		Total			
				Amount	Percent	Amount	Percent	Amount	Percent	Amount	Percent
Towns	1, 280	1, 086, 944	37. 1	$5, 485, 159	14. 7	$13, 071, 972	23. 8	$16, 481, 150	10. 2	$21, 966, 309	11. 1
Incorporated places having a population of 1-1,000	334	159, 279	5. 4	391, 330	1. 0	2, 395, 970	4. 4	4, 051, 512	2. 5	4, 442, 842	2. 2
Rural areas	1, 614	1, 246, 223	42. 5	5, 876, 489	15. 7	15, 467, 942	28. 2	20, 532, 662	12. 7	26, 409, 151	13. 3
Incorporated places having a population of—											
1,001-2,500	88	137, 613	4. 7	429, 117	1. 2	2, 269, 350	4. 1	4, 702, 805	2. 9	5, 131, 922	2. 6
2,501-5,000	36	128, 990	4. 4	453, 731	1. 2	2, 139, 826	3. 9	4, 155, 669	2. 6	4, 609, 400	2. 3
5,001-10,000	20	141, 905	4. 8	412, 970	1. 1	2, 388, 745	4. 4	4, 816, 365	3. 0	5, 229, 335	2. 6
10,001-25,000	14	223, 821	7. 6	392, 249	1. 1	3, 997, 845	7. 3	8, 745, 482	5. 4	9, 137, 731	4. 6
25,001-50,000	9	305, 175	10. 4	678, 083	1. 8	5, 129, 930	9. 4	12, 332, 469	7. 6	13, 010, 552	6. 6
50,001-100,000	3	175, 703	6. 0	246, 954	. 7	3, 578, 688	6. 5	7, 898, 974	4. 9	8, 145, 928	4. 1
Milwaukee	1	577, 083	19. 6	1, 462, 027	3. 9	10, 826, 334	19. 8	26, 624, 041	16. 5	28, 086, 068	14. 1
Urban places	171	1, 690, 290	57. 5	4, 075, 131	11. 0	30, 330, 718	55. 4	69, 275, 805	42. 9	73, 350, 936	36. 9
Counties				11, 255, 374	30. 3	1, 100, 576	2. 0	53, 995, 482	33. 4	65, 250, 856	32. 8
State				15, 974, 942	43. 0	7, 885, 648	14. 4	17, 700, 349	11. 0	33, 675, 291	17. 0
Grand total	1, 785	2, 936, 513	100. 0	37, 181, 936	100. 0	54, 784, 884	100. 0	161, 504, 298	100. 0	198, 686, 234	100. 0

[1] From Wisconsin State-wide Highway Planning Survey, basic analysis form, line 37.

Table 3.—*Expenditures in Wisconsin towns selected for preliminary sample*

Assigned serial No.	County	Town	Total net expenditures[1]	
			Actual	Nearest $1,000
22	Ashland	Gordon	$11, 726	12
30	do	White River	16, 673	17
75	Bayfield	Pilsen	6, 257	6
106	Buffalo	Dover	13, 865	14
113	do	Mondovi	9, 759	10
124	Burnett	La Follette	7, 770	8
139	Calumet	Brillion	20, 382	20
151	Chippewa	Birch Creek	6, 555	7
182	Clark	Hixon	19, 659	20
199	do	Washburn	9, 978	10
205	Columbia	Caledonia	23, 655	24
217	do	Newport	6, 030	6
242	Dane	Burke	49, 523	50
304	Door	Nasewaupee	17, 966	18
322	Douglas	Summit	16, 270	16
334	Dunn	Otter Creek	8, 990	9
351	Eau Claire	Fairchild	12, 110	12
371	Forest	Blackwell	6, 833	7
381	do	Wabeno	36, 327	36
393	Fond du Lac	Lamartine	20, 399	20
396	do	Oakfield	16, 073	16
432	Grant	Watterstown	3, 451	3
486	Jackson	Albion	36, 375	36
506	Jefferson	Aztalan	14, 795	15
511	do	Ixonia	15, 717	16
524	Juneau	Cutler	12 850	13
532	do	Lisbon	10, 308	10
558	Kewaunee	West Kewaunee	20, 407	20
565	La Crosse	Hamilton	26, 022	26
608	Lincoln	Harding	6, 404	6
623	Manitowoc	Cooperstown	28, 173	28
638	do	Two Rivers	24, 202	24
693	Marinette	Porterfield	15, 893	16
708	do	Oxford	8, 903	9
712	do	Westfield	9, 094	9
724	Monroe	Glendale	12, 384	12
726	do	Greenfield	5, 355	5
730	do	Leon	12, 314	12
770	Oneida	Hazelhurst	5, 371	5
776	do	Pelican	22, 514	23
851	Polk	Johnstown	13, 089	13
852	do	Laketown	10, 541	11
863	Portage	Almond	15, 784	16
879	Price	Catawba	6, 300	6
893	do	Prentice	9, 655	10
899	Racine	Mount Pleasant	114, 049	114
918	Richland	Sylvan	20, 464	20
987	Sauk	Bear Creek	19, 289	19
995	do	Honey Creek	19, 936	20
1017	Sawyer	Meteor	5, 660	6
1018	do	Ojibwa	12, 307	12
1023	do	Weirgor	10, 128	10
1037	Shawano	Hutchins	11, 470	11
1071	Taylor	Ford	6, 675	7
1072	do	Goodrich	9, 161	9
1081	do	Medford	26, 479	26
1097	Trempealeau	Pigeon	23, 731	24
1164	Washburn	Gull Lake	4, 823	5
1169	do	Spooner	5, 564	6
1172	do	Stone Lake	5, 778	6
1179	do	Hartford	19, 242	19
1198	Waukesha	Ottawa	10, 924	11
1212	Waupaca	Larrabee	18, 656	19
1254	Winnebago	Utica	19, 692	20

[1] Data taken from basic analysis form.

A general discussion of random sampling methods is given in the appendix, page 207.

The numbering of the towns was the first task in commencing the actual selection of a sample. The 1,280 Wisconsin towns[8] were arranged alphabetically by and within counties, and then numbered consecutively. There was no important reason for an alphabetical arrangement of the towns. It rendered the location of data for the selected towns somewhat easier, but it was not prerequisite and the numbering could have followed any other scheme without affecting the remainder of the procedure.

Before the actual selection could be commenced it was necessary to make some decision relative to the size of the sample to be taken. The size of the ultimate sample, as will be emphasized later, must be predicated upon the probable accuracy which it is desired to attribute to the resultant expansions. This decision cannot very well be made without some knowledge of the characteristics of the universe being sampled. Specifically it is necessary to have some idea of the dispersion of the individual members about their mean. In these circumstances it is convenient to draw a small sample and compute certain statistics which facilitate the determination of final sample size. The initial sample in this instance consisted of 64 towns or approximately 5 percent of the total number of such units in Wisconsin. This preliminary sample was fixed at 5 percent instead of some other proportion because previous investigation had disclosed some facts relative to the range of town data. If the universe had been differently constituted, either quantitatively or qualitatively, or both, the choice of the initial sample would have been altered accordingly. In other words, familiarity with the general nature of the data being analyzed is a practical advantage for which there is no entirely satisfactory substitute.

[8] The seven Milwaukee County towns were numbered in regular order although they could have as easily been skipped inasmuch as they were not subjected to sampling for reasons outlined above.

IMPORTANT FORMULAS EXPLAINED

It was necessary to consider total net expenditures only in the preliminary computations since the stability of the other data was believed to be of approximately the same order. A given sample will not, of course, yield exactly the same reliability in all the different categories of receipts and expenditures. The individual reliability of these statistics varies with their respective dispersions. The size of the final sample, therefore, will depend upon the degree of reliability that is believed necessary in estimating the most widely dispersed of the items to be tabulated. It follows that the less widely dispersed items will be estimated with correspondingly greater reliability.

Table 3 includes the planning survey expenditure data for the initial sample just described. In this

Table 4.—*Computations required in the calculation of standard deviation and arithmetic mean from table 3*

Total net expenditures x	Frequency f	$(1)-M_a$ d	$(2)\times(3)$ fd	$(3)\times(4)$ fd^2
(1)	(2)	(3)	(4)	(5)
$1,000				
3	1	−12	−12	144
5	3	−10	−30	300
6	7	−9	−63	567
7	3	−8	−24	192
8	1	−7	−7	49
9	4	−6	−24	144
10	5	−5	−25	125
11	3	−4	−12	48
12	5	−3	−15	45
13	2	−2	−4	8
14	1	−1	−1	1
15	1	0		
16	5	1	+5	5
17	1	2	+2	4
18	1	3	+3	9
19	3	4	+12	48
20	7	5	+35	175
23	1	8	+8	64
24	3	9	+27	243
26	2	11	+22	242
28	1	13	+13	169
36	2	21	+42	882
50	1	35	+35	1, 225
114	1	99	+99	9, 801
	64		$\left\{ \begin{matrix} -217 \\ +303 \end{matrix} \right. = +86 \}$	14, 490

table the total net expenditures are shown in the last column to the nearest thousand dollars. This forms a preliminary step in the transition to table 4 and in addition provides a convenient tabulation of the computed values necessary for substitution in the formulas for the standard deviation and arithmetic mean computed by the so-called "short-cut" method. The necessary notation follows:

X=variable.
M_x=mean value of X.
M_a=assumed mean class interval.
$f(x)$=frequency of occurrence of X.
$d=(x-M_a)$ deviation of each value of X from class interval of assumed mean.
$N=\Sigma f$=total frequency.
σ=standard deviation.
σ_M=standard error of the mean.

The important formulas are:

$$M_x=M_a+\frac{\Sigma fd}{N} \quad (1)$$

$$\sigma=\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2} \quad (2)$$

$$\sigma_M=\frac{\sigma}{\sqrt{N}} \quad (3)$$

In equation 2 the assumed mean should, for purposes of efficient calculation, be located as near the actual mean as possible while the correction term $\frac{\Sigma fd}{N}$ must be an algebraic summation with due care observed as to the sign of the individual terms, since the correction may be either positive or negative depending upon the location of the assumed mean.

Substituting the values from table 4 in these equations yields the following results:

$$M_x=M_a+\frac{\Sigma fd}{N} \quad (1)$$

$$=15+\frac{86}{64}=16.344.$$

$$\sigma=\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2} \quad (2)$$

$$=\sqrt{\frac{14490}{64}-\left(\frac{86}{64}\right)^2}=14.987.$$

$$\sigma_M=\frac{\sigma}{\sqrt{N}} \quad (3)$$

$$=14.987/8=1.873.$$

If the coefficient of variation be used as a measure of reliability and defined as follows, it will provide a convenient index for comparing various size samples.

Let V=coefficient of variation (percent).

$$V=\left(\frac{3\sigma_M}{M_x}\right)100 \quad (4)$$

In this case

$$V=\left(\frac{3\times 1.873}{16.344}\right)100=34.4 \text{ percent.}$$

The interpretation to be placed upon such a result is that in repeated trials, randomly drawn samples of the same size (5 percent) will seldom [9] yield means varying by more than 34.4 percent from 16.344. It is practically certain [10] that the true mean of the parent population lies between 16.344±5.619 or between 10.725 and 21.963.

