PUBLIC ROADS

A JOURNAL OF HIGHWAY RESEARCH

FEDERAL WORKS AGENCY
PUBLIC ROADS ADMINISTRATION

VOL. 20, NO. 12 ▽ FEBRUARY 1940

TRAFFIC DURING A STUDY OF VEHICLE PASSING PRACTICES

For sale by the Superintendent of Documents, Washington, D. C. - - - - - - - - - - - - - - See page 2 of cover for prices

PUBLIC ROADS ▸▸▸ *A Journal of Highway Research*

Issued by the

FEDERAL WORKS AGENCY

PUBLIC ROADS ADMINISTRATION

D. M. BEACH, *Editor*

Volume 20, No. 12 February 1940

The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

In This Issue

THE PUBLIC ROADS ADMINISTRATION - - - - - - Willard Building, Washington, D. C.

REGIONAL HEADQUARTERS - - - - - - - - - - - - - - - Federal Building, Civic Center, San Francisco, Calif.

▼

DISTRICT OFFICES

DISTRICT No. 1. Oregon, Washington, and Montana.
Post Office Building, Portland, Oreg.

DISTRICT No. 2. California, Arizona, and Nevada.
Federal Building, Civic Center, San Francisco, Calif.

DISTRICT No. 3. Colorado, New Mexico, and Wyoming.
254 New Customhouse, Denver, Colo.

DISTRICT No. 4. Minnesota, North Dakota, South Dakota, and Wisconsin.
907 Post Office Building, St. Paul, Minn.

DISTRICT No. 5. Iowa, Kansas, Missouri, and Nebraska.
711 U. S. Courthouse, Kansas City, Mo

DISTRICT No. 6. Arkansas, Louisiana, Oklahoma, and Texas.
Room 502, United States Courthouse, Fort Worth, Tex.

DISTRICT No. 7. Illinois, Indiana, Kentucky, and Michigan.
South Chicago Post Office Building, Chicago, Ill.

DISTRICT No. 8. Alabama, Georgia, Florida, Mississippi, and Tennessee.
Post Office Building, Montgomery, Ala.

DISTRICT No. 9. Connecticut, Maine, Massachusetts, New Hampshire, New Jersey, New York, Rhode Island, and Vermont.
505 Post Office Building, Albany, N. Y.

DISTRICT No. 10. Delaware, Maryland, Ohio, Pennsylvania, and District of Columbia.
Willard Building, Washington, D. C.

DISTRICT No. 11. Alaska.
Room 419, Federal and Territorial Building, Juneau, Alaska.

DISTRICT No. 12. Idaho and Utah.
Federal Building, Ogden, Utah.

DISTRICT No. 14. North Carolina, South Carolina, Virginia, and West Virginia.
Montgomery Building, Spartanburg, S. C.

Because of the necessarily limited edition of this publication it is impossible to distribute it free to any person or institution other than State and county officials actually engaged in planning or constructing public highways, instructors in highway engineering, and periodicals upon an exchange basis. At the present time additions to the free mailing list can be made only as vacancies occur. Those desiring to obtain PUBLIC ROADS can do so by sending $1 per year (foreign subscription $1.50), or 10 cents per single copy, to the Superintendent of Documents, United States Government Printing Office, Washington, D. C.

CERTIFICATE: By direction of the Commissioner of Public Roads, the matter contained herein is published as administrative information and is required for the proper transaction of the public business.

PROGRESS IN STUDY OF MOTOR-VEHICLE PASSING PRACTICES[1]

BY THE DIVISION OF HIGHWAY TRANSPORT, PUBLIC ROADS ADMINISTRATION

Reported by **O. K. NORMANN**, Associate Highway Economist

KNOWLEDGE of the manner in which highways are used is a prerequisite to improving their design so that they will more adequately serve highway users. A study of passing practices of motor vehicles is part of a research program recently initiated by the Public Roads Administration to supply information on the normal driving habits of vehicle operators.

During the fall of 1938 studies of the passing practices of motor vehicles were conducted on four sections of highway in Maryland and Virginia with special equipment developed by the Public Roads Administration. A report describing in detail the methods and equipment used, and the purposes of the passing-practice studies, has been published.[2]

In cooperation with State highway officials, studies were conducted during the summer of 1939 in Massachusetts, Ohio, and Illinois, and studies are now being conducted in Texas. The program also includes studies in California and Oregon next spring. Upon completion of the field work, data will be available for normal passing practices under a wide variety of road conditions, geographically distributed to include any major differences in driving habits.

There has been so much interest manifested wherever the equipment has been in operation that, in addition to supplying most of the personnel for the field work, different State officials have felt that the information obtained will be of such immediate value to them that they have desired to supply the personnel necessary for a complete analysis of the data.

Several improvements have been made in the equipment to reduce the time required to install it on the study sections and to permit operation at night and on rainy days. The most important improvement has reduced the amount of work required in transcribing the field records. This is the major item of expense in the studies and has been reduced to less than two-thirds of its former cost.

The detailed data for 1,635 passing maneuvers recorded during 37½ hours of operation on the four study sections in Maryland and Virginia are now ready to be placed on tabulating machine cards prior to starting the comprehensive analysis. Although these 1,635 passings are but a small portion of the total number that will be obtained during this study, they will be used to illustrate the method of analysis and some of the facts with respect to passing practices and driver behavior that are being obtained.

OVER HALF OF PASSINGS ACCOMPLISHED BY MULTIPLE MANEUVERS

The first classification of the passing maneuvers was made by separating them into the single- and multiple-passing types. In the single-passing maneuvers, one vehicle passed one other vehicle, while in the multiple-passing maneuvers, two or more vehicles either passed or were passed by one or more vehicles.

Table 1 shows that 57.3 percent of the passings were accomplished by multiple maneuvers although there were only about half as many multiple maneuvers as there were single maneuvers (one vehicle passing two other vehicles accounts for two passings). These figures illustrate the importance of including in a study of passing distances and practices a study of multiple-passing maneuvers as well as single-passing maneuvers.

TABLE 1.—*Types of passing maneuvers observed (average traffic volume 375 vehicles per hour)*

Type of maneuver	Maneuvers made		Passings accomplished	
	Number	*Percent*	*Number*	*Percent*
Single	1,096	67.0	1,096	42.7
Multiple:				
1 vehicle passing 2 vehicles	181	11.1	362	14.1
2 vehicles passing 1 vehicle	161	9.8	322	12.6
1 vehicle passing 3 vehicles	63	3.9	189	7.4
2 vehicles passing 2 vehicles	42	2.6	168	6.5
3 vehicles passing 1 vehicle	30	1.8	90	3.5
1 vehicle passing 4 to 6 vehicles	31	1.9	136	5.3
2 vehicles passing 3 to 5 vehicles	13	.8	102	4.0
All other multiple passings	18	1.1	99	3.9
Total multiple	539	33.0	1,468	57.3
Grand total	1,635	100.0	2,564	100.0

The most important multiple-passing maneuvers are those in which one vehicle either passes or is passed by two vehicles. They compose 63.5 percent of the multiple maneuvers and 46.6 percent of the passings accomplished by multiple maneuvers. Three vehicles passing four other vehicles was the most complicated multiple-passing maneuver recorded.

Figure 1 shows, for various hourly volumes, the percentage of the total number of maneuvers and passings accomplished by multiple-passing maneuvers. At an hourly traffic volume of 200 vehicles, 35 percent of the total passings were accomplished by multiple maneuvers. At traffic volumes above 450 vehicles per hour, this figure exceeds 60 percent.

Figure 2 shows the average number of maneuvers and passings observed per hour on the four half-mile study sections during various hourly traffic volumes. As expected, there is a marked increase in the number of passings as the volume increases. These are the number of passings accomplished per hour and not the number of passings that would have been made had all vehicle operators that desired to pass been able to make passing maneuvers.

One vehicle usually passes another vehicle because the driver wants to travel faster than the other vehicle is moving. Within the half-mile study section it was generally possible to determine the speed that the driver of the passing vehicle desired to travel by noting his speed either before slowing down prior to making the passing maneuver or after the maneuver was completed.

Table 2 shows that in 55 percent of the passings the passed vehicle was traveling from 31 to 40 miles per hour. The speeds of the passed vehicles in nearly all of the remaining passings were almost equally distrib-

---

[1] Paper presented at the Nineteenth Annual Meeting of the Highway Research Board, December 6, 1939.

[2] Procedure Employed in Analyzing Passing Practices of Motor Vehicles, by E. H. Holmes, PUBLIC ROADS, vol. 19, No. 11, January 1939.

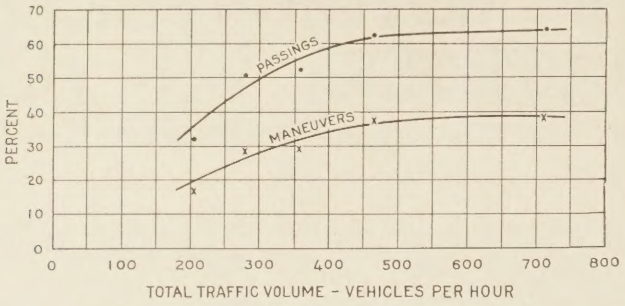

Figure 1.—Percentages of Total Passing Maneuvers and Total Passings That Were Accomplished by Multiple-Passing Maneuvers at Various Traffic Volumes.

uted between the 21 through 30 and 41 through 50 mile-per-hour groups.

This table also shows that 51.4 percent of the drivers that passed desired to travel less than 11 miles per hour faster than the passed vehicle and that the desired speed of 21.2 percent was less than 6 miles per hour faster. There is a marked decrease in the average difference between the speed of the passed vehicle and the desired speed of the passing vehicle as the speed of the passed vehicle increases. Beside showing the frequency distribution of the speeds of passed vehicles, these data indicate that drivers desiring to travel at a slightly higher speed than the vehicle ahead would rather pass the preceding vehicle when the opportunity presents itself than reduce their speed slightly and stay behind.

Table 2.—*Single passings classified by the speed of the passed vehicle and the desired speed of the passing vehicle*

Desired speed of passing vehicle in miles per hour faster than speed of passed vehicle	Speed of passed vehicle in miles per hour					
	20 and under	21–30	31–40	41–50	Over 50	Total
	Percent	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
5 and under		1.9	11.2	7.8	0.3	21.2
6–10		4.0	18.8	7.1	.3	30.2
11–15	0.4	6.7	17.6	5.5	.3	30.5
16–20	.7	5.0	5.7	.8		12.2
21–30	.3	2.9	1.6	.3	.1	5.2
Over 30	.3	.2	.1	.1		.7
Total	1.7	20.7	55.0	21.6	1.0	100.0
Average difference in speed between passed and passing vehicle (miles per hour)						
	20.6	14.2	10.5	8.6	11.1	10.9

Of all the drivers that were able to accomplish single passing maneuvers on the study sections, table 3 shows that 84.4 percent had to slow down before they could start to pass; 53.7 percent slowed down to practically the same speed as the vehicle they were going to pass, and 16 percent slowed down to within 5 miles per hour of the speed of the vehicle they were going to pass. About one-third had to slow down and stay behind the preceding vehicle until they could see that the road was clear for a sufficient distance ahead to permit them to pass, and 50.9 percent had to slow down and wait for an oncoming vehicle to pass before they could start the passing maneuver. The other 15.6 percent were not required to slow down prior to starting the maneuver. They may have had to slow down after completing the maneuver but they started the maneuver at their normal speed.

When the drivers of the passing vehicles had completed the passing maneuvers and returned to the right-hand lane, table 4 shows that the left lane was clear for a distance of less than 500 feet in 27 percent of the passings and that there was an oncoming vehicle less than 500 feet away in 16.8 percent of the passings. The data for the passings in which the passing vehicle was not forced back into the right lane by oncoming traffic or limited sight distance may not be very useful in determining minimum passing distances, but they will show actual driving practices during unrestricted conditions. Driving practices during unrestricted as well as restricted conditions must be known when designing highways to fit the normal driving habits of vehicle operators.

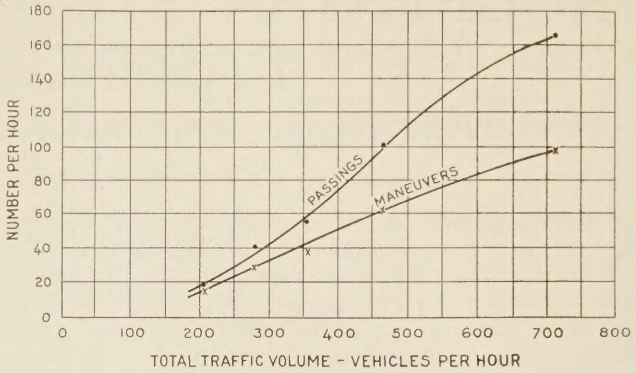

Figure 2.—Total Number of Maneuvers and Passings Accomplished per Hour on ½-Mile Study Sections at Various Hourly Traffic Volumes.

Table 3.—*Percentage of vehicles making single-passing maneuvers that were delayed before starting to pass (average traffic volume 375 vehicles per hour)*

	Delayed by insufficient sight distance	Delayed by oncoming vehicle	Total
	Percent	*Percent*	*Percent*
Slowed down to same speed as vehicle to be passed	18.0	35.7	53.7
Slowed down to within 5 miles per hour of the speed of the vehicle to be passed	6.6	9.4	16.0
Slowed down, but not to within 5 miles per hour of speed of vehicle to be passed	8.9	5.8	14.7
Total delayed in starting maneuver	33.5	50.9	84.4
Total not delayed in starting maneuver			15.6
			100.0

Table 4.—*Distance that drivers of passing vehicles could see that left lane was clear at the time the passing maneuver was completed (average traffic volume 375 vehicles per hour)*

Distance that left lane was clear (feet)	Sight distance limiting factor	Oncoming car in view	Total
	Percent	*Percent*	*Percent*
Less than 500	10.2	16.8	27.0
500 to 1,000	19.0	16.0	35.0
Over 1,000	30.5	7.5	38.0
Total	59.7	40.3	100.0

ILLUSTRATION OF DATA OBTAINED FOR PASSING MANEUVERS

The data obtained for each passing maneuver are illustrated by figures 3 to 9 inclusive, each representing one of seven critical positions. All speeds, distances, time intervals, and relative positions of each vehicle with respect to the other vehicles as shown in these figures were obtained from the data sheet for one passing maneuver.

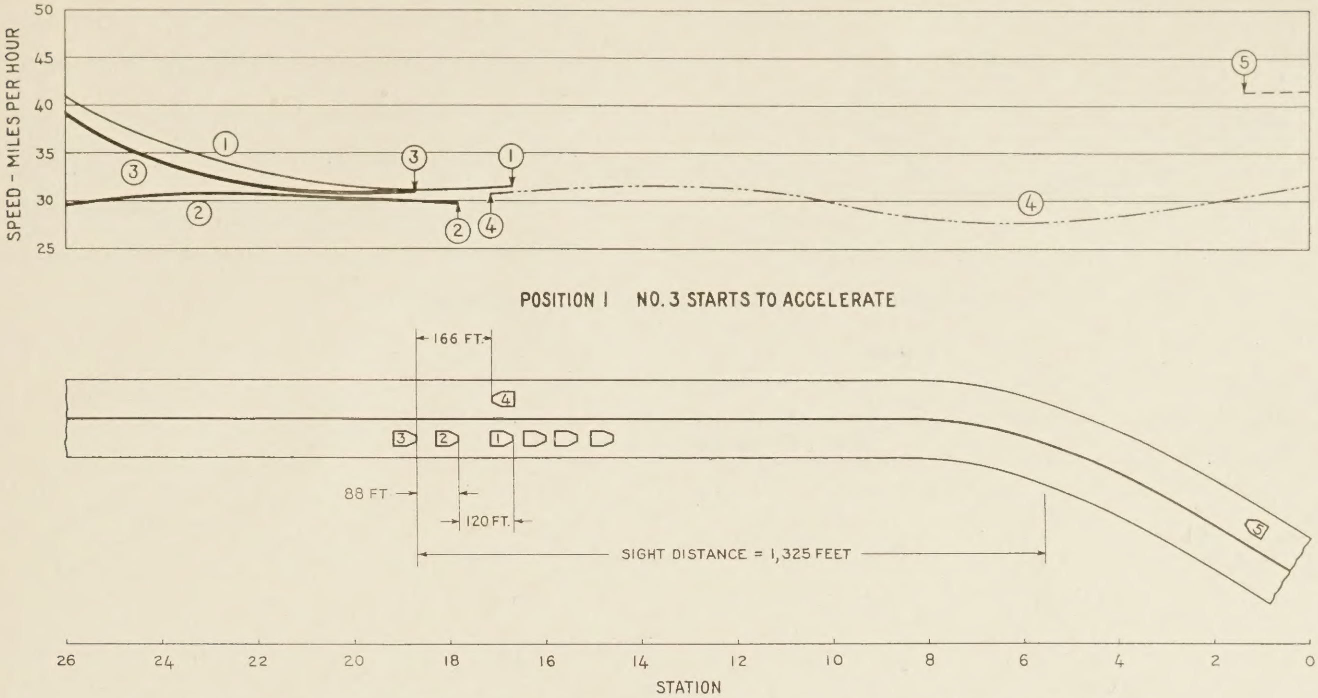

FIGURE 3.—CRITICAL POSITIONS AND SPEEDS OF VEHICLES AT START OF PASSING MANEUVER.

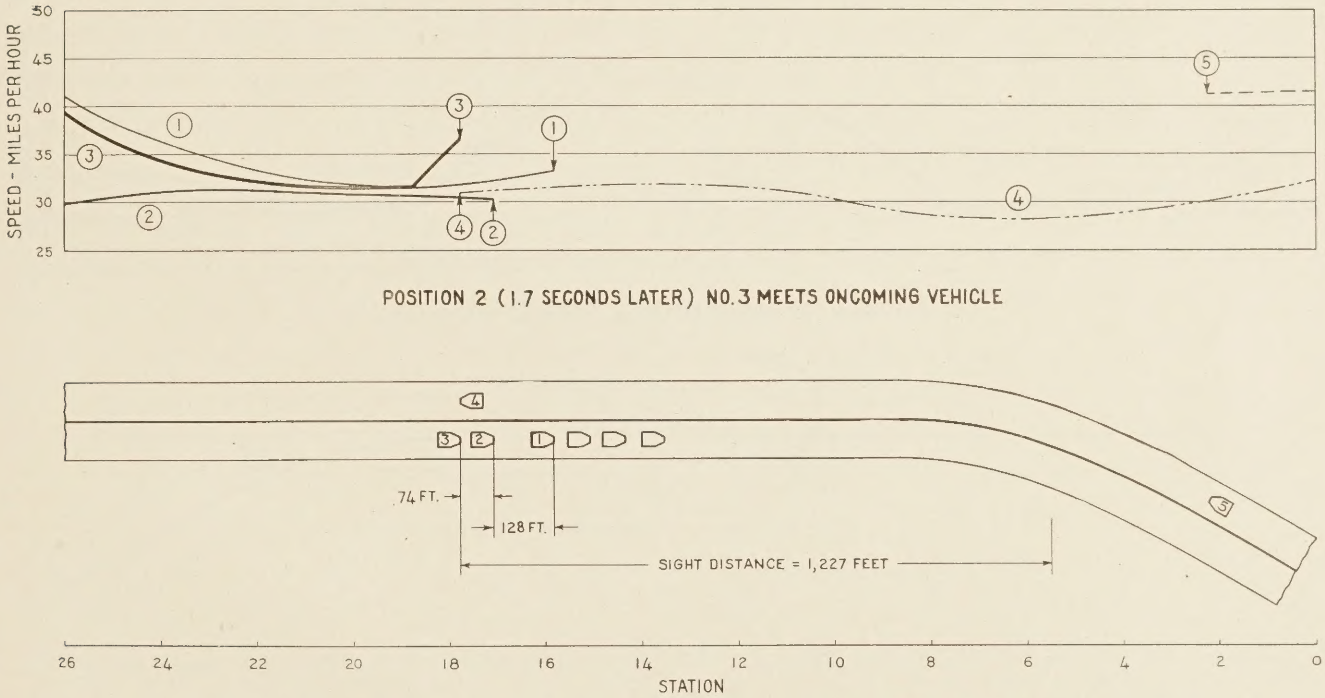

FIGURE 4.—CRITICAL POSITIONS AND SPEEDS OF VEHICLES AT SECOND STAGE OF PASSING MANEUVER.