The practical problem which arises at this point is the determination of coefficients of variation for samples of different size. This first 5-percent sample produced a reliability as measured by this statistic of 34.4 percent. It is possible to effect a slight transformation in the basic equation for the coefficient of variation and derive an equation which will facilitate the calculation of acceptable estimates of the reliability which can be expected from larger samples randomly drawn from the same universe.

The original equation was:

$$V=\left(\frac{3\sigma_M}{M_x}\right)100 \quad (4)$$

Substituting for σ_M the value given in equation 3,

$$V=\left(\frac{3\sigma/\sqrt{N}}{M_x}\right)100$$

which can be written

$$V=\left(\frac{3\sigma}{M_x}\div\sqrt{N}\right)100$$

$$=\frac{300\sigma}{M_x}\div\sqrt{N} \quad (5)$$

This equation affords an expeditious means of calculation since the numerator can through reasonable assumptions be made a constant for a given problem, and the denominator is a direct function of the number contained in the sample. The assumptions necessary are (1) that the σ computed for the initial sample is a satisfactory estimate of the dispersion of the parent population from which the sample was drawn, and (2) that the value of the mean obtained from this sample

TABLE 5.—*Coefficient of variation of 5 to 75 percent samples of total net expenditures of Wisconsin towns, 1935, as calculated from initial sample of 64 towns*

Number in sample, N	Coefficient of variation [1] $300\frac{\sigma}{M_x}\div\sqrt{N}$
64	34.4
81	30.6
100	27.5
121	25.0
144	22.9
169	21.2
196	19.6
225	18.3
256	17.2
289	16.2
324	15.3
361	14.5
400	13.8
625	11.0
900	9.2

[1] $V=\frac{300\sigma}{M_x}\div\sqrt{N}=\frac{300\times 14.987}{16.344}\div\sqrt{N}=275.0917767\div\sqrt{N}$

[9] Presuming an infinite number of trials the variation would in the limit be greater only three times in a thousand.

[10] Observe a literal construction of "practically certain" as opposed, for example, to absolutely certain. Practically certain as here used does not imply the impossibility of an adverse result; it merely is indicative of the relatively infrequent occurrence of such events.

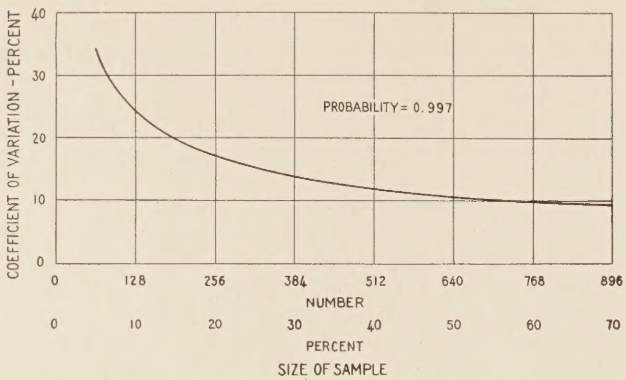

FIGURE 1.—RELATION BETWEEN COEFFICIENT OF VARIATION AND SIZE OF SAMPLE.

will suffice temporarily as an estimate of the population mean. Both of these assumptions are justified.

RELATIONSHIP BETWEEN RELIABILITY AND SAMPLE SIZE EXAMINED

The necessary calculations were arranged in this case as shown in table 5. The range of sample sizes was limited to 5 percent increments from 5 percent to 75 percent, since the only purpose of these computations was to provide sufficient points upon which to base a freehand curve of the probable relationship between reliability and sample size for this particular inquiry. This curve is reproduced as figure 1.

The responsibility for determining the size of an acceptable sample is not a proper function of the statistician. It is an administrative problem the proper solution of which will depend upon the use to be made of the resultant expansions. The construction of a curve such as that shown in figure 1 is helpful in making a decision as to proper size of sample. It provides a graphical means of appraising various sample sizes in terms of their probable reliability. For purposes of illustration it was assumed in this case that acceptable accuracy called for a coefficient of variation of less than 20 percent.[11] It appeared, therefore, that a sample of 200 towns or approximately 15 percent would be required.

The additional towns selected to raise the total sample to 200 are listed with their expenditure data in table 6. These towns were selected in the same manner as the first 64 towns, but, it should be noted, without duplication. The final sample, therefore, was precisely the same as though it had been randomly selected at one time instead of in two portions. The geographical distribution of the initial and final samples is shown in figure 2.

Table 7 combines the rounded-off expenditure figures from tables 3 and 6 while table 8 fulfills the same function as table 4 in providing data necessary to the calculations which follow:

Again substituting in the basic equations,

$$M_x = M_a + \frac{\Sigma fd}{N} \qquad (1)$$

$$= 15 + \frac{255}{200} = 16.275.$$

[11] This is a purely abstract assumption and could as well have been any other percentage.

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \qquad (2)$$

$$\sqrt{\frac{28059}{200} - \left(\frac{255}{200}\right)^2} = 11.772.$$

$$\sigma_M = \frac{\sigma}{\sqrt{N}} \qquad (3)$$

$$= 11.772 \div \sqrt{200} = 0.832$$

$$V = \left(\frac{3\sigma_M}{M_x}\right)100 \qquad (4)$$

$$= \frac{300 \times 0.832}{16.275} = 15.34 \text{ percent.}$$

TABLE 6.—*Expenditures of additional Wisconsin towns selected to complete final sample*

Assigned serial No.	County	Town	Total net expenditures [1]	
			Actual	Nearest $1,000
17	Adams	Strongs Prairie	$3, 596	4
29	Ashland	Shanagolden	5, 214	5
32	Barron	Arland	11, 939	12
38	do	Crystal Lake	22, 181	22
39	do	Cumberland	22, 472	22
61	Bayfield	Cable	9, 391	9
66	do	Hughes	11, 694	12
67	do	Iron River	28, 633	29
73	do	Orienta	7, 350	7
91	Brown	Humboldt	16, 311	16
99	do	Suamico	30, 178	30
104	Buffalo	Canton	13, 246	13
112	do	Modena	17, 622	18
116	do	Nelson	28, 927	29
121	Burnett	Dewey	9, 910	10
126	do	Meenon	9, 316	9
130	do	Sand Lake	7, 461	7
144	Calumet	New Holstein	19, 919	20
187	Clark	Lynn	10, 818	11
188	do	Mayville	18, 991	19
190	do	Mentor	14, 984	15
221	Columbia	Scott	13, 235	13
224	do	Wyocena	17, 945	18
229	Crawford	Haney	16, 330	16
230	do	Marietta	20, 512	21
231	do	Prairie du Chien	6, 505	7
237	Dane	Berry	13, 878	14
239	do	Blooming Grove	67, 560	68
257	do	Perry	13, 374	13
261	do	Rutland	23, 950	24
297	Door	Claybanks	8, 095	8
310	Douglas	Bennett	9, 267	9
323	do	Superior	29, 646	30
327	Dunn	Eau Galle	16, 028	16
329	do	Grant	12, 133	12
330	do	Hay River	10, 969	11
344	do	Tiffany	11, 200	11
357	Eau Claire	Union	19, 555	20
358	do	Washington	23, 687	24
376	Forest	Laona	47, 180	47
379	do	Popple River	8, 715	9
389	Fond du Lac	Empire	18, 559	19
391	do	Forest	15, 500	16
401	do	Taycheedah	14, 429	14
406	Grant	Cassville	15, 559	16
411	do	Glen Haven	10, 413	10
418	do	Little Grant	8, 515	9
425	do	Paris	13, 508	14
436	Green	Adams	16, 538	17
449	do	Sylvester	17, 090	17
458	Green Lake	Marquette	11, 855	12
464	Iowa	Clyde	13, 634	14
465	do	Dodgeville	36, 443	36
466	do	Eden	8, 583	9
468	do	Linden	17, 380	17
473	do	Ridgeway	19, 318	19
493	Jackson	Franklin	9, 792	10
501	do	Melrose	14, 301	14
510	Jefferson	Hebron	13, 377	13
516	do	Oakland	18, 332	18
522	Juneau	Armenia	13, 027	13
523	do	Clearfield	10, 751	11
525	do	Finley	7, 749	8
530	do	Lemonweir	9, 280	9
535	do	Necedah	32, 979	33
544	Kenosha	Pleasant Prairie	71, 446	71
552	Kewaunee	Franklin	19, 326	19
569	La Crosse	Washington	5, 080	5

[1] Data taken from basic analysis form, line 37, column J.

TABLE 6.—*Expenditures of additional Wisconsin towns selected to complete final sample*—Continued

Assigned serial No.	County	Town	Total net expenditures: Actual	Total net expenditures: Nearest $1,000
570	Lafayette	Argyle	13,293	13
572	do	Benton	15,991	16
581	do	New Diggings	19,876	20
582	do	Seymour	13,948	14
584	do	Wayne	28,879	29
605	Lincoln	Birch	6,811	7
609	do	Harrison	8,464	8
615	do	Schley	15,471	15
634	Manitowoc	Newton	20,017	20
641	Marathon	Bern	7,911	8
663	do	Knowlton	14,178	14
725	Monroe	Grant	5,688	6
746	Oconto	Bagley	5,543	6
754	do	Little River	26,911	27
756	do	Maple Valley	18,592	19
784	Oneida	Woodboro	9,005	9
798	Outagamie	Kaukauna	9,102	9
799	do	Liberty	9,315	9
809	Ozaukee	Grafton	14,813	15
829	Pierce	Martell	13,772	14
831	do	River Falls	24,738	25
838	Polk	Alden	20,621	21
848	do	Farmington	12,050	12
859	do	St. Croix Falls	18,340	18
868	Portage	Dewey	10,678	11
894	Price	Spirit	7,209	7
898	Racine	Dover	19,580	20
914	do	Orion	12,737	13
916	do	Richwood	12,428	12
917	do	Rockbridge	16,460	16
929	Rock	Johnstown	14,628	15
931	do	Lima	20,453	20
932	do	Magnolia	16,615	17
947	Rusk	Grant	17,602	18
957	do	Strickland	10,174	10
958	do	Stubbs	12,911	13
959	do	Thornapple	17,213	17
967	St. Croix	Cylon	10,736	11
971	do	Forest	14,773	15
984	do	Troy	16,046	16
986	Sauk	Baraboo	23,041	23
1004	do	Washington	25,910	26
1010	Sawyer	Draper	20,464	20
1014	do	Hunter	14,100	14
1020	do	Round Lake	15,576	16
1028	Shawano	Bartelme	6,678	7
1040	do	Morris	11,307	11
1052	Sheboygan	Holland	35,947	36
1069	Taylor	Pershing	8,979	9
1076	do	Holway	13,654	14
1079	do	McKinley	10,807	11
1091	Trempealeau	Chimney Rock	14,645	15
1095	do	Hale	24,295	24
1117	Vernon	Sterling	22,397	22
1125	Vilas	Boulder Junction	11,906	12
1128	do	St. Germain	8,963	9
1152	Walworth	Whitewater	16,907	17
1165	Washburn	Long Lake	9,341	9
1166	do	Madge	7,025	7
1175	Washington	Barton	11,880	12
1184	do	Trenton	16,893	17
1206	Waupaca	Dupont	17,287	17
1218	do	Royalton	15,651	16
1220	do	Scandinavia	12,366	12
1225	Waushara	Aurora	14,734	15
1241	do	Warren	11,469	11
1250	Winnebago	Omro	20,120	20
1277	Wood	Seneca	8,965	9

It is now possible to say that in repeated trials, randomly drawn samples of 200 towns will seldom yield means varying by more than 15.34 percent from 16.275. It is practically certain, therefore, that the population mean lies between 16.275±3(0.832) or between 13.779 and 18.771.