Similar data have been compiled for the 1,635 maneuvers recorded on the study sections in Maryland and Virginia and also for 500 of the maneuvers recorded during the Massachusetts studies. It is not intended that this much detail be obtained for all the thousands of passings that will have been recorded when the scheduled field work is completed, but the factors that appear to be the most important as the analysis progresses will be taken from the field records for enough maneuvers to obtain a representative sample of each type of maneuver at a series of speeds for each available road condition.

Figure 3 shows the position of each vehicle that is likely to affect the manner in which the passing maneuver is made. At this instant, vehicle No. 3 starts to accelerate in order to pass vehicle No. 2 and possibly the four vehicles ahead of vehicle No. 2, the closest one being vehicle No. 1, a distance of 120 feet ahead of No. 2. All dimensions between vehicles represent the distances between the front of vehicles. Vehicles Nos.

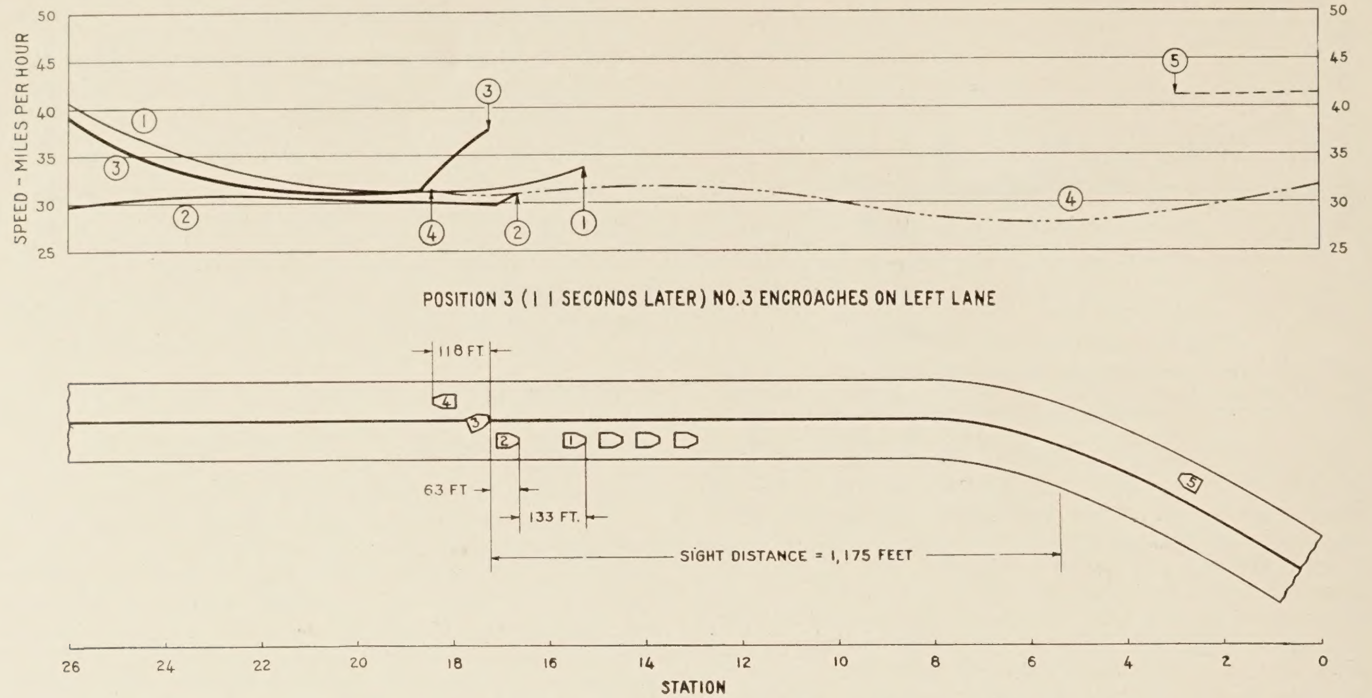

Figure 5.—Critical Positions and Speeds of Vehicles at Third Stage of Passing Maneuver.

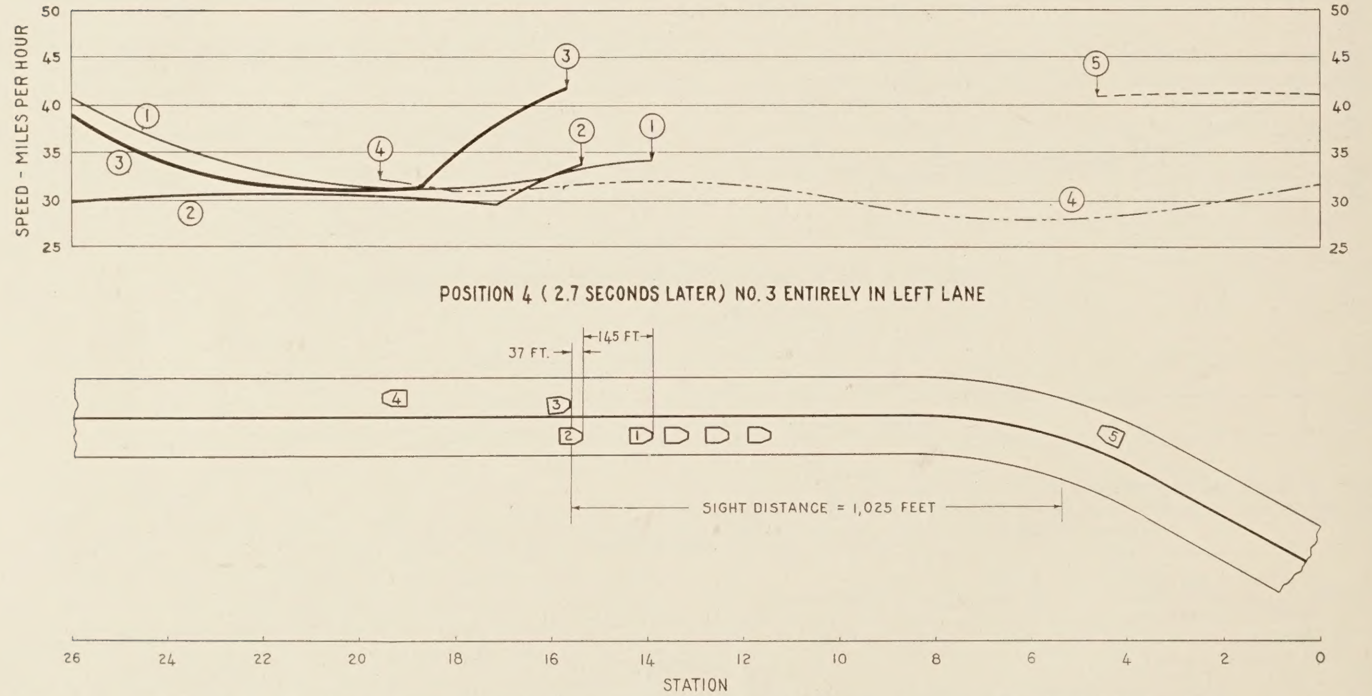

Figure 6.—Critical Positions and Speeds of Vehicles at Fourth Stage of Passing Maneuver.

4 and 5 are oncoming vehicles in the opposing lane of traffic, No. 4 being the vehicle met by No. 3 before encroaching on the left lane, and No. 5 being the first oncoming vehicle met by No. 3 after completing the maneuver. The space between No. 4 and No. 5 represents the "hole" available in the opposing lane of traffic.

At the top of figure 3 is shown the speed of each of the five vehicles over the portion of the study section traversed up to this point. Vehicle No. 3 entered the section traveling about 40 miles per hour but has been required to slow down to 31 miles per hour, the approximate speed that No. 2 has maintained. (Table 3 indicated that 53.7 percent of the observed passing maneuvers were started after the passing vehicle had slowed down to the same speed as the vehicle to be passed.)

The data sheets for another passing maneuver indicate that vehicle No. 1 has just finished passing No. 2 and has also slowed down to a speed of about 31 miles per hour. Vehicle No. 4 is approaching at a speed of 31 miles per hour and No. 5 at a speed of 42 miles per

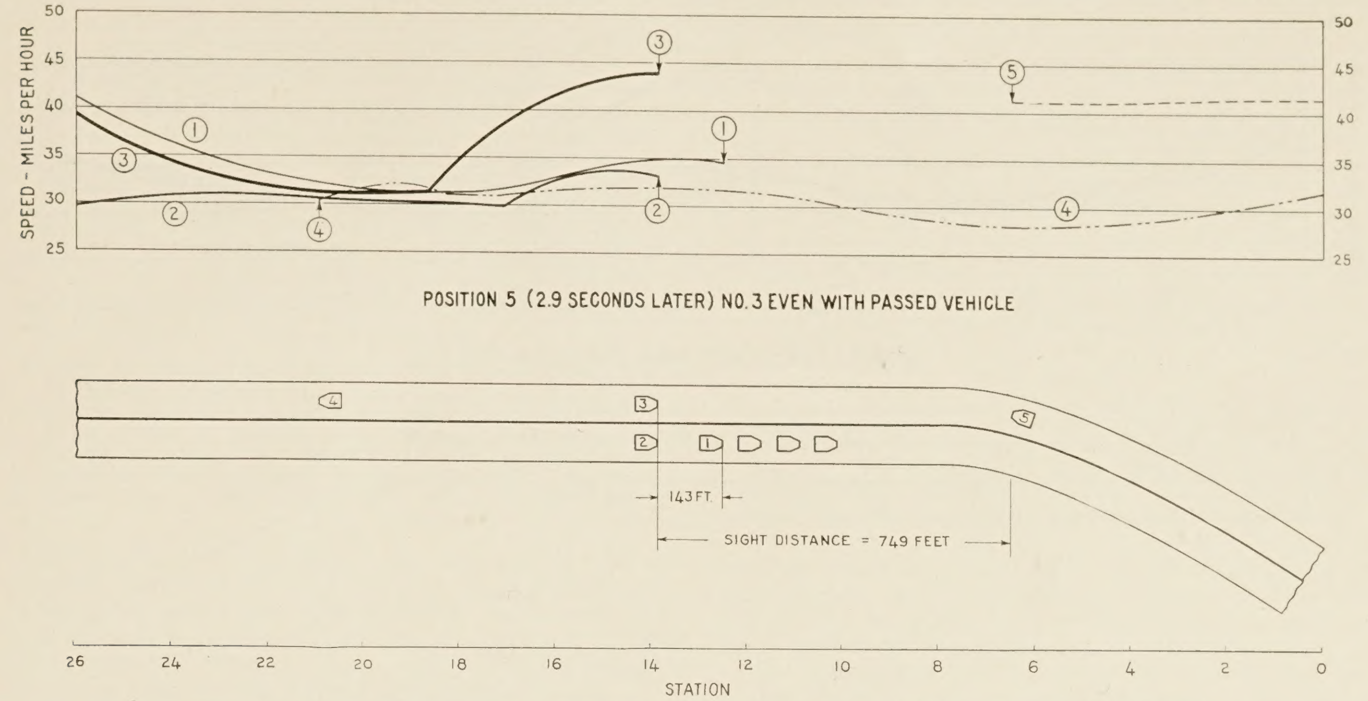

FIGURE 7.—CRITICAL POSITIONS AND SPEEDS OF VEHICLES AT FIFTH STAGE OF PASSING MANEUVER.

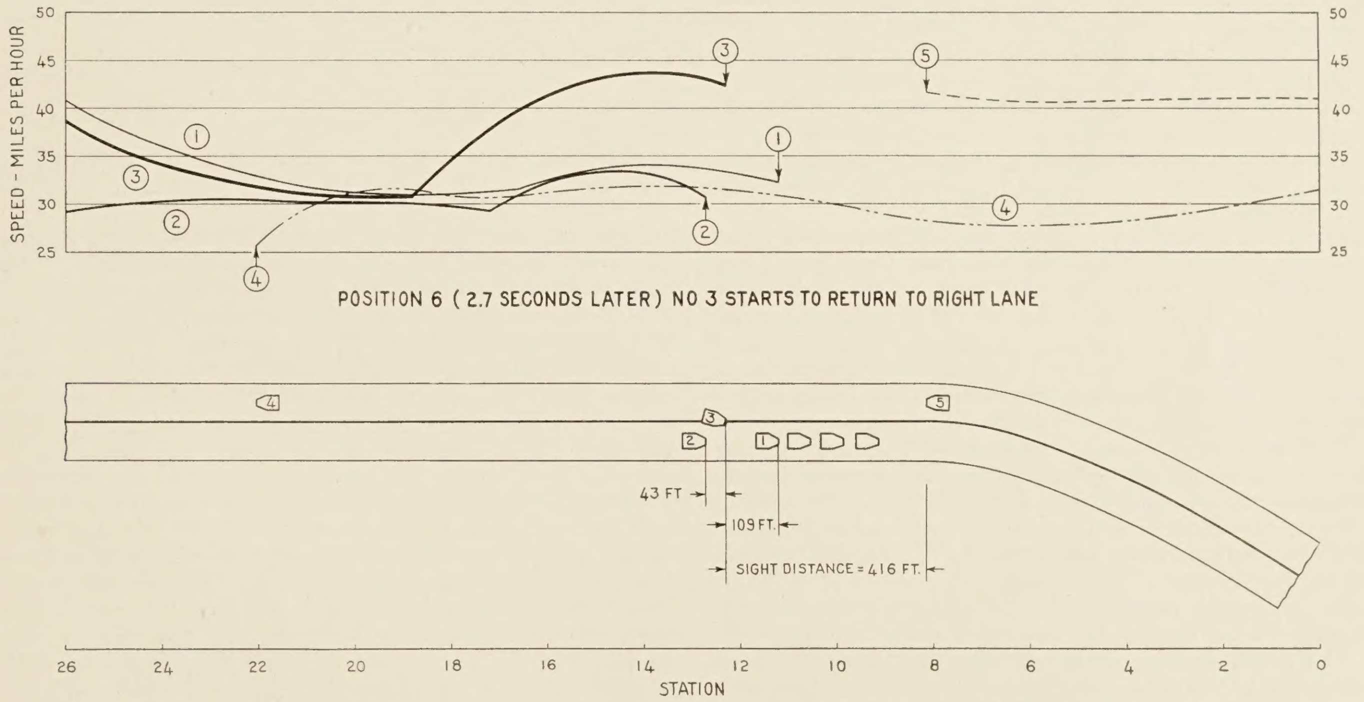

FIGURE 8.—CRITICAL POSITIONS AND SPEEDS OF VEHICLES AT SIXTH STAGE OF PASSING MANEUVER.

hour, but No. 5 cannot be seen by vehicle No. 3. The fact that vehicle No. 3 starts to accelerate at this point indicates that the driver has already decided to attempt to pass even though No. 4 is still 166 feet away.

In the second position, occurring 1.7 seconds later and shown by figure 4, the fronts of No. 3 and No. 4 are parallel; No. 3 has accelerated to 36 miles per hour and is now 74 feet behind vehicle No. 2. It is immediately after this instant that the driver of vehicle No. 3 has his first opportunity to enter the left lane without hindrance from oncoming traffic (vehicle No. 5 still being out of sight).

In the third position, occurring 1.1 seconds later and shown by figure 5, vehicle No. 3 first encroaches on the left lane while 63 feet behind vehicle No. 2 and traveling 7 miles per hour faster than the vehicle to be passed.

In the fourth position, taking place 2.7 seconds later (fig. 6), No. 3 is entirely in the left lane for the first time and is still 37 feet behind vehicle No. 2. In the meantime vehicle No. 2, which has been traveling at a uniform speed throughout the first 900 feet of the section, starts to accelerate. The driver evidently does not like the idea of being passed or unintentionally steps on the accelerator. He cannot accelerate very

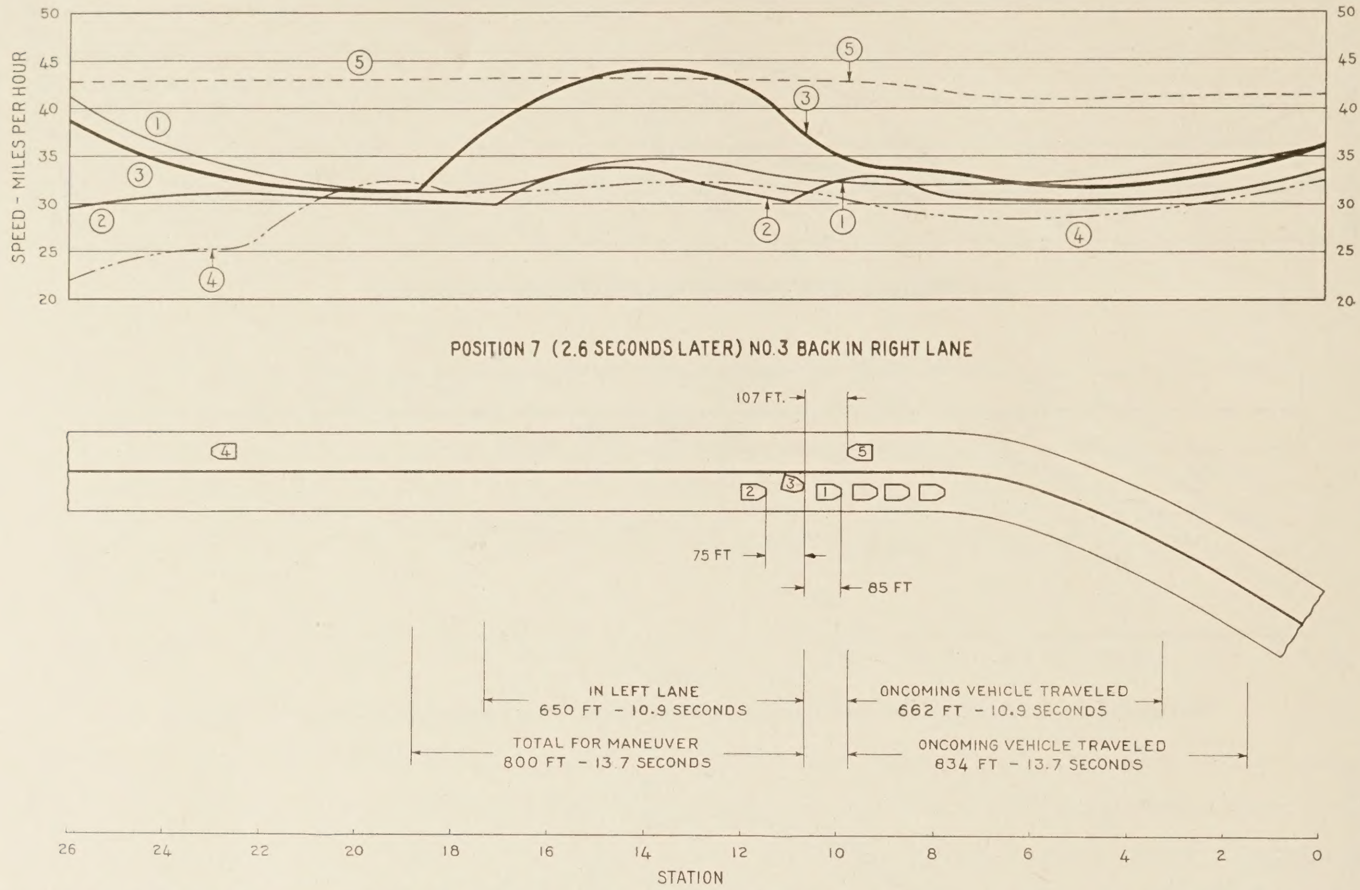

Figure 9.—Critical Positions of Vehicles After Completion of Passing Maneuver and Speeds of Vehicles Through Entire Study Section.

long without hitting No. 1 but he can reduce the space between his car and vehicle No. 1 so that No. 3 will be required to crowd his way back to the right lane. Neither No. 2 nor No. 3 can as yet see the oncoming car No. 5.

Position 5 (fig. 7) occurs 2.9 seconds after position 4, and the passing vehicle is now even with No. 2 and is no longer accelerating. Vehicle No. 2 has decelerated a little and the oncoming vehicle No. 5 is now in view.

From figure 8, representing the sixth position of the maneuver, it is seen that the driver of the passing vehicle has decided not to try to pass more than one vehicle and is now cutting back into the "hole" between No. 1 and No. 2. This occurs 2.7 seconds after No. 2 and No. 3 were parallel. Vehicle No. 3 is traveling 10 miles per hour faster than vehicle No. 1.