The formulas which were applied in the calculation of the sampling error in total net expenditures are equally applicable to the determination of error in estimating any other statistics. Ordinarily the same sample should be used in estimating all data pertaining to the same class of governmental units for a given fiscal period. The dispersion of the various items may vary considerably, and consequently, as was previously emphasized, the choice of sample size should be governed by the minimum reliability required in the estimation of the most widely dispersed of the statistics tabulated. This procedure will necessarily result in a higher degree of reliability in certain items than might otherwise be required, but it is nevertheless the only practicable way sampling can be efficiently employed. If separate samples were to be used for different statistics the clerical work involved would increase and could easily dissipate the savings that would otherwise accrue through the use of sampling procedures. It was not feasible in the present instance to present complete calculations for statistics other than total net expenditures. Table 9 has been included, however, to provide some indication of the relative stability of other important receipt and disbursement classifications.

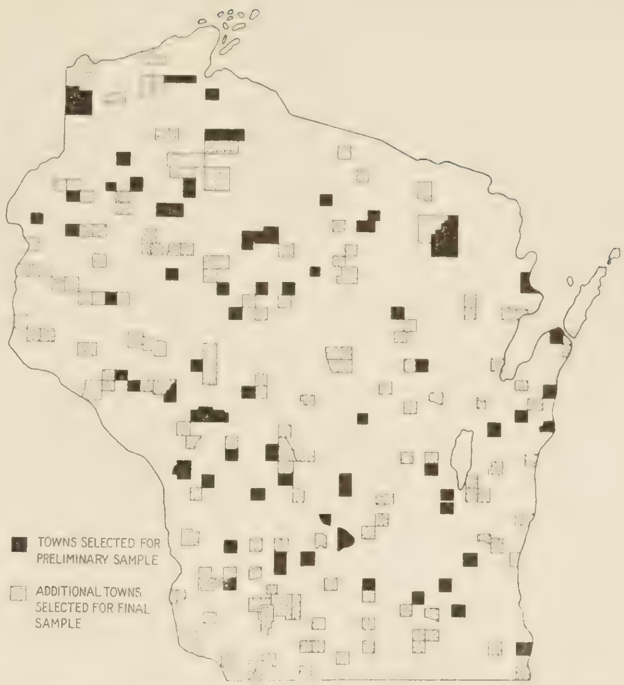

FIGURE 2.—GEOGRAPHICAL DISTRIBUTION OF INITIAL AND FINAL SAMPLES

The method of obtaining the results shown in table 9 are as follows:

The mean (M_x) of total net expenditures of the 200 Wisconsin towns selected for the final sample was found to be $16,275. There were 1,273 towns in the class to

TABLE 7.—*Frequency distribution of net expenditures of initial and final samples of Wisconsin towns taken from tables 3 and 6*

Total net expenditures	First 64 towns	Second 136 towns	Total 200 towns	Total net expenditures	First 64 towns	Second 136 towns	Total 200 towns
$1,000				*$1,000*			
3	1		1	21		2	2
5	3	2	5	22		3	3
6	7	2	9	23	1	1	2
7	3	7	10	24	3	3	6
8	1	4	5	25		1	1
9	4	14	18	26	2	1	3
10	5	4	9	27		1	1
11	3	9	12	28	1		1
12	5	9	14	29		3	3
13	2	8	10	30		2	2
14	1	11	12	33		1	1
15	1	7	8	36	2	2	4
16	5	10	15	47		1	1
17	1	8	9	50	1		1
18	1	5	6	68		1	1
19	3	5	8	71		1	1
20	7	8	15	114	1		1

which the 200 yielding this mean expenditure belonged. The expanded figure representing the over-all expenditures of the group is $20,718,075 and results from multiplying the mean, $16,275 by the number of towns, 1,273. The dissimilarity of the seven Milwaukee County towns was noted at the outset and it was remarked that it would be necessary to obtain actual data for all of them. The total net expenditures of these towns as shown in column 4 of table 9 was $1,423,660 which, added to the expanded figure for the 1,273 towns described above, provides a State total of $22,141,735 as shown in column 5. This latter amount was more than the known State total of $21,966,309 by $175,426 which facts are shown in columns 6 and 7 respectively. As shown in column 8 this is a relative error of +0.8 percent.

Table 8.—*Computations required in the calculation of standard deviation and arithmetic mean from table 7*—Continued

Total net expenditures x	Frequency f	$(1)-M_a$ d	(2)×(3) fd	(3)×(4) fd^2
(1)	(2)	(3)	(4)	(5)
$1,000				
3	1	−12	−12	144
5	5	−10	−50	500
6	9	−9	−81	729
7	10	−8	−80	640
8	5	−7	−35	245
9	18	−6	−108	648
10	9	−5	−45	225
11	12	−4	−48	192
12	14	−3	−42	126
13	10	−2	−20	40
14	12	−1	−12	12
15	8	0		
16	15	1	15	15
17	9	2	18	36
18	6	3	18	54
19	8	4	32	128
20	15	5	75	375
21	2	6	12	72
22	3	7	21	147
23	2	8	16	128
24	6	9	54	486
25	1	10	10	100
26	3	11	33	363
27	1	12	12	144
28	1	13	13	169
29	3	14	42	588
30	2	15	30	450
31		16		
32		17		
33	1	18	18	324
34		19		
35		20		
36	4	21	84	1,764
37		22		
38		23		
39		24		
40		25		
47	1	32	32	1,024
50	1	35	35	1,225
68	1	53	53	2,809
71	1	66	66	4,356
114	1	99	99	9,801
	200		−533 +788 = +255	28,059

SUMMARY

It is taken for granted that sampling would not ordinarily be undertaken where complete data are already available in the form desired. Consequently, in practical operations, it would be impossible to ascertain the actual error in expansions like those illustrated above. The only practical indices of error which can be derived are those predicated upon theories of probability. In the case of the present expansions complete data were available and it is, therefore, possible to compare the error which mathematical reasoning had indicated as a maximum which it was practically certain would not be exceeded, with the actual error which resulted.

In table 2 it was shown that the total net expenditures of Wisconsin towns for 1935 as developed from the reports of the entire 1,280 units amounted to $21,966,309. In table 9 it is shown that the corresponding figure resulting from this particular sampling experiment was $22,141,735. This difference of $175,426 or approximately 0.8 percent was much less than the coefficient of variation calculated from the sample of 200 units. The careful interpretation of this difference is of utmost importance. With such a result as a precedent there might be a tendency to assume that too great conservatism had been injected into the procedure, and that in reality much smaller samples could be relied upon to yield satisfactory expansions. This would be an unfortunate attitude to cultivate, for it overlooks certain fundamentals of probability theory.

In random sampling the most unbiased method will occasionally produce the most biased selection possible, while conversely, the most biased of sampling methods will now and then yield a sample that would satisfy every test for freedom from bias. It is a question of the frequency of particular results when an infinite, or at least a very large number, of trials are made. Even when a theoretically unbiased sampling method is employed it should be evident that the exact error can neither be foretold, nor indeed measured at all, save by tabulating all of the data, which procedure would, of course, completely vitiate the whole sampling attempt. The important consideration is, or should be, the relative frequency with which a biased selection can be

Table 9.—*Expansion of means obtained from sample of 200 Wisconsin towns together with a comparison of actual and relative errors resulting from the procedure*

Item	Average (from sample of 200 towns)	Expansion excluding Milwaukee County, 1,273×col. 2	Milwaukee County	Expanded to State total, col. 3+col. 4	Actual State total	Deviation of expanded from actual State total: Absolute, col. 5−col. 6	Deviation of expanded from actual State total: Relative, col. 7÷col. 6
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
							Percent
Local revenues:							
(13D) Highway	$845	$1,075,685	$24,167	$1,099,852	$1,140,305	−$40,453	−3.5
(13J) Total	8,180	10,413,140	688,645	11,101,785	10,672,104	+429,681	+4.0
Nonlocal revenues:							
(25D) Highway	1,600	2,036,800	23,618	2,060,418	2,152,891	−92,473	−4.3
(25J) Total	7,750	9,865,750	674,530	10,540,280	10,626,656	−86,376	−.8
All current receipts:							
(29D) Highway	2,430	3,093,390	47,785	3,141,175	3,333,196	−192,021	−5.8
(29J) Total	16,710	21,271,830	1,366,028	22,637,858	22,395,653	+242,205	+1.1
Net expenditures:							
(37D) Highway	3,875	4,932,875	326,096	5,258,971	5,485,159	−226,188	−4.1
(37J) Total	16,275	20,718,075	1,423,660	22,141,735	21,966,309	+175,426	+.8

expected through the use of a given sampling procedure on the particular data available. It should be obvious that the frequency theory is only important as it affects the chances of bias in the single selection which is ordinarily drawn. The actual error in a single selection may be any size whatever, and this fact must be recognized if erroneous conclusions are to be avoided. Theories of probability are helpful, however, even though but one sample is drawn instead of an infinite number.

The problem may be likened to one of the betting odds against the occurrence of a certain event such as the toss of a coin yielding a head, or the rolling of a die producing a four. Assuming freedom from bias in each case the odds against a head in the toss of a coin would be even or 1 : 1, and against a four in the roll of a die 5 : 1. Similarly, the odds against the occurrence of errors larger than certain specified departures from actuality can be calculated for the present sampling inquiry in exactly the same fashion in which it is done for coin tossing and die rolling experiments. The data in table 10 have been arranged to demonstrate these relationships.