Figure 9, the seventh position, 2.6 seconds later, shows that the passing vehicle completed the maneuver when 107 feet from the oncoming car. After returning to the right lane, the speed of No. 3 decreased until it was below the speed of vehicle No. 1 and then increased to the same speed. All vehicles slowed down slightly going around the curve.

The acceleration and deceleration curve for the passing vehicle during the maneuver, as shown in figure 9, indicates a maximum acceleration of 2.3 miles per hour per second. This is considerably lower than any rate that would be assumed in calculating the minimum passing distance under similar conditions.

It required 13.7 seconds or 800 feet to complete the maneuver. The passing vehicle spent 10.9 seconds and traveled 650 feet in the left lane. The approaching vehicle traveled 834 feet during the maneuver and 662 feet while the passing vehicle was in the left lane. The net result of the passing maneuver is that vehicles No. 2 and No. 3 have reversed their respective positions. Providing there are sufficient passing sections and "holes" in oncoming traffic along the remaining portion of this highway, vehicle No. 3 might be able to pass two or three more of the cars of this group before getting to the next town, thereby arriving a little sooner than if no attempt were made to pass. It may be possible for No. 3 to pass all the preceding vehicles in this group at the next section of highway with a long sight distance, providing oncoming traffic does not interfere. Vehicle No. 3 could then continue at the desired speed until catching up with the next group of cars and might reach its destination somewhat sooner than by staying in line. It is evident that this section of highway with a sight distance of 1,900 feet provided practically no relief from restricted travel conditions for this particular vehicle.

MOST PASSINGS STARTED WITH SIGHT DISTANCE OF 1,000 FEET OR MORE

Such a detailed analysis of each passing necessarily takes more time than it would to obtain only such factors as the average vehicle speeds and the total passing distances, but the relative value of the results will more than compensate for the increased work.

There are so many variable factors involved in each passing that it would require almost an unlimited number of passings to obtain a representative sample of each type of passing maneuver possible by different

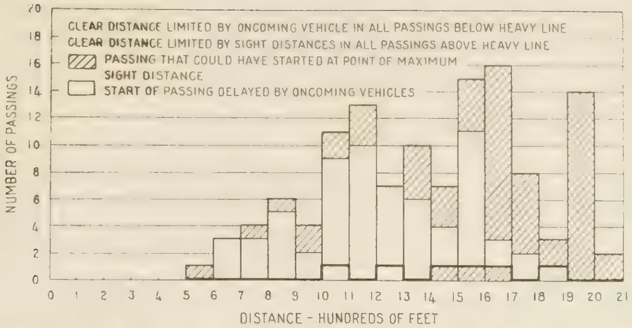

Figure 10.—Maximum Distance That Drivers of Passing Vehicles Could See That Left Lane was Free of Oncoming Traffic at the Time They Entered the Left Lane During Passing Maneuvers. (Including Only Those Vehicles That Slowed Down to Same Speed as Vehicles To Be Passed Prior to Starting the Passing Maneuvers on Study Section.)

combinations of variables. As the same variable appears in many different types of passing maneuvers, a break-down of each passing into its component parts will permit a representative sample of each variable to be obtained from a much smaller total number of passings than if the break-down is not made. The variables may then be recombined to form composite passings covering all types. It is in this respect that the method of analysis, made possible by the complete field records, will differ from other analyses of passing distances and practices that have been made.

The determination of distances involved in a passing maneuver is a relatively simple, though laborious, operation, but the determination of the effect of highway alinement and of driver psychology upon future design requirements is far more difficult. Even though sections of sufficient sight distance to complete individual passing maneuvers in safety are provided, they will not serve their purpose unless the drivers take advantage of their opportunities to pass. Figures 10 and 11 have been constructed from the data for single passings that took place at one particular study location, where vehicles traveling in either direction had maximum and minimum sight distances of 1,900 and 200 feet, respectively. To eliminate a number of variables, only the passings in which the passing vehicle started the maneuver while traveling at the same speed as the vehicle to be passed have been used.

In figure 10, the passings have been classified by the maximum distance that the driver of the passing vehicle could see that the oncoming traffic lane was clear at the time he encroached on it. Three of the drivers encroached on the left lane when this distance was between 1,800 and 1,900 feet; eight encroached when this distance was between 1,700 and 1,800 feet, etc. In 85.5 percent of the passings, the distance was over 1,000 feet and no maneuvers were started when this distance was less than 500 feet.

Sixteen vehicles encroached on the left lane before reaching the point of maximum sight distance. Fourteen were within 100 feet of this point and two were between 100 and 200 feet away. As the measured sight distance decreased to 250 feet immediately before reaching the point of maximum sight distance, the drivers of the 16 vehicles either could see farther than the measured sight distance because of unusual vehicle construction, or the drivers started the passing maneuvers knowing or hoping that the sight distance would increase while they still had a chance to return to the right lane in case an oncoming vehicle came into view. All 16 vehicles reached the point of maximum sight distance before getting entirely in the left lane.

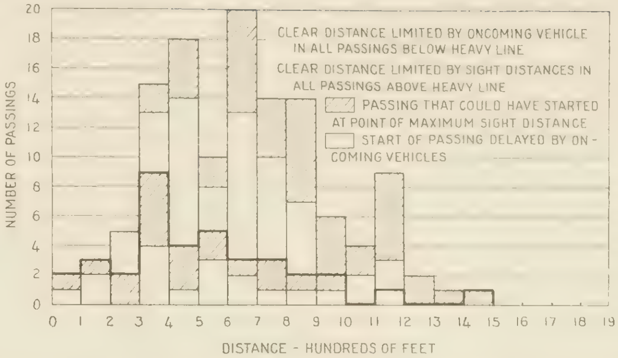

Figure 11.—Maximum Distance That Drivers of Passing Vehicles Could See That Left Lane Was Free of Oncoming Traffic at Time Passing Maneuvers Were Completed. (Including Only Those Vehicles That Slowed Down to Same Speed as Vehicles To Be Passed Prior To Starting the Passing Maneuvers on Study Section.)

The passings in each of these distance groups have been divided by a heavy horizontal line into two groups. The sight distance limited the distance that the driver could see a clear left lane in all the passings above the heavy horizontal line. There was an oncoming vehicle in sight when the driver encroached on the left lane in all the passings shown below the heavy line. In only six of the passings did the driver encroach on the left lane when there was an oncoming vehicle in view and not a passing was started when the driver could see an oncoming vehicle within 1,000 feet. With no oncoming vehicle in view, passings were started when the sight distance was as low as 500 feet but relatively few were started below 1,000 feet, there being a marked drop at this point.

OPPORTUNITIES TO PASS NOT UTILIZED TO MAXIMUM EXTENT

The passings are further classified into two groups as determined by whether or not the driver had to wait for an oncoming vehicle before starting the maneuver. All of the passings that are cross-hatched represent maneuvers in which the passing vehicle was immediately behind the vehicle to be passed upon reaching the point of maximum sight distance and could have started the passing maneuver immediately. The areas that are not cross-hatched represent passings that could not have been started at the point of maximum sight distance, because the driver had to wait for oncoming traffic to pass before encroaching on the left lane. Sixty-one (49.2 percent) of the 124 passings that were made could have been started immediately upon reaching the point of maximum sight distance. In the other 50.8 percent, the passing vehicle waited for an oncoming car after reaching the point of maximum sight distance before starting to encroach.

This one figure in itself contains an enormous quantity of information regarding driver behavior. The number of drivers that did not attempt to pass until after reaching a point where the sight distance was considerably less than it was where they could have started to pass is surprisingly large and offers excellent data for a study of combined perception and judgment time.

(Continued on page 237)

SAMPLE OF DATA OBTAINED IN STUDY OF MOTOR-VEHICLE PASSING PRACTICES IN ILLINOIS

By F. N. BARKER, Engineer of Highway Research, Illinois Division of Highways

A COMMON and one of the most annoying conditions that tend to reduce the traffic capacities of two-lane rural highways is the presence on these roads of vehicles moving at slow speeds. Where sight distances are restricted, where oncoming traffic is heavy, or where other conditions are such that passing maneuvers cannot be executed safely, these slow-moving vehicles restrict vehicle drivers who wish to travel at greater speeds. Under such conditions passenger vehicles move slowly because they are compelled to, not because of any mechanical limitation of the vehicle itself. Motortrucks, however, are often loaded to a weight in excess of that which the engine is able to move at a reasonable road speed, especially on the steeper grades.

During September and October 1939, the Illinois Division of Highways cooperated with the Public Roads Administration in conducting studies of truck performance on medium and heavy grades, using equipment developed by the Public Roads Administration and described in the January 1939 issue of PUBLIC ROADS. These studies were made on typical two-lane roads carrying normal traffic volumes and were conducted so that the presence of the observers and equipment had no effect on the performances of the vehicles. Detectors spaced at 50-foot intervals over the test section were the only part of the equipment visible to the drivers. These detectors were small rubber tubes, connected through pneumatic switches and electrical circuits to recording instruments. Vehicles passing through the course actuated successive pens on the instruments, which recorded the time of passage of each vehicle through successive segments of the course. Travel on each of the two lanes was recorded on separate charts, so that a vehicle straddling the center line was recorded on both charts. The license numbers of commercial vehicles were recorded as they entered the course and these vehicles were stopped and weighed a considerable distance beyond the section.

Analysis of the data collected in the Illinois study is yet in a preliminary stage. Until this analysis is completed for all of the data collected in different sections of the State under various conditions of traffic and road alinement so as to provide a complete cross section of vehicle behavior on grades, and until these results are compared with the results of similar studies made in other States, no definite conclusions can be drawn or recommendations made. It is already apparent, however, that studies of this nature will ultimately provide a wealth of reliable data which will be useful in establishing design standards and in determining reasonable and desirable standards of motor-vehicle performance.

As a demonstration of the type of basic data obtained in truck performance studies, figure 1 has been developed. This is a chart of the passage of a typical group of vehicles through a course located on U. S. Route 66 near Edwardsville, Ill., between 3 : 09 : 45 p. m. and 3 : 14 : 55 p. m. (a period of 310 seconds) on October 4, 1939. This location was on a 20-foot pavement consisting of an 18-foot brick surface, with 1-foot concrete edges and a light gutter on each side. The rate of grade was 6 percent within the limits of the course. Conditions were ideal on the day of the observations—pavement dry, weather clear, and visibility excellent.

PROGRESS OF EACH VEHICLE THROUGH TEST SECTION CHARTED

For convenience, vehicles ascending the grade are numbered on figure 1 in the order of their entrance into the section. Opposing vehicles are not numbered because there were few of them and their paths through the section are easily followed. The vertical scale shows the distance the vehicle has traveled from the bottom of the grade at the times shown on the horizontal scale.

Information readily taken from the chart indicates:

1. The position of each vehicle at any time with respect to any other vehicle on the section at the same time, regardless of direction of travel.
2. The time and point on the course at which each vehicle first encroached on the opposing traffic lane, the duration of this encroachment in time and distance, and the time and point at which the vehicle returned to the proper lane.
3. The speed of each vehicle through each 50-foot section of the course (indicated by the slope of the line representing the progress of the vehicle).
4. The acceleration or deceleration of each vehicle through any portion of the course (indicated by the rate of change of the slope of the line representing the vehicle's progress).

Vehicle No. 1 was a tractor-truck semitrailer combination with a manufacturer's rated capacity of 1½ tons and a gross weight of 16,500 pounds. This combination entered the course at 20 miles an hour, but slowed to about 5½ miles an hour at 450 feet, or after 25 seconds of travel on the course. This speed gradually decreased to 4.9 miles an hour at which speed the vehicle left the test section.

Vehicle No. 2, which was a passenger car following this combination as it entered the course, crossed into the opposing traffic lane after traveling 250 feet on the course, with the apparent intention of passing vehicle No. 1. An oncoming vehicle prevented this maneuver so that vehicle No. 2 was forced to return to its proper lane. The driver of the oncoming vehicle decreased his speed appreciably until he was certain that the dirver of vehicle No. 2 had abandoned his attempt to pass, after which he accelerated and left the course at 30 miles an hour. After meeting the descending vehicle, the driver of vehicle No. 2 accelerated to a speed of 25 miles an hour, passed vehicle No. 1, and left the course at a speed of 35 miles an hour. During this passing maneuver vehicle No. 2 encroached on the opposing traffic lane for a distance of 205 feet.

Vehicle No. 3 was a tractor-truck semitrailer combination, with a manufacturer's rated capacity of 2 tons

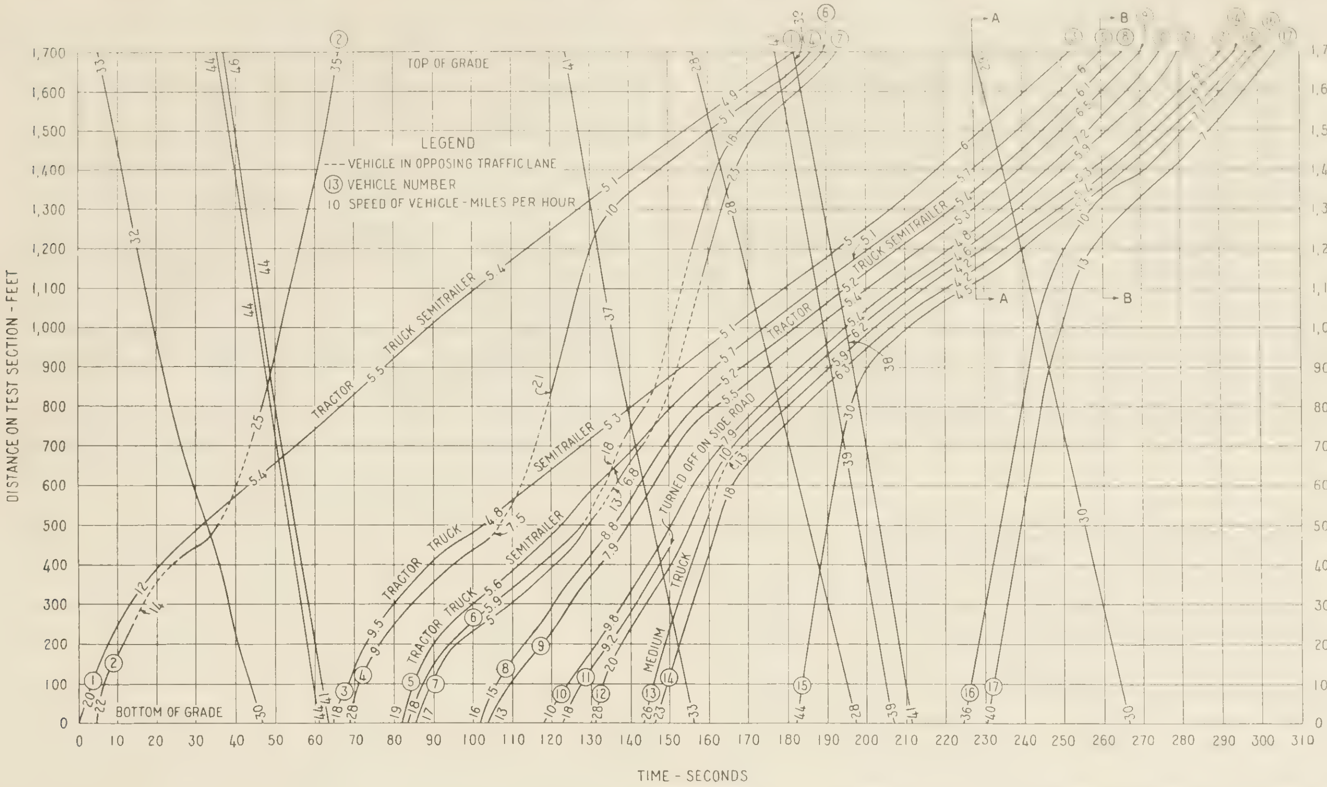

Figure 1.—Time-Distance Curves Showing Progress of Vehicles Through Test Section. Curves Sloping to Right Indicate Vehicles Ascending the Hill; Curves Sloping to Left Indicate Descending Vehicles. Ordinate Between Any Two Curves Indicates Distance in Feet Between Front Axles of Respective Vehicles. Vehicles Nos. 1, 3, 5, and 8 Are Tractor Truck Semitrailer Combinations; Vehicle No. 13 Is a Medium Truck; All Other Vehicles Are Passenger Cars.

and a gross weight of 17,000 pounds. This vehicle entered the course at 18 miles an hour, but after traveling about 450 feet its speed had decreased to 5 miles an hour, which was approximately maintained throughout the remainder of the course.

Vehicle No. 4, a passenger car, passed this combination after following it for about 475 feet on the course but was prevented by oncoming vehicles from passing vehicle No. 1. Vehicle No. 4 was forced, therefore, to follow vehicle No. 1 out of the course at a speed of about 5 miles an hour.

Vehicles Nos. 6 and 7 were successful in passing vehicles Nos. 5 and 3, vehicle No. 7 accomplishing this on the second attempt, but both were prevented by oncoming traffic from passing vehicles Nos. 1 and 4. It is apparent from the varying slope of the charted course of vehicle No. 6 that the driver considered abandoning his attempt to pass, but accelerated and completed the maneuver. It is interesting to note the small clearance between vehicle No. 6 and the oncoming vehicle as the former passed vehicle No. 5. The vertical distance between the upper end of the dotted section of his path as he passed vehicle No. 5 and the position of the oncoming vehicle at the same time indicates that vehicle No. 6 returned to its own lane with only about 100 feet of clearance from the oncoming vehicle.

The remaining ascending vehicles numbered 8 to 17, which entered the section during the period of observation covered by the chart, were all prevented by oncoming traffic from passing any of the vehicles ahead of them and were forced to follow in line behind vehicle No. 3 at its speed of about 5 miles an hour.

Vehicles Nos. 15 and 16 are the only ones recorded on this diagram entering the course at speeds which were apparently unrestricted by preceding vehicles or heavy loads. For vehicle No. 15 this speed was 44 miles an hour and for vehicle No. 16 the speed was 36 miles an hour. These are reasonable speeds for this location and it may be assumed that they are the speeds the drivers of these two vehicles desired to maintain throughout the course. As they approached the line of vehicles following vehicle No. 3, however, they were forced to decelerate to a speed approximating that of vehicle No. 3.

Pictures were taken of vehicles on the test section at points on the time axes designated "AA" and "BB" in figure 1. Figure 2 shows the vehicles at the instant designated "AA" and figure 3 the vehicles at "BB." These two photographs show a condition that is frequently encountered on sections having short sight distances, that of a group of vehicles which have accumulated behind a slow-moving vehicle and are prevented from passing by inadequate sight distances or oncoming traffic. In figure 2, vehicle No. 15 is the last car in line, while in figure 3 vehicle No. 17 is catching up with the group of vehicles ahead.

SEVERAL REMEDIES AVAILABLE TO RELIEVE SUCH CONGESTION

Two of the five commercial vehicles included in figure 1, vehicles Nos. 1 and 13, did not carry Illinois registration plates. Vehicle No. 1 was obviously loaded in excess of the weight that would allow a reasonable performance on this grade, but vehicle No. 13 carried a load well within its capacity. Examination of the charted path of this vehicle through the

Figure 2.—Vehicles Climbing Hill at Instant Designated "AA" in Figure 1.

course indicates that the driver desired to proceed up the grade at a higher speed than that which he was forced to maintain. At a point 525 feet from the start of the course vehicle No. 13 moved into the opposing traffic lane and accelerated in an attempt to pass the two vehicles ahead. Oncoming traffic prevented the passing maneuver and vehicle No. 13 was forced to remain in line.

Trucks are registered in Illinois according to graduated gross weight classifications. There is no restriction other than the higher fee which would tend to discourage registration of a vehicle of low weight carrying capacity in a high weight classification. Vehicles Nos. 3 and 5 carried consecutive Illinois license numbers, which would indicate they were probably registered by the same owner. Both carried loads within their legal classification, as they were licensed to operate in Illinois with a total gross weight of 24,000 pounds. Nevertheless, it is obvious that they were both overloaded insofar as their ability to perform satisfactorily on this grade was concerned. These two vehicles traveled several hundred feet on the section with a space between vehicles of less than 40 feet. For only a very short distance at the beginning of the section did they travel at the 300-foot spacing required by the Illinois Uniform Act Regulating Traffic on Highways.