TABLE 10.—*Probability of occurrence of theoretical errors in statistics abstracted from sample of 200 Wisconsin towns in comparison with actual errors*

Classification of item	Probable error				Actual error
	3σ (0.9974)	2σ (0.9546)	σ (0.6826)	P. E. (0.5000)	
(13)					
Local revenues:	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
Highway	34.4	23.0	11.5	7.7	−3.5
Total	22.8	15.2	7.6	5.1	+4.0
(25)					
Nonlocal revenues:					
Highway	18.0	12.0	6.0	4.0	−4.3
Total	12.4	8.3	4.1	2.8	−.8
(29)					
All current receipts:					
Highway	19.8	13.2	6.6	4.4	−5.8
Total	16.5	11.0	5.5	3.7	+1.1
(37)					
Net expenditures:					
Highway	15.5	10.3	5.2	3.5	−4.1
Total	15.3	10.2	5.1	3.4	+.8

The important conclusion to be obtained from this presentation is that the reliability of a sample cannot be improved by using less standard errors of the mean as a criterion for an acceptable sample. It is true that by being less conservative and permitting a choice of sample size to rest upon calculations using a single standard error of the mean, the resultant expansions will, apparently, be closer approximations of the actual totals. This improved accuracy is only apparent, however, since the odds against the error being of greater magnitude decrease with the number of standard errors used in determining the range. In the final analysis it is entirely a question of point of view; and a preference for one, two, or three standard errors of the mean is a matter of individual choice without substantial significance, providing it is understood that the probabilities are correspondingly altered and over-optimism is not engendered to the extent that too small a sample is selected.

APPENDIX

The method suggested in this article of selecting a sample by means of "Tippett's Numbers" opens up a field of inquiry that in itself is sufficiently extensive to require separate treatment. It is appropriate to discuss here a few of the reasons for the use of random sampling numbers.

By way of introduction it is pertinent to inquire exactly what is meant by a "random" sample. The statistical concept of randomness cannot be defined merely as the absence of design or purpose. It is not, as its name appears to suggest, the result of caprice. A definition sufficiently rigorous to satisfy the mathematician would fail in most respects to appease the lay reader. If, as is usually true, a random sample is taken to mean an unbiased sample, then the concept of randomness must be approached through a consideration of sampling methods rather than individual samples. It is the method that is biased or unbiased rather than particular samples drawn by that method.

An unbiased method is merely one which, repeated a very large number of times (theoretically an infinite number), rarely produces a biased sample. It is clear that defined in this manner the question of whether methods are biased or unbiased largely depends upon the frequency theory of probability. If it can be established either empirically or inductively that a given method of sampling produces a biased sample very infrequently, then by definition, such method may be termed a random method and samples produced through use of the method will be random samples. While all samples, as thus defined, will be random samples, they will by no means all be unbiased. The apparent inconsistency is not real since the definition for an unbiased method in nowise precludes the possibility of a biased drawing, but merely stipulates that such occurrences will be experienced relatively infrequently.

There is still another aspect of unbiased or random methods of selection that should be fully appreciated; and this is that a biased result may occur at any point in an infinite series of trials. There are no mathematical propositions upon which to base prognostications of the point at which a biased drawing will occur. It is important to remember this because in actual practice nothing approaching an infinite number of samples is drawn. In fact, usually but a single drawing is made and conclusions derived therefrom are attributed to the entire population from which the drawing was made.

A random sample, then, is one produced by a random method of selection. A random method is usually taken to mean an unbiased method, that is, a method which infrequently yields a biased result. A biased result, however, may as easily occur one time as another (including the first time) in an infinite number of trials, the exact incidence of occurrence being utterly unpredictable.

With meticulous regard for the foregoing distinctions, consideration may be given to the problem of drawing randomly. Stated in mathematical form, a random method is one in which every selection of M objects from an original N is equally probable. This is a task that is deceptive in its apparent simplicity. If rigorous mathematical treatment is adhered to, there are few statistical problems more difficult of practical accomplishment. It is clear that whatever other expedients are resorted to it will not suffice to leave the matter of selection to human discretion. This is true even when the individuals concerned are imbued with a conscious desire to avoid bias, are unaware of predispositions of any kind, and are above suspicion so far as intellectual honesty is concerned. The remarks of two leading English authorities, Kendall and Smith, are worth quoting in this particular connection.

* * * House-to-house sampling, the sampling of crop yields, even ticket drawing have all been found to give results widely divergent from expectation. Apart from theoretical considerations, there is thus practical evidence to show that it is insufficient to define a random method as one free from purposive selection. The criterion of randomness in selection must be of a more objective kind.

For the purpose of the discussion we require, at this point, a notion of independence. For the present we take this concept to be undefined, merely noting that it may be expressed in terms of probability. With its aid we may define a random method of selection, applied to the characteristic C of a Universe U, as a method which is independent of C in U.

It is important to notice that this definition of random selection relates to a particular characteristic which is under consideration. There is no such thing as a random method of selection *per se,* considered apart from the universe whose members are being selected. A method which would be random for one universe is not necessarily random for another, and even within the same universe a method which is random in respect of one characteristic is not necessarily random in respect of another.

This accords with general ideas on the subject. For example, a possible method of sampling inhabitants of a street is to take, say, every tenth house. This may give a random sample, but if every tenth house is a corner house, the sample may, or may not, lose its randomness. To decide this point, we shall have to consider the properties of the universe which are under investigation. If we were inquiring into the proportion of inhabitants with blue eyes, it might be sufficient to take the corner houses, on the assumption that the color of eyes was independent of geographical location. On the other hand, if we were sampling for income, the method might fail, since corner houses have, in general, higher rents and rates than others, and we should therefore expect to find them inhabitated by people with larger incomes.

A practical question of great importance which arises in this connection is: How are we to determine whether a given method is independent of a given characteristic? The answer is that we cannot determine it without doubt, for to do so would require a full knowledge of the universe; and this is almost always in practice denied us, for otherwise there would be no point in a sampling inquiry. The assumption of independence must therefore be made with more or less confidence on *a priori* grounds. It is part of the hypothesis on which our ultimate expression of opinion is based.[1]

Ample evidence to substantiate Kendall and Smiths' recommendation against the use of so-called random methods of sampling involving the selection of every nth variate of an array has accumulated during the progress of the State-wide Highway Planning Surveys. To mention but one instance, the sampling of motor-vehicle registrations by taking each license number ending in naught was found quite unsatisfactory. It is useless to speculate upon the reasons for the bias which occurred, but important to note that it did occur in spite of a popular belief that the method was entirely adequate and practical.

Virtually the only situation in which the selection of every nth variate would fulfill the general requirements for an unbiased method would be where the variates were arrayed in random order at the outset. The sampling process would then consist merely in choosing the necessary number of variates, taking them in a block from any part of the array. This presupposes the existence of randomness in the arrayed order of the variates prior to selection, a condition seldom if ever satisfied. In fact the entire problem arises precisely because raw data as they are usually assembled are not randomly arrayed. Data tend to become what is termed "packaged" or grouped together in various and sundry ways. Sometimes packaging in data is readily discovered merely by inspection, while at other times its detection is extremely difficult.

Granted that the method of drawing every nth variate lacks virtue, consideration may be given the alternative chiefly resorted to prior to the advent of random sampling numbers, that is, lottery devices. These methods involve the same initial step necessary in the case of sampling by random numbers, the superimposing of an additional and independent characteristic upon the members of the universe. This is accomplished by numbering the members in any convenient way. Tickets, cards, marbles, beads, capsules, and an infinitude of similar media are numbered to correspond and placed in a variety of contrivances that supposedly effect thorough shuffling. Practical experiments, however, have demonstrated that it is impossible to mix balls or shuffle cards sufficiently to effect randomness in their arrangement. Speaking of this problem Karl Pearson of the University of London says,

* * * The dice of commerce are always loaded, however imperceptibly. The records of whist, even those of long experienced players, show how the shuffling is far from perfect, and to get theoretically correct whist returns we must deal the cards without playing them. In short, tickets and cards, balls and beads fail in large scale random sampling tests; it is as difficult to get artificially true random samples as it is to sample effectively a cargo of coal or of barley.[2]

It is evident that what is needed is an unbiased method of sampling that will overcome the deficiencies of the alternative methods that have been discussed. A method is needed that will overcome the theoretical objections surrounding the taking of every nth variate of an array, and at the same time one that will avoid the practical difficulties involved in devising and operating an adequate shuffling mechanism for the randomizing of tickets, capsules, beads, etc. Fortunately, both objectives may be accomplished by the use of random sampling numbers.

Random sampling numbers are tables of numbers, the digits of which have been selected by unbiased methods. Presumably they represent a random set of possible ordinals. Until recently, relatively few tables have been offered, but a number of methods of producing satisfactory sets have been devised, including some very refined processes. There are a number of technical requirements to be satisfied in constructing a set of numbers, and there is still a measure of disagreement among students as to the necessary and sufficient tests that must be applied. The fact that no set of numbers has received the unqualified endorsement of all investigators is a circumstance of little consequence insofar as the present use of certain of these tables is concerned. The numbers of Tippett, Kendall and Smith, and possibly others, are entirely satisfactory.

One of the conditions to be satisfied by a set of random sampling numbers is "local randomness". The concept of local randomness arises from the necessity of distinguishing between a random table of numbers and a table of random numbers. Any set of numbers are random in the sense that they could have resulted from a random selection. A set of one million zeros might even have been produced by an unbiased method. Subsets of numbers drawn from such tables would not necessarily, in fact almost certainly would not, be

(Continued on page 212)

[1] M. G. Kendall and B. Babington Smith, Journal of the Royal Statistical Society, vol. 101, p. 151, 152.

[2] Karl Pearson, in foreword to Random Sampling Numbers, by L. H. C. Tippett, No. XV, Tracts for Computers. Cambridge University Press, London, 1927

EFFECT OF GLASSY SLAG ON THE STABILITY AND RESISTANCE TO FILM STRIPPING OF BITUMINOUS PAVING MIXTURES

REPORTED BY THE DIVISION OF TESTS, PUBLIC ROADS ADMINISTRATION

In the various States that use blast-furnace slag as a road-building material, there appears to be considerable difference of opinion as to the amount of restriction that should be placed on the percentage of glassy particles in slag aggregates for bituminous paving mixtures. Glassy particles have been considered detrimental because it was feared that their smooth surface texture would impair the stability of bituminous mixtures and that the bituminous film would not adhere to them in the presence of water.

From an investigation to determine whether the stability and resistance to film stripping of bituminous paving mixtures prepared from slag aggregates were affected materially by the glassy-particle content of the slag, it was concluded:

1. The susceptibility to film stripping of the mixtures containing the six slags that were tested was not affected materially by variations in the content of glassy particles.

2. For bituminous mixtures containing 0, 15, and 30 percent of glassy particles in the fraction retained on the No. 4 sieve, the percentage of glassy material did not have a significant effect on stability.

3. The tests furnish no indication that specification requirements placing drastic limitations on the glassy-particle content of slag aggregates for bituminous concrete are necessary.

THERE appears to be a considerable difference of opinion in the various States that use blast-furnace slag as a road-building material as to the amount of restriction that should be placed on the percentage of glassy particles in slag aggregates for bituminous paving mixtures. This difference of opinion is reflected in the specification requirements of the States concerned. Several States that use slag place no restriction whatsoever on the amount of glassy material. Others allow various maximum percentages from 20 percent in some cases down to 10 percent in others. A third group of States words its specifications in such a way as virtually to prohibit the inclusion of any appreciable percentage of glassy particles.

As generally understood, glassy particles are those that by visual inspection appear to be composed of more than 50 percent of glassy material. They are characterized by a vitreous to waxy luster and are therefore easily distinguished from the rough-textured cellular material that usually comprises the major portion of blast-furnace slags. Glassy particles have been considered detrimental because it was feared that their smooth surface texture would impair the stability of bituminous mixtures and that the bituminous film would not adhere to them in the presence of water.