Vehicle No. 8, although carrying a gross weight of 26,800 pounds, was also operating within its legal classification of 40,000 pounds gross weight.

From the data presented in figure 1 it may be concluded:

1. That the performances of the tractor-truck semitrailer combinations designated as vehicles Nos. 1 and 3 were not satisfactory under the loading and highway conditions under which they were operating. The same is true of vehicles Nos. 5 and 8, although this is not so strikingly apparent from an examination of the chart.
2. That the extremely slow speeds at which vehicles Nos. 1 and 3 ascended the grade resulted in inconvenience and delay to a number of vehicles which were forced to follow them through the test section.
3. That drivers of vehicles Nos. 5 and 8 violated the Illinois statute requiring that commercial vehicles traveling on rural highways maintain an interval between vehicles of not less than 300 feet.
4. That at least one passing maneuver, that when vehicle No. 6 passed No. 5, was executed under hazardous conditions.

Figure 3.—Vehicles Climbing Hill at Instant Designated "BB" in Figure 1.

5. That at least three of the drivers of passenger vehicles included in this group desired to travel through the course at a speed of 35 to 45 miles an hour.

There are a number of possible remedies which might be suggested to provide a less restricted movement of vehicles over this section of highway. A third lane might be constructed for the use of slow-moving vehicles; the grade might be reduced so that heavily loaded vehicles could maintain a more reasonable speed; or the loading of vehicles might be limited so that with their available power a satisfactory performance would be obtained. The same remedies might be applied to numerous locations throughout the State, but until analysis of all available data relating to the problem is completed, and these data are carefully considered by highway officials, no recommendations can be made.

PUBLICATIONS ON BRIDGE FLOOR DESIGN AVAILABLE

Three new publications that present the results of tests made to verify theoretical analyses of bridge floor designs are now available. These publications, all University of Illinois Bulletins, are No. 313, "Tests of Plaster-Model Slabs Subjected to Concentrated Loads;" No. 314, "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads;" and No. 315, "Moments in Simple Span Bridge Slabs with Stiffened Edges." The bulletins are the result of a cooperative investigation by the Engineering Experiment Station of the University of Illinois, the Public Roads Administration of the Federal Works Agency, and the Illinois Division of Highways.

The results of this investigation will have direct application to practically all modern highway bridges and to many other structural design problems and will lead to more satisfactory structures.

The Public Roads Administration has a limited number of these bulletins for free distribution. Requests should be addressed to the Public Roads Administration, Federal Works Agency, Washington, D. C.

STRESSES UNDER A LOADED CIRCULAR AREA[1]

BY THE DIVISION OF TESTS, PUBLIC ROADS ADMINISTRATION

Reported by **L. A. PALMER**, Associate Chemist

THE main purpose of this paper is to bring to the attention of engineers the fact that a precise analysis of the complete system of stresses under a uniformly loaded circular area at the surface of a semi-infinite, elastically isotropic material has been in published form for more than 10 years. Another purpose of the paper is to indicate approximate methods of checking numerical values of the stresses that have been computed by the precise methods.

Still another purpose is to indicate the possibility of using the results of the analyses as a rough guide in experimentation involved in the study of the design of flexible types of highway surfaces. In this latter connection, it is realized generally that the condition of elastic isotropy does not exist in the mass of material under an automobile tire. Hence, it would be a mistake to attempt to apply directly the analytical results, and the fact that they can serve only as a rough guide in experimentation needs to be emphasized.

Engineers are confronted with many earth problems for which no theory other than the one of elasticity has been proposed, and the treatment of earth problems apart from adequate theory is likely to lead to as many solutions as there are varieties of earth. The experimental study of such problems without reference to any theoretical basis whatsoever is an aimless procedure at best.

The problem of computing stresses at any point within a semi-infinite, elastically isotropic mass produced by a uniform load over a circular area at the plane boundary has been completely solved by A. E. H. Love[2] and S. D. Carothers.[3] The results of these two investigators are in general in good agreement and suggest a means of estimating stresses under wheel loads, since it has been shown by Teller and Buchanan[4] that the pressure distribution of a pneumatic tire on a flexible-type pavement is nearly uniform.

PROBLEM SIMPLIFIED BY MAKING REASONABLE ASSUMPTIONS

In such problems as the determination of the rate of settlement (by soil consolidation) of a foundation (fig. 1A), the vertical stresses at the upper and lower boundaries of a clay layer are computed from formulas derivable from those of Boussinesq by assuming that the entire mass of earth below the foundation is elastically isotropic. That is, the complications that may be involved, owing to the fact that the underground is comprised of alternate strata of dissimilar materials and is, therefore, not elastically isotropic, are ignored completely. Despite the questionable procedure of assuming isotropy in this case, the analysis of settlements by consolidation is one of the best established theoretical methods of soil mechanics and the present writer believes that in this instance the assumption of isotropy is as reasonable as any other assumption and is to be preferred in the interest of simplicity.

No doubt there is some refraction of the stress trajectories at the boundary of two dissimilar earth materials, that is, this boundary very likely acts as a plane of discontinuity. But any assumption as to the amount of friction between two layers of dissimilar materials such as clay and sand (fig. 1A) or as to what extent discontinuity of stresses may exist is likely to be no more valid than the assumption of isotropy in the entire earth mass below the foundation.

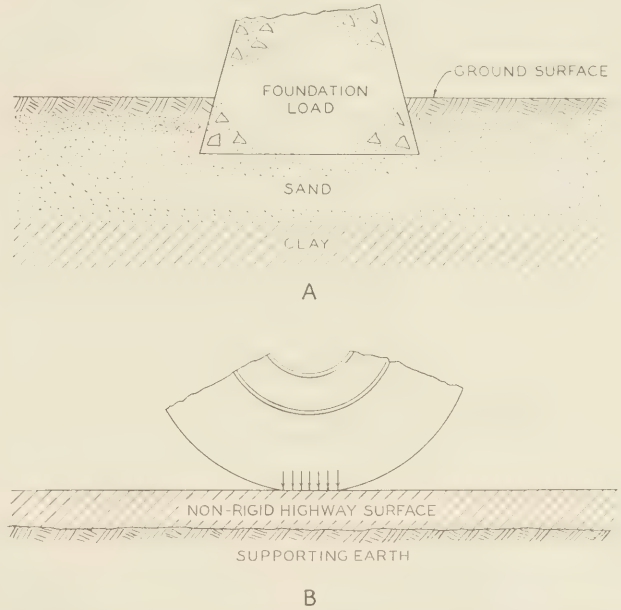

FIGURE 1.—A, FOUNDATION RESTING ON SAND UNDERLAID BY CLAY; B, WHEEL LOAD RESTING ON NONRIGID HIGHWAY SURFACE SUPPORTED BY EARTH.

For the same reason, the entire mass of materials below a wheel load (fig. 1B) may be considered as being elastically isotropic.

On the basis of such an assumption, no precise solution can be expected. Nevertheless, the theoretical development meets a definite requirement in that it shows, qualitatively at least, what happens under wheel loads and, therefore, may serve as a guide in planning methods of experimentation. All that is sought is a suggested trace of the transmission of stresses by the wheel load since rigorous results or exact formulas are out of the question.

A. C. Benkelman[5] has presented in considerable detail the present status of knowledge concerning the design of flexible pavements. It is indeed surprising to note that the conclusions reached by Carothers and Love have not been used in the technical publications discussed in Benkelman's report. However, with practically no exception, the various authors mentioned in his report have based their experimental procedures on

---

[1] Paper presented at the Nineteenth Annual Meeting of the Highway Research Board, December 5, 1939.

[2] The Stress Produced in a Semi-Infinite Solid by Pressure on Part of the Boundary, by A. E. H. Love, Philosophical Transactions of the Royal Society, series A, vol. 228, 1929.

[3] Test Loads on Foundations as Affected by Scale of Tested Area, by S. D. Carothers, Proceedings International Mathematical Congress, Toronto, 1924, pp. 527–549.

[4] Determination of Variation in Unit Pressure over the Contact Area of Tires, by L. W. Teller and James A. Buchanan, PUBLIC ROADS, vol. 18, No. 10, Dec. 1937.

[5] Present Knowledge of the Design of Flexible Pavements, by A. C. Benkelman, PUBLIC ROADS, vol. 18, No. 11, Jan. 1938.

theoretical considerations involving numerous simplifying assumptions which are similar in many respects to the classical theories of Love, Carothers, and Hencky.[6]

The simplest three-dimensional problem considered in the theory of elasticity is the one involving axial symmetry. Hence, the problem of a wheel load on a pavement is simplified by the assumption that the area of contact between a rubber tire and the pavement is circular, although tests indicate that the contact area is elliptic. Thus an "equivalent circular area" with uniform pressure has been used generally by investigators. The equivalent circle is one having an area equal to the elliptic area of contact and its radius is the square root of the quantity: Area of contact divided by π.

The subject of pressure distribution over a circular contact area in connection with foundation design has been discussed at length by Cummings,[7] Krynine,[8] and various others. The simplest problem is the one of uniform pressure over the contact area. In foundation problems, the actual pressure distribution depends on the relative rigidities of the structure (or loading member) and the earth mass, and in the light of present knowledge no one knows how to express these relative rigidities in quantitative terms. Another difficulty is that any formula expressing the pressure distribution over the contact area can be only approximately true when the deformations are within the elastic limits of the materials and cannot be applicable in any sense as the deformations increase more and more and become characteristic of those attending plastic yield. If an attempt is made to include all of these departures from simplicity in a theoretical development of the subject, any result would be so hopelessly complicated as to be of little practical use.

GREATEST SHEAR NEAR PERIMETER OF A UNIFORMLY LOADED CIRCULAR AREA

The outstanding conclusion of Love and Carothers was that the greatest value of the principal stress difference, S, is very close to the perimeter of the uniformly loaded circular area. This difference is twice the maximum shearing stress at any point, that is,

$$S = 2\, s_{\max.} \qquad (1)$$

Table 1 is taken from Love's article and shows values of $\frac{S}{p}$, p being the unit contact pressure over the circular area, corresponding to various positions of the point Q, figure 2. The ratio, $\frac{r_1}{r_2}$, of the two radial lines, figure 2, and the magnitudes of the quantities, angle B, $\frac{z}{a}$, and $\frac{r}{a}$, a being the radius of the circle, z being the depth to the point, and B the angle between r_1 and a, determine the position of any point Q.

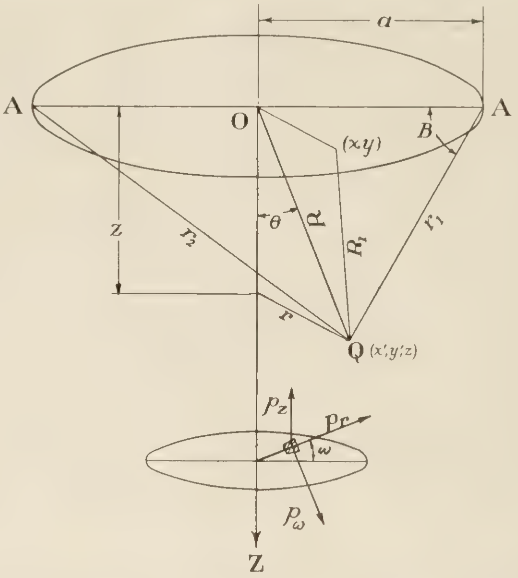

FIGURE 2.—PROBLEM OF THE PRINCIPAL STRESS DIFFERENCE, S, UNDER A UNIFORMLY LOADED CIRCULAR AREA.

As the point Q moves in such a manner that the ratio, $\frac{r_1}{r_2}$, approaches zero as a limit, then according to Love the value of S depends on the angle B, or in other words, S at the point A depends on the direction of approach of point Q to point A. Love has shown that for $B=71°$ the limiting value of S as $\frac{r_1}{r_2}$ approaches zero is $0.723p$ (table 1). The $\frac{S}{p}$ values of table 1 were computed by Love for $\mu=\frac{1}{4}$ and are "partial maxima stress differences," a term that will be considered later in this report.

TABLE 1.—*Values of $\frac{S}{p}$ for different positions of point Q in figure 2*

[Values given by A. E. H. Love for $\mu=\frac{1}{4}$] [1]

	$\frac{r_1}{r_2}$ tends to zero	$\frac{r_1}{r_2}$=sin 5°	$\frac{r_1}{r_2}$=sin 15°	$\frac{r_1}{r_2}$=sin 30°	$\frac{r_1}{r_2}$=sin 50°	$\frac{r_1}{r_2}$ tends to unity
$\frac{S}{p}$	0.723	0.714	0.704	0.695	0.690	0.689
B	71°	67°	56°	44°	37°	32°
$\frac{r}{a}$	1	0.934	0.742	0.446	0.184	0
$\frac{z}{a}$	0	0.156	0.383	0.536	0.615	0.620

[1] The values, $\frac{1}{4}$ and 0.45 for μ, appearing in this paper are values selected by Love and Carothers, respectively, in their computations.

From table 1, the value for s_g, the greatest value of $s_{\max.}$ at any point Q in the supporting earth, is $\frac{0.723p}{2} = 0.36p = \frac{S}{2}$. This is an important fact because it shows that the greatest shearing stress is at the surface where, due to lack of confinement, there is the least resistance to yield under stress. Love has given in table 1 more data than are necessary for locating the point Q, figure 2.

In texts dealing with the theory of elasticity, it is shown that when the point Q (fig. 2) is restricted to move only along the axis of symmetry, OZ, the greatest value of $s_{\max.}$ occurs at a considerable depth, z, from the surface. Thus Timoshenko[9] shows that for $\mu=0.3$, the greatest value of $s_{\max.}$ at any point on OZ is $0.33p$

---

[6] Über einige statische bestimmte Fälle des Gleichgewichts in Plastischen Körpen by von Heinrich Hencky, Zeitschrift für Angewandte Mathematik und Mechanik, Bd. 3, Heft 4, Aug. 1923, pp. 241–251 inc.

[7] Distribution of Stresses Under a Foundation, by A. E. Cummings, Proceedings American Society of Civil Engineers, vol. 61, No. 6, Aug. 1935.

[8] Pressures Beneath a Spread Foundation, by D. P. Krynine, Proceedings American Society of Civil Engineers, vol. 63, No. 4, pt. I, Apr. 1937.

[9] Theory of Elasticity by S. Timoshenko, Engineering Societies Monographs, first edition 1934, McGraw-Hill Book Company, Inc. (See pp. 336–337.)

and at a depth equal to $\frac{2a}{3}$. The earth at this point is confined in all directions.

Love's solution was obtained by the application of potential theory[10] and involved various elliptic integrals.

Carother's procedure is only very briefly described and therefore requires some discussion. He considers the logarithmic potential[11] of a uniform distribution of matter over a circular area of radius a, expressed by the relation,

$$\psi=\frac{p}{2\pi}\int\int\log\,(z+R_1)dx\,dy \text{-----------} (2)$$

where dx and dy refer to the coordinates of any point x, y, on the surface and R_1 is the distance of this point to the point Q, figure 2.

By differentiating equation 2 under the integral sign,

$$\frac{\partial^2\psi}{\partial z^2}=\frac{p}{2\pi}\int\int\frac{z}{R_1^3}\,dx\,dy=\frac{pw}{2\pi} \text{-----------} (3)$$

where w is the solid angle[12] subtended at Q by the circular area.

By use of the equations of equilibrium and the compatibility equations[13] all of the stresses at any point, Q, may be expressed in terms of w, the solid angle subtended at point Q. Thus it is found, for example, that

$$s_{rz}=z\frac{\partial}{\partial r}\frac{\partial^2\psi}{\partial z^2}=\frac{p}{2\pi}z\frac{\partial w}{\partial r} \text{-----------} (4)$$

and

$$p_z=\frac{p}{2\pi}\left(z\frac{\partial w}{\partial z}-w\right) \text{-----------} (5)$$

p_z being the pressure at Q that is normal to the horizontal plane and s_{rz} the shearing stress.

Similarly, p_r and p_ω (fig. 2) may be expressed as functions of the solid angle, w.

For a uniformly loaded circular area, the maximum shearing stress is obtained from the expression,

$$s^2_{\text{max.}}=\frac{(p_r-p_z)^2}{4}+s_{rz}{}^2 \text{-----------} (6)$$

Love's very comprehensive tables give values for p_r, p_z, and s_{rz}, for $\mu=¼$, at a great many points. Values for $\frac{S}{p}$ in addition to those given in table 1, may be computed by means of equation 6 and the other tables of Love.

The stresses at any point, Q, were computed by Carothers by expanding the various functions of w in zonal harmonics. Some of his computed values for $\frac{S}{p}$ are given in table 2. These values were computed by taking $\mu=0.45$. It is seen from this table that the greatest value of $s_{\text{max.}}$ is near the perimeter of the loaded circular area where $R=a$ and θ has values ranging from 80° to 90° (fig. 2). Love and Carothers are in good agreement as to the region of greatest shear.

TABLE 2.—*Values of $\frac{S}{p}$ for different positions of point Q, figure 2, according to Carothers, for $\mu=0.45$*

θ (degrees)	$\frac{S}{p}$ for $R=a$	$\frac{S}{p}$ for $R=\frac{2a}{3}$
0	0. 54	0. 60
30	. 55	. 60
45	. 57	. 53
60	. 58	. 50
75	. 62	. 40
80	. 63	. 30
85	. 63	. 18
90	. 63	. 05

MEANS GIVEN FOR CHECKING THE VALUES OF LOVE AND CAROTHERS

The method of Love, although enormously complicated, is presented in great detail and it is possible to check his numerical values, using his method. Carothers, on the other hand, gives no clue as to what particular functions he expanded in zonal harmonics in making his numerical computations. It would seem desirable then to find some means of checking the values of both Carothers and Love without using exactly the same method of either of these two investigators. This is an exceedingly difficult undertaking for the reason that Love's solution is complete, all of the stresses, p_r, p_z, and s_{rz}, being found throughout the entire region within a radial distance, a, of the loaded area. On the basis solely of his numerical values and without regard to how they were obtained, the same is true of Carothers' solution.

For the case of a uniform pressure, p, on a circular area it is easy to show[9] that for any point on the axis of symmetry and for $\mu=¼$,

$$\frac{S}{p}=\frac{1}{4}+\frac{1}{4}\left[\frac{5a^2z-z^3}{(a^2+z^2)^{3/2}}\right] \text{-----------} (7)$$

This expression may be expanded by the binomial theorem to give the infinite series,

$$\frac{S}{p}=\frac{1}{4}+\frac{5}{4}\left[\frac{z}{a}-\frac{1\times3}{2}\frac{z^3}{a^3}+\frac{1\times3\times5}{2\times4}\frac{z^5}{a^5}-\frac{1\times3\times5\times7}{2\times4\times6}\frac{z^7}{a^7}+,-\right]\text{---}$$
$$-\frac{1}{4}\left[\frac{z^3}{a^3}-\frac{1\times3}{2}\frac{z^5}{a^5}+\frac{1\times3\times5}{2\times4}\frac{z^7}{a^7}+,-\right]\text{---------------} (8)$$

if a is greater than z, and

$$\frac{S}{p}=\frac{1}{4}+\frac{5}{4}\left[\frac{a^2}{z^2}-\frac{1\times3}{2}\frac{a^4}{z^4}+\frac{1\times3\times5}{2\times4}\frac{a^6}{z^6}-\frac{1\times3\times5\times7}{2\times4\times6}\frac{a^8}{z^8}+\text{--}\right]$$
$$-\frac{1}{4}\left[1-\frac{1\times3}{2}\frac{a^2}{z^2}+\frac{1\times3\times5}{2\times4}\frac{a^4}{z^4}-,+,\right]\text{---------------} (9)$$

if z is greater than a.

From a mathematical standpoint, it is not correct to expand a function in zonal harmonics if it does not satisfy Laplace's equation. In potential theory, a Newtonian potential function, known to be harmonic (satisfies Laplace's equation) may first be computed on an axis of symmetry and thereafter it may be computed at any point not on the axis, by expanding in zonal harmonics or Legendrian polynomials.

---

[10] For an adequate description of potential theory, see, for example, Pt. III of Dynamics by A. G. Webster, Text, G. E. Stechert and Co., New York, second edition, 1922.

[11] See, for example, pp. 385 et seq., of Dynamics by A. G. Webster.

[12] See, for example, p. 351 of Dynamics by A. G