The investigation which is the subject of this report was undertaken to determine whether the stability and resistance to film stripping of bituminous paving mixtures prepared from slag aggregates were affected materially by the glassy-particle content of the slag.

The seven samples of slag used in the investigation were furnished by the National Slag Association and were obtained from representative plants in Ohio, Pennsylvania, and New York. They were normal materials for the plants that produced them and while not necessarily representative of slag aggregates for the entire country, they were typical of material produced in the areas from which they came. Samples 1 to 6 were composed of various proportions of glassy and nonglassy particles all retained on the No. 4 sieve. The percentage of glassy particles in each was determined by hand sorting the entire samples and weighing the glassy and nonglassy fractions. The fractions were then stored in separate containers and used either separately or recombined in definite proportions for the various tests to be described. Sample 7 was a slag sand sized to pass the No. 8 sieve. No attempt was made to determine its glassy-particle content. It was used as the fine aggregate in preparing all the mixtures for the stability tests but was not used in the mixtures for the film-stripping test since these mixtures contained only material passing the ⅜-inch sieve and retained on the No. 4 sieve.

The percentages of glassy particles contained in plant samples 1 to 6 inclusive, as determined in the Public Roads laboratory, were as follows:

Slag No.:	*Glassy particles, percent*
1	22
2	22
3	17
4	21
5	22
6	25

FILM-STRIPPING TESTS MADE ON INDIVIDUAL FRACTIONS AND ON BLENDS

From each of the six slags listed above, three classes of bituminous mixtures were prepared for the film-stripping test. The first contained only the nonglassy fraction; the second contained the same proportions of nonglassy and glassy particles as were found in the plant samples; and the third contained only glassy particles. A complete series of mixtures was made with each of four bituminous materials, namely, RC–3 cut-back asphalt, 85–100 penetration asphalt, and road tars RT–6 and RT–9. All the mixtures contained 5 percent by weight of bituminous material and 95 percent by weight of aggregate. The aggregates were sized to pass the ⅜-inch sieve and be retained on the No. 4 sieve. All the mixtures were oven-cured for 24 hours at a temperature of 140°F. before testing.

The stripping test was made in an apparatus similar to that described by Victor Nicholson in the Proceedings of the Association of Asphalt Paving Technologists, January 1932, page 43.[1] Certain modifications of the described machine were made such as the installation of an electric heater and thermostat in the bath and the provision of a variable-speed motor and odometer to permit variation of the number of turns obtained during the 60-minute normal test period. The rate of rotation used in these tests was 45 revolutions per minute.

The amount of stripping was recorded at 15-minute intervals during the 60-minute test period. The test temperatures were: For the first two 15-minute periods, 77° F.; for the third 15 minutes, 100° F. and for the

[1] See also Report of Committee on Characteristics of Asphalts, by E. F. Kelley, Chairman. Proceedings of The Highway Research Board, 1937, pp. 329 to 333.

fourth and final 15 minutes, 120° F. The degree of stripping at each time of observation was recorded as follows:

N=no stripping;
VS=very slight stripping (minute breaks showing in film);
S=slight stripping (dots of bare stone showing);
B=bad stripping (¼ of the aggregate surface exposed); and
VB=very bad stripping (more than ¼ of aggregate surface exposed).

In order to facilitate comparison of the behavior of the various test mixtures, two methods of assigning numerical ratings based on the test results were devised. In both methods the numerical values, 100, 90, 70, 40, and 0 were assigned to the five degrees of stripping—N, VS, S, B, and VB, respectively.

In the first method of rating (see table 1) the observed condition of the samples after 30 minutes of rotation at 77° F. was taken as the criterion of behavior and the numerical value previously assigned to the degree of stripping observed was used as the rating index.

In the second method (see table 2) the numerical values corresponding to the various degrees of stripping observed at the four stages of the test were averaged to give a rating index. By either method, the best possible rating would be 100 corresponding, in the first, to "no stripping" after 30 minutes rotation at 77° F. and, in the second, to "no stripping" after 15, 30, 45, and 60 minutes while the temperatures at the respective observation times were 77° F., 77° F., 100° F. and 120° F.

The results of the stripping tests on the entire series of mixtures using the first method of rating, as shown in table 1, indicate that the type and consistency of the bituminous material affected the amount of stripping to a much greater extent than did the variations in the glassy-particle content of the slag aggregate. In fact, no consistent and significant difference in the resistance to stripping appeared to result from the complete exclusion of glassy particles or the inclusion of 100 percent glassy particles in the mixtures with any of the bituminous materials. The most startling result of the tests was the fact that, in 22 out of 24 sets of samples where the only variable considered within the set was the glassy-particle content, the mixtures containing blends of glassy and nonglassy particles showed resistance to stripping equal to or greater than that shown by the mixtures containing all nonglassy particles; and that in 20 out of 24 cases, the mixtures containing all glassy particles resisted stripping as well as, or slightly better than those containing all nonglassy particles.

The ratings obtained by method 2, as shown in table 2, are generally similar in the group relationships to those of table 1, although the numerical values are nearly all lower. It should be realized that the ratings in both tables are based on visual estimates of the extent of the stripping. Thus, they are, at best, only approximately quantitative and for that reason, slight differences in the ratings of individual samples should not be given too much consideration. However, the test results by groups of samples show a fair degree of consistency and, from these group relationships, it is concluded that the percentage of glassy particles in the slag aggregate has no important influence on stripping.

TABLE 1.—*Results of stripping tests for one test condition*[1] (*rating method 1*)

Slag No.	Glassy-particle content	Rating in the stripping test for slag aggregates mixed with—								Average of all tests
		RC-3		85-100 penetration asphalt		RT-6		RT-9		
		Observed	Index	Observed	Index	Observed	Index	Observed	Index	
1	0	Slight	70	Very slight	90	Bad	40	Slight	70	57
2	0	do	70	do	90	do	40	Bad	40	
3	0	do	70	Slight	70	Very bad	0	do	40	
4	0	do	70	do	70	do	0	do	40	
5	0	Very slight	90	do	70	Bad	40	Slight	70	
6	0	Slight	70	Very slight	90	do	40	Bad	40	
Average			73		80		27		50	
1	22	Slight	70	Very slight	90	Very slight	90	Bad	40	70
2	22	Very slight	90	do	90	Slight	70	do	40	
3	17	do	90	do	90	do	70	do	40	
4	21	do	90	do	90	Bad	40	do	40	
5	22	do	90	do	90	Slight	70	do	40	
6	25	Slight	70	do	90	do	70	do	40	
Average			83		90		68		40	
1	100	Slight	70	Very slight	90	Bad	40	Bad	40	55
2	100	do	70	do	90	do	40	do	40	
3	100	do	70	do	90	Very bad	0	do	40	
4	100	do	70	do	90	do	0	do	40	
5	100	Very slight	90	do	90	do	0	do	40	
6	100	do	90	do	90	do	0	do	40	
Average			77		90		13		40	

[1] Test condition: Temperature, 77° F.; time of rotation, 30 minutes.

TABLE 2.—*Results of stripping tests for four test conditions*[1] (*rating method 2*)

Slag No.	Glassy-particle content	Rating in the stripping test for slag aggregates mixed with—				
		RC-3	85-100 penetration asphalt	RT-6	RT-9	Average of all tests
1	0	58	78	28	60	47
2	0	58	78	28	33	
3	0	50	50	0	33	
4	0	60	50	0	33	
5	0	73	68	28	50	
6	0	68	73	33	33	
Average		61	66	20	40	
1	22	58	85	63	28	55
2	22	78	85	50	28	
3	17	63	63	50	28	
4	21	63	73	20	28	
5	22	63	85	45	43	
6	25	58	85	45	43	
Average		64	79	46	33	
1	100	50	78	20	28	41
2	100	50	78	28	28	
3	100	40	63	10	28	
4	100	50	63	0	28	
5	100	63	73	0	43	
6	100	63	78	0	28	
Average		53	72	10	31	

[1] Condition observed at 15-minute intervals while test temperature is 77° F. for 30 minutes, 100° F. for 15 minutes, and 120° F. for 15 minutes.

STABILITY TESTS MADE ON MIXTURES CONTAINING 0, 7.5, AND 15 PERCENT GLASSY PARTICLES

Both roller stability and Stanton-Hveem stability tests were made on a series of slag-asphalt concrete mixtures containing one 50–60 penetration asphalt and three different proportions of glassy and nonglassy particles. All six slags were brought to the following grading and 7.5 percent by weight of asphalt and 92.5 percent by weight of aggregate were used in all the mixtures.

Sieve size:	Total amount Passing percent
½-inch	100
⅜-inch	92
No. 4	50
No. 8	42

For all the mixtures, the material passing the No. 4 sieve and retained on the No. 8 sieve was obtained by crushing and sieving a part of the nonglassy material from the appropriate plant sample. In all cases, the fraction passing the No. 8 sieve consisted of a portion of slag sand, sample No. 7. As shown above, the sum of these two fractions, or the total amount passing the No. 4 sieve, comprised 50 percent of the aggregate in each test mixture.

Variations were made in the glassy-particle content of the 50 percent of the aggregate retained on the No. 4 sieve. In one set of six mixtures (one from each slag sample) no glassy particles were included in the fraction retained on the No. 4 sieve. In the next set, 15 percent of the material retained on the No. 4 sieve, or 7.5 percent of the total aggregate, consisted of glassy particles. In the third set, 30 percent of the coarse fraction or 15 percent of the total aggregate was glassy.

One 2½- by 4- by 8-inch specimen was prepared from each mixture and tested in the Public Roads roller stability machine.[2] Each specimen was compacted to develop the same aggregate density in the bituminous mixture as that obtained by vibrating the dry aggregate.[3] The compacted unit weights of the mixtures in grams per cubic centimeter and the roller stability values obtained on them are given in table 3.

The roller stability test is a simulated small-scale traffic test in which a series of small steel rollers are passed over the specimen, which is immersed in water maintained at the test temperature of 140° F. The measure of stability is taken as the number of roller passages required to produce an elongation in the specimen of 0.3 inch.

After the specimens had been tested in the roller stability machine, they were broken up and a portion of each was molded into a cylindrical specimen 4 inches in diameter by 2½ inches high for the Stanton-Hveem stability test. These specimens were molded under direct compression of 3,000 pounds per square inch by the double-plunger method. They were tested at a temperature of 77° F. The results of the Stanton-Hveem stability tests are given in table 4 and are shown graphically in figure 1.

TABLE 3.—*Results of roller stability tests on slag-asphaltic concrete*

Slag No.	Unit weight of test specimen			Roller stability at 140° F.		
	0 percent glassy	7.5 percent glassy	15 percent glassy	0 percent glassy	7.5 percent glassy	15 percent glassy
	Gm. per C. C.	*Gm. per C. C.*	*Gm. per C. C.*			
1	2.21	2.23	2.26	15	24	20
2	2.22	2.24	2.27	28	25	19
3	2.33	2.34	2.34	55	51	27
4	2.21	2.23	2.24	22	24	14
5	2.23	2.24	2.24	25	33	28
6	2.19	2.21	2.22	25	25	21
Average	2.23	2.25	2.26	28	30	22

[2] Apparatus and method of test described by E. L. Tarwater in PUBLIC ROADS, September 1935, p. 134.
[3] Apparatus and method of test described by J. T. Pauls and J. F. Goode in PUBLIC ROADS, May 1939, p. 55.

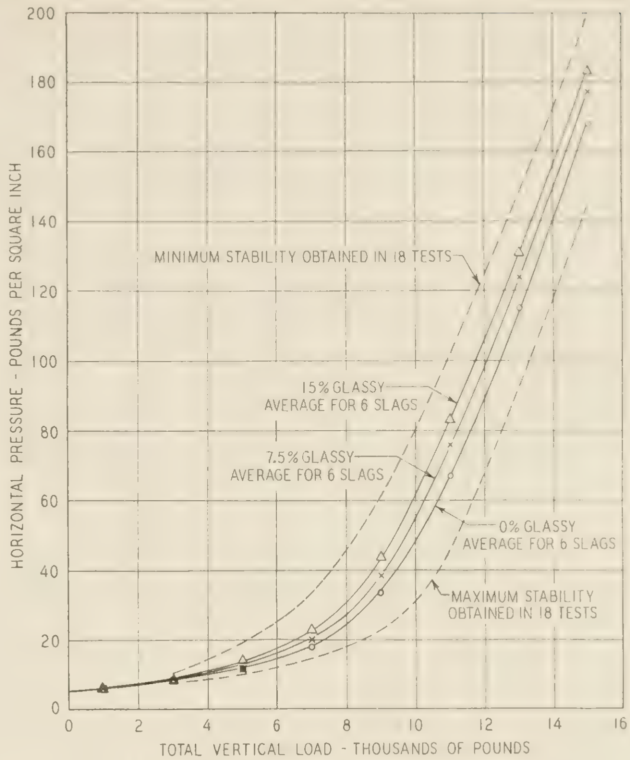

FIGURE 1.—MAXIMUM, MINIMUM, AND AVERAGE STANTON-HVEEM STABILITY CURVES FOR THE 18 TEST MIXTURES OF TABLE 4.

This stability test is a direct compression test in which a measurement is made of the lateral pressure generated in a cylindrical test specimen by the axial load. High lateral pressure for a given load indicates relatively low stability while low lateral pressure for the same axial load indicates relatively high stability.

Neither the roller nor the Stanton-Hveem test results indicated that the stability of this group of mixtures

TABLE 4.—*Results of Stanton-Hveem stability tests on slag-asphaltic concrete*

Slag No.	Glassy-particle content	Unit weight of test specimen	Horizontal pressure, when total vertical load was—								
			0	1,000 pounds	3,000 pounds	5,000 pounds	7,000 pounds	9,000 pounds	11,000 pounds	13,000 pounds	15,000 pounds
	Percent	*Gm. per c. c.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*
1	0	2.31	5	6	8	11	17	31	66	119	175
2	0	2.32	5	6	9	11	14	22	45	92	[1] 145
3	0	2.36	5	7	11	16	25	47	85	132	185
4	0	2.26	5	7	10	14	20	38	74	122	174
5	0	2.31	5	6	8	11	15	30	67	115	169
6	0	2.26	5	6	9	12	19	34	65	109	159
Average			5	6	9	13	18	34	67	115	168
1	7.5	2.30	5	6	9	13	21	38	74	122	176
2	7.5	2.33	5	6	9	12	17	30	63	112	165
3	7.5	2.38	5	7	11	16	27	51	90	139	190
4	7.5	2.27	5	6	9	13	20	41	81	129	182
5	7.5	2.31	5	6	9	12	19	35	74	121	174
6	7.5	2.28	5	6	9	11	17	34	72	122	174
Average			5	6	9	13	20	38	76	124	177
1	15	2.34	5	7	10	14	22	41	81	130	185
2	15	2.34	5	6	9	12	18	34	73	122	175
3	15	2.38	5	7	12	19	33	61	102	148	[2] 197
4	15	2.28	5	6	10	15	25	51	91	139	190
5	15	2.32	5	6	9	13	20	38	76	124	174
6	15	2.29	5	6	9	14	21	40	77	123	174
Average			5	6	10	15	23	44	83	131	183

[1] Most stable of the 18 mixtures.
[2] Least stable of the 18 mixtures.

was materially affected by the glassy-particle content of the slag aggregate. Considering individual slag samples, the roller stability test indicated that the mixtures containing 7.5 percent glassy particles might be slightly more stable than those containing 15 percent (see table 3). However, in two cases, the mixtures containing no glassy particles appeared to be slightly less stable than either of the mixtures containing glassy fractions and there were only two cases in which the nonglassy mixtures showed even slight superiority over both the corresponding glassy mixtures.

The Stanton-Hveem tests were quite consistent in showing a slight advantage for the all nonglassy mixtures over those containing 7.5 percent of glassy material and essentially the same advantage for the 7.5 percent mixtures over those containing 15 percent glassy material. These slight differences are not believed to be particularly significant since, without exception, the stability values for all 18 mixtures, whether obtained by the roller or Stanton-Hveem method, are well within the range considered necessary to assure satisfactory resistance to displacement under traffic.

CONCLUSIONS

1. The susceptibility to film stripping of the mixtures containing the six slags that were tested was not affected materially by variations in the content of glassy particles.

2. For bituminous mixtures containing 0, 15, and 30 percent of glassy particles in the fraction retained on the No. 4 sieve, the percentage of glassy material did not have a significant effect on stability.

3. The tests described furnish no indication that specification requirements placing drastic limitations on the glassy-particle content of slag aggregates for bituminous concrete are necessary.

(Continued from page 208)

random. Hence, a table of numbers from which randomly arrayed subsets may be drawn is designated "locally random".

The subject of local randomness cannot receive a full exposition here but some discussion of its fundamental importance in a set of random sampling numbers is believed to be desirable. The remarks of Kendall and Smith relative to tests for local randomness are worthy of quotation.

> For practical purposes in deciding whether a given set is locally random, we have found that the following four tests are useful and searching. They are, however, not sufficient to establish the existence of local randomness, although they are necessary.
>
> *a.* The first and most obvious test to be applied is that all the digits shall occur an approximately equal number of times. This test we call the frequency test;
>
> *b.* Secondly, if the series is locally random, no digit shall tend to be followed by any other digit. If therefore we form a bivariate table showing the distribution of pairs of digits in the series, arranged in the rows according to the first digit, and in the columns according to the second digit, we should get frequencies which are approximately equal in all the cells. This test we refer to as the serial test;
>
> *c.* Thirdly, if the digits are arranged in blocks of, say, five, there will be certain expectation of the numbers in which the five digits are all the same, the numbers in which there are four of one kind, and so on. This test we refer to as the poker test, from an analogy with the card game;
>
> *d.* Finally, there are certain expectations in regard to the gaps occurring between the same digits in the series. For instance, if we take one digit, say, zero, in about one-tenth of the cases the first zero will be followed immediately by a second zero, and there will be no gap. In about nine-hundredths of the cases there will be one digit between two zeros. In about eighty-one thousandths of the cases there will be a gap of two digits between successive zeros, and so on. This we call the gap test.
>
> These four tests taken together are very powerful. It is comparatively easy to form series that evade the first three. For example, the recurring series, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, evades the frequency test, the series 1–2–3–4–, etc., evades the frequency test, and the serial test if the dashes are filled with random digits. We have, however, not succeeded in constructing a series which would certainly evade the gap test. Such a series would, it appears, have to have a very peculiar bias indeed, such as would hardly ever rise in practice.
>
> The gap test may be extended. Not only will there be an expectation of the frequency of the gaps, but there will also be an expectation of the gaps between gaps of the same kind; these in turn will have expected gaps between them, and so on. There is thus an infinite sequence of the gap tests no one of which includes the others. All these tests are necessary for local randomness, though we have not established their sufficiency. It appears, therefore, that there is no finite set of tests of this character which is sufficient to demonstrate the local randomness of all finite sets of numbers.[3]

There remains to be mentioned the manner of using a set of random numbers in drawing a sample. If the topics just discussed are kept in mind, the use of a table of random numbers is extremely simple and straightforward. As already indicated, the first step is to number the members of the universe being sampled in any convenient order. Then, recalling that the digits of a proper set of random numbers are locally random, it is seen that all that is required is the choice of some pattern for selecting subsets of the proper number of digits each of which will constitute the sample to be drawn from the universe in question.

Naturally, to maintain theoretical validity the pattern chosen to select subsets should be followed consistently until a sufficient number of ordinals are obtained or until the possibilities of this pattern are exhausted. Most sets are arranged in columns of four or five digit numbers. Assuming the set being used to be that of Tippett which contains columns of four digit numbers, the selection of a 10 percent sample, for example, from a universe of 1,000 variates becomes a matter merely of selecting, starting at any point and not retracing, the first 100 numbers less than 1,000 which appear. This will be taken as following an unbiased method of selection. Any other method of putting digits together vertically or horizontally, forwards or backwards to make numbers of the desired size will be equally satisfactory, providing only that the pattern of selecting subsets once chosen is followed consistently.

[3] M. G. Kendall and B. Babington Smith, Journal of the Royal Statistical Society vol. 101, pp. 154, 155.

HIGHWAY RESEARCH BOARD MEETS IN BALTIMORE DECEMBER 2–5, 1941

Changing a custom of twenty years' standing on account of the need for conserving Washington facilities for urgent defense needs, the Highway Research Board announces that its Twenty-fisrt Annual Meeting will be held at The Johns Hopkins University, Baltimore, Maryland, the first week of December 1941.

On Tuesday, December 2 several Departments of the Board will hold open meetings for the consideration of papers and reports.

Sessions of the Board for the discussion of topics relating to highway finance, economics, design, materials, construction, maintenance, traffic, and soils investigations will be held on Wednesday, Thursday, and Friday, December 3–5.

STATUS OF FEDERAL-AID HIGHWAY PROJECTS

AS OF OCTOBER 31, 1941

STATE	COMPLETED DURING CURRENT FISCAL YEAR			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	
Alabama	$ 2,537,337	$ 1,261,780	92.8	$ 6,038,031	$ 2,996,115	193.3	$ 1,880,004	$ 934,850	49.2	$ 897,957
Arizona	531,710	383,026	33.1	1,825,980	1,226,780	67.8	558,314	372,325	12.2	875,129
Arkansas	2,710,335	1,258,509	49.6	845,756	422,558	29.0	575,078	286,701	37.8	393,967
California	4,980,409	2,639,950	88.0	5,149,495	2,808,374	61.1	3,496,621	1,752,090	61.4	1,725,973
Colorado	2,051,555	1,160,808	122.3	2,150,528	1,247,015	150.6	1,898,858	1,072,095	56.1	1,113,203
Connecticut	711,753	347,250	7.2	1,839,282	898,833	22.8	1,121,015	502,187	14.9	493,167
Delaware	214,010	104,617	4.9	811,153	399,662	21.2	268,040	134,020	8.4	979,278
Florida	772,915	386,458	52.4	1,537,392	794,925	30.5	1,892,110	922,393	16.1	2,236,146
Georgia	2,125,531	1,062,765	62.1	6,566,643	3,293,572	273.6	3,919,851	1,959,926	164.8	4,849,951
Idaho	1,361,855	839,942	72.8	1,168,465	720,832	63.7	604,926	373,507	20.4	942,750
Illinois	2,725,111	1,354,897	71.3	8,198,935	4,099,392	136.3	1,258,998	628,794	6.3	3,310,226
Indiana	2,600,929	1,296,650	39.4	6,951,227	3,233,098	108.3	1,567,842	783,921	20.7	909,495
Iowa	1,875,347	884,343	85.5	5,629,674	2,520,158	183.2	1,218,516	335,300	44.5	28,022
Kansas	3,205,430	1,621,883	161.5	4,777,500	2,392,926	277.2	3,360,461	1,389,584	128.5	2,936,917
Kentucky	2,398,662	1,203,144	93.6	6,920,394	3,296,653	155.1	2,932,980	1,466,490	20.3	70,280
Louisiana	800,645	400,301	22.3	2,030,120	1,006,642	39.7	2,568,556	1,259,086	56.3	3,053,749
Maine	708,657	356,578	20.2	1,893,742	972,831	22.3	270,170	135,085	3.9	364,973
Maryland	1,703,200	851,000	18.9	3,370,162	1,574,678	19.7	1,145,000	242,500	5.9	951,489
Massachusetts	2,253,419	1,129,370	17.3	2,316,145	1,189,333	14.9	1,175,721	584,274	8.4	2,593,405
Michigan	5,902,470	2,936,758	131.7	4,320,270	2,160,135	74.5	2,083,100	1,041,550	19.5	737,564
Minnesota	3,290,769	1,641,307	310.2	9,765,332	4,841,298	410.1	1,363,844	679,537	58.7	729,780
Mississippi	2,802,960	1,400,620	160.1	5,549,952	2,710,986	308.9	659,300	327,900	32.5	661,219
Missouri	3,923,098	1,936,666	141.6	10,254,546	4,659,107	190.1	3,162,480	757,629	44.5	3,045,965
Montana	2,002,944	1,132,347	98.9	2,847,172	1,617,899	136.6	753,264	428,306	66.9	3,080,650
Nebraska	756,448	373,489	109.9	7,319,146	3,694,565	647.0	710,611	355,306	38.5	2,176,430
Nevada	1,774,181	1,543,338	92.2	1,036,310	899,711	34.3	458,057	397,569	9.4	97,664
New Hampshire	292,328	144,253	5.3	1,223,881	585,889	14.3	18,224	7,312		770,981
New Jersey	2,348,574	1,174,287	20.5	3,562,648	1,781,244	22.2	22,270	11,135		1,759,929
New Mexico	943,125	580,478	71.0	1,404,425	868,207	87.2	16,182	10,463	5.0	1,507,962
New York	5,116,556	2,551,583	76.8	10,931,119	5,432,161	123.4	862,200	411,850	8.4	2,466,895
North Carolina	2,392,919	1,180,290	105.9	4,067,552	2,036,635	157.0	577,927	285,555	26.2	1,736,439
North Dakota	2,696,932	1,509,463	238.4	3,068,110	1,587,484	235.4	2,086,670	1,046,705	179.4	2,976,805
Ohio	5,211,504	2,599,677	43.2	14,054,902	6,786,461	114.4	4,064,080	1,691,485	27.3	2,164,705
Oklahoma	1,329,205	650,460	62.8	3,108,341	1,640,449	85.2	2,183,370	1,143,558	72.9	4,056,099
Oregon	1,142,629	689,896	37.6	4,344,409	2,283,840	97.6	449,649	221,190	14.7	371,582
Pennsylvania	3,082,916	1,535,850	41.8	13,941,192	6,869,809	112.1	2,234,606	1,070,884	18.0	1,669,073
Rhode Island	533,402	266,095	5.5	1,280,255	638,753	9.0	28,060	14,030	.3	773,816
South Carolina	891,504	415,437	64.7	3,876,970	1,793,057	96.7	1,594,553	687,327	37.8	1,250,646
South Dakota	1,732,723	1,008,810	224.0	4,565,593	2,892,779	484.1	1,435,890	898,660	171.6	1,369,857
Tennessee	1,583,516	789,774	60.1	5,322,162	2,661,081	96.4	1,821,164	910,582	52.0	2,312,934
Texas	5,393,989	2,645,034	286.0	12,971,773	6,328,224	495.8	5,484,880	2,244,670	182.1	3,776,213
Utah	648,455	484,518	42.1	2,214,987	1,670,431	43.7	75,033	55,188	5.4	251,387
Vermont	333,669	165,120	12.5	1,695,292	843,996	37.0	41,081	20,540	.3	57,640
Virginia	1,243,087	623,172	30.0	5,272,146	2,462,923	94.7	889,310	441,605	7.7	1,076,104
Washington	362,625	194,000	6.0	3,239,392	1,731,404	46.0	539,413	260,085	5.2	764,214
West Virginia	1,278,460	637,233	30.4	3,570,460	1,770,293	44.7	464,766	230,633	6.8	1,230,577
Wisconsin	908,049	446,770	46.8	6,329,197	2,988,288	194.3	1,939,320	731,759	62.8	2,393,602
Wyoming	1,414,675	903,980	148.3	1,148,529	754,151	78.9	737,311	474,086	44.6	239,335
District of Columbia	463,514	231,540	2.5	732,536	343,000	1.0	140,642	70,300	.9	247,214
Hawaii	140,808	70,395	1.1	701,887	518,147	9.5	41,366	21,894	.3	1,694,653
Puerto Rico	176,100	87,085	2.6	1,420,268	701,150	15.4	1,082,599	534,660	5.1	336,513
TOTALS	98,382,944	51,092,996	3,825.7	225,161,428	113,647,984	6,467.8	69,734,283	32,619,081	1,940.9	76,510,535

STATUS OF FEDERAL-AID SECONDARY OR FEEDER ROAD PROJECTS

AS OF OCTOBER 31, 1941

STATE	COMPLETED DURING CURRENT FISCAL YEAR			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	
Alabama	$ 832,405	$ 414,568	35.8	$ 991,152	$ 506,990	51.9	$ 165,900	$ 77,280	5.9	$ 294,164
Arizona	67,371	48,972	4.5	194,180	143,267	9.5	68,387	47,600	4.0	307,364
Arkansas	330,055	164,276	27.4	235,449	117,623	17.4	331,004	165,436	14.3	68,426
California	522,510	318,740	12.4	1,032,408	738,548	9.9	129,976	74,930	3.6	409,896
Colorado	142,204	70,526	20.7	129,755	72,649	2.1	4,602	2,598	1.9	262,976
Connecticut	105,456	49,907	1.8	458,826	202,361	9.1				76,648
Delaware	31,959	15,265		274,043	135,122	12.3	102,873	37,617	3.9	158,720
Florida	118,966	59,483	4.4	1,046,553	528,727	7.4				232,834
Georgia	293,032	131,516	26.7	993,558	589,129	61.8	833,048	416,524	73.5	697,801
Idaho	163,914	97,878	12.0	188,523	116,006	17.7	144,882	63,255	7.0	145,326
Illinois	680,650	337,436	28.6	1,434,510	717,255	76.4	31,700	12,600	7.1	286,246
Indiana	90,900	45,450	2.8	1,648,805	791,246	83.7				599,828
Iowa	508,912	240,491	124.3	363,952	153,398	71.6	245,880	115,426	43.0	252,470
Kansas	348,387	176,679	54.6	1,944,652	974,493	132.3	443,636	221,159	33.6	668,146
Kentucky	413,140	97,595	15.0	1,377,687	355,843	82.0	786,685	199,174	32.4	110,785
Louisiana	558,248	227,059	20.6	6,460	3,230		289,362	138,761	21.5	446,956
Maine	14,200	7,100	.8	251,758	125,879	11.7	63,650	26,114	2.2	9,491
Maryland	135,000	67,500	8.1	585,000	292,325	13.7	82,000	41,000	.8	202,116
Massachusetts	163,235	85,298	4.1	651,150	342,642	10.1				361,339
Michigan	323,068	161,491	27.2	1,307,660	653,830	61.0	727,370	363,685	23.0	71,980
Minnesota	829,633	420,567	113.5	1,562,365	783,990	171.8	539,972	255,078	48.8	103,952
Mississippi	477,700	238,850	20.6	1,012,161	491,346	49.7	1,008,600	367,376	45.2	112,643
Missouri	285,184	142,198	36.0	747,240	359,779	78.7	420,266	168,538	54.2	583,029
Montana	311,504	176,926	50.7	241,662	137,374	26.7	65,917	37,480	4.8	577,423
Nebraska	102,684	50,743	13.5	681,717	348,749	68.5	83,106	41,553	41.1	338,426
Nevada	118,591	103,169	12.8	129,863	93,360	9.7	159,363	138,486	10.8	2,981
New Hampshire				338,140	167,149	8.2	52,105	3,572	.1	89,289
New Jersey	246,870	123,355	5.1	624,812	329,865	16.5	82,910	41,455	1.8	353,335
New Mexico	413,533	259,915	42.6	189,504	122,533	15.1	167,514	101,129	5.1	6,907
New York	751,962	378,224	20.8	1,053,146	534,664	21.3	75,000	37,500	1.0	502,500
North Carolina	129,990	64,995	11.1	742,847	401,478	60.1	69,820	20,000	5.0	254,103
North Dakota	49,569	26,558	2.4	3,434	3,434		808,050	793,860	42.7	485,901
Ohio	845,072	422,280	30.2	1,709,240	901,810	37.1	115,000	57,500	4.7	795,592
Oklahoma	246,780	130,349	9.7	127,338	67,173	11.9	884,486	467,008	68.3	723,546
Oregon	324,677	162,124	26.6	487,315	260,529	30.8	134,541	51,670	14.1	120,289
Pennsylvania	620,078	310,039	14.1	1,887,581	932,724	33.9	72,000	36,000	1.8	43,101
Rhode Island	88,194	44,040	.9	139,310	73,157	1.7	3,610	1,805		56,220
South Carolina	310,932	100,066	33.9	690,000	284,224	21.2				165,627
South Dakota	32,130	18,006	15.2	3,622	3,622		1,143,430	1,047,600	114.5	490,587
Tennessee	230,843	114,430	8.1	1,479,414	739,707	47.9	234,030	117,015	8.7	409,820
Texas	561,809	272,011	59.3	1,047,627	504,517	95.1	429,420	201,550	40.8	1,168,978
Utah	186,949	123,241	17.0	62,245	32,943	2.5	79,538	36,884	1.0	172,273
Vermont	36,231	18,109	1.2	2,558	1,279		168,097	75,058	7.8	9,258
Virginia	339,398	155,485	11.1	374,996	171,246	9.1	59,050	29,525	4.5	351,901
Washington	151,705	92,113	16.2	565,560	265,814	15.4				149,770
West Virginia	86,300	43,150	2.4	623,774	311,274	23.8	15,700	7,850		313,609
Wisconsin	698,433	350,225	34.5	1,815,193	809,663	56.2	76,393	36,920	.8	124,441
Wyoming	364,162	157,111	18.8	464,237	204,529	31.7	43,601	16,400	2.6	721
District of Columbia	80,772	39,924	.9	2,558	1,279					76,120
Hawaii				2,375	2,375					249,562
Puerto Rico	45,960	22,305	2.5	185,404	90,550	8.1				134,883
TOTALS	14,811,257	7,377,738	1,063.5	34,113,319	16,992,699	1,694.3	11,442,474	6,191,971	807.9	14,630,299

U. S. GOVERNMENT PRINTING OFFICE: 1941

PUBLICATIONS of the PUBLIC ROADS ADMINISTRATION

Any of the following publications may be purchased from the Superintendent of Documents, Government Printing Office, Washington, D. C. As his office is not connected with the Agency and as the Agency does not sell publications, please send no remittance to the Federal Works Agency.

ANNUAL REPORTS

Report of the Chief of the Bureau of Public Roads, 1931. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1932. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1933. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1934. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1935. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1936. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1937. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1938. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1939. 10 cents.

Work of the Public Roads Administration, 1940.

HOUSE DOCUMENT NO. 462

Part 1 . . . Nonuniformity of State Motor-Vehicle Traffic Laws. 15 cents.

Part 2 . . . Skilled Investigation at the Scene of the Accident Needed to Develop Causes. 10 cents.

Part 3 . . . Inadequacy of State Motor-Vehicle Accident Reporting. 10 cents.

Part 4 . . . Official Inspection of Vehicles. 10 cents.

Part 5 . . . Case Histories of Fatal Highway Accidents. 10 cents.

Part 6 . . . The Accident-Prone Driver. 10 cents.

MISCELLANEOUS PUBLICATIONS

No. 76MP . . The Results of Physical Tests of Road-Building Rock. 25 cents.

No. 191MP . . Roadside Improvement. 10 cents.

No. 272MP . . Construction of Private Driveways. 10 cents.

No. 279MP . . Bibliography on Highway Lighting. 5 cents.

Highway Accidents. 10 cents.

The Taxation of Motor Vehicles in 1932. 35 cents.

Guides to Traffic Safety. 10 cents.

An Economic and Statistical Analysis of Highway-Construction Expenditures. 15 cents.

Highway Bond Calculations. 10 cents.

Transition Curves for Highways. 60 cents.

Highways of History. 25 cents.

Specifications for Construction of Roads and Bridges in National Forests and National Parks. 1 dollar.

DEPARTMENT BULLETINS

No. 1279D . . Rural Highway Mileage, Income, and Expenditures, 1921 and 1922. 15 cents.

No. 1486D . . Highway Bridge Location. 15 cents.

TECHNICAL BULLETINS

No. 55T . . . Highway Bridge Surveys. 20 cents.

No. 265T. . . Electrical Equipment on Movable Bridges. 35 cents.

Single copies of the following publications may be obtained from the Public Roads Administration upon request. They cannot be purchased from the Superintendent of Documents.

MISCELLANEOUS PUBLICATIONS

No. 296MP. . Bibliography on Highway Safety.

House Document No. 272 . . . Toll Roads and Free Roads.

Indexes to PUBLIC ROADS, volumes 6–8 and 10–21, inclusive.

SEPARATE REPRINT FROM THE YEARBOOK

No. 1036Y . Road Work on Farm Outlets Needs Skill and Right Equipment.

TRANSPORTATION SURVEY REPORTS

Report of a Survey of Transportation on the State Highway System of Ohio (1927).

Report of a Survey of Transportation on the State Highways of Vermont (1927).

Report of a Survey of Transportation on the State Highways of New Hampshire (1927).

Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio (1928).

Report of a Survey of Transportation on the State Highways of Pennsylvania (1928).

Report of a Survey of Traffic on the Federal-Aid Highway Systems of Eleven Western States (1930).

UNIFORM VEHICLE CODE

Act I.—Uniform Motor Vehicle Administration, Registration, Certificate of Title, and Antitheft Act.

Act II.—Uniform Motor Vehicle Operators' and Chauffeurs' License Act.

Act III.—Uniform Motor Vehicle Civil Liability Act.

Act IV.—Uniform Motor Vehicle Safety Responsibility Act.

Act V.—Uniform Act Regulating Traffic on Highways.

Model Traffic Ordinances.

A complete list of the publications of the Public Roads Administration, classified according to subject and including the more important articles in PUBLIC ROADS, may be obtained upon request addressed to Public Roads Administration, Willard Bldg., Washington, D. C.

STATUS OF FEDERAL-AID GRADE CROSSING PROJECTS

AS OF OCTOBER 31, 1941

STATE	COMPLETED DURING CURRENT FISCAL YEAR					UNDER CONSTRUCTION					APPROVED FOR CONSTRUCTION					BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
	Estimated Total Cost	Federal Aid	NUMBER			Estimated Total Cost	Federal Aid	NUMBER			Estimated Total Cost	Fed. Aid	NUMBER			
			Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise			Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise			Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise	
Alabama	$ 5,739	$ 5,739			2	$ 421,822	$ 419,800	6	5	1	$ 154,235	$ 154,235	3	1	6	$ 729,700
Arizona						309,635	300,944	1	2		13,255	13,255			2	92,976
Arkansas	107,311	107,256			1	511,114	509,130	6			33,786	33,786			11	313,381
California	376,504	190,789	1		1	1,315,838	1,309,865	9	1		20,630	20,630			7	1,501,452
Colorado	5,685	5,646				590,186	590,186	6			21,042	21,042			10	547,822
Connecticut	166,222	165,415	2			61,712	60,676		1		231,374	222,740		1		352,566
Delaware	4,380	4,380			1	94,135	94,135	1			692,694	504,341	3	1		73,707
Florida	33,447	33,447			6	767,560	762,376	8		14	203,082	203,021			19	610,361
Georgia	403,084	403,084	6	1	1	978,537	978,537	6	6	1	934,287	934,287	2	5	27	1,141,222
Idaho	11,301	11,301				302,225	293,553	3	1		34,621	34,621	1		4	262,324
Illinois	187,801	148,267			73	2,161,884	1,978,650	10	1	8	420,414	401,664	1	1	35	1,531,566
Indiana	173,001	173,001	1		8	775,513	763,026	6	1	10	234,376	228,667	1		44	644,992
Iowa	213,010	203,589	2	1	5	1,267,169	1,019,232	10	2	10	495,404	485,475	1		70	120,383
Kansas	34,334	34,229	1		2	569,456	569,456	8			348,854	304,969	5		14	874,766
Kentucky	163,064	161,519	2		1	1,075,361	1,069,248	8	1		319,265	319,265	3			48,565
Louisiana						588,415	588,415	8			473,319	472,152	4		2	610,118
Maine						390,447	390,447	2			8,680	8,680			3	109,934
Maryland	481,550	449,757	2	2	8	330,159	330,159	2		1	590,703	446,905	2		13	205,511
Massachusetts						1,120,671	1,109,293	6	2		763,830	763,830	2	1		878,999
Michigan	394,700	394,700		1	5	972,739	972,739	3	3	3	527,954	497,793	2		27	667,292
Minnesota	338,950	338,635	2	3	1	1,026,634	1,026,634	7	3		270,324	270,324	1	3	4	697,343
Mississippi	202,300	202,300	2		1	811,084	811,084	9	1		98,708	98,708	1		11	327,264
Missouri	55,240	55,240		1		1,979,682	1,524,262	6	3		330,551	223,526	1	2		1,147,479
Montana	56,726	56,726	1			174,019	174,019	2			6,645	6,645			2	474,600
Nebraska	142,718	142,714		1	12	1,187,289	1,187,289	23		6	26,502	26,502			7	116,137
Nevada	119,580	119,580	2			56,484	56,484	2			21,703	21,703			7	103,916
New Hampshire	63,682	62,862	2			306,045	305,745	4	2		9,464	9,464			4	194,348
New Jersey	214,360	214,360	2			1,254,863	1,129,314	5	2	1	354,985	295,560	1	2		554,080
New Mexico						68,342	68,342	1			259,103	252,068	3		1	338,650
New York	1,345,993	1,318,867		8		2,812,436	2,763,787	5	11		750,945	613,285		3		2,420,776
North Carolina	412,365	412,365	2	4	17	256,906	254,028		3	4	251,793	251,793	3		24	782,113
North Dakota	80,242	79,789	2			697,230	684,293	8			223,120	223,120	2			336,428
Ohio	324,301	323,904	1	1	1	3,457,893	3,376,850	15		4	541,407	282,350	2		7	1,049,956
Oklahoma	128,076	124,660	1		17	849,622	846,212	6		3	412,316	354,274	3		18	1,123,686
Oregon	302,166	278,255	3		2	125,127	84,757	2			4,733	4,733			3	391,816
Pennsylvania	873,683	873,355	7	1		3,243,392	3,203,414	17			1,147,255	1,142,789	6			1,330,260
Rhode Island	205,241	205,241		1		3,655	3,655									176,421
South Carolina	169,540	166,310	2		10	342,673	330,278	5	3	5	300,248	166,574	1	2	16	691,260
South Dakota	348,243	348,243	10		2	677,142	661,192	12		7	41,200	41,200	2			577,780
Tennessee	225,916	216,452	1	3		1,176,346	1,176,346	8			102,106	102,106		1	2	845,415
Texas	717,706	708,673	8			1,716,525	1,708,791	17			156,650	148,350	3		5	1,371,886
Utah	42,014	41,272	2		13	77,264	77,264	1		12	69,025	69,025			26	226,082
Vermont	9,723	9,711			2	331,830	302,051	2		2						4,381
Virginia	92,292	92,292		2		718,013	717,553	5	3		106,935	87,435	2		5	568,313
Washington	55,443	55,443		1	1	333,418	333,418	3	1		12,151	12,151			4	453,333
West Virginia	10,640	10,640			4	807,142	801,522	9	1	2	93,190	93,190			3	510,406
Wisconsin	156,539	128,261	1		14	840,100	838,394	4	2	21	61,692	61,692			13	1,159,125
Wyoming	477,151	477,150	5			4,929	4,929				8,199	8,199			6	286,556
District of Columbia	2,193	2,193				1,462	1,462				298,213	273,744		1		5,851
Hawaii	192,574	192,566	2			214,170	213,655	2								180,308
Puerto Rico	103,629	102,980	1			639,340	632,516	9			150,994	149,890	2			188,708
TOTALS	10,230,359	9,853,158	76	31	211	40,795,640	39,409,407	298	61	115	12,631,962	11,361,758	63	24	462	29,952,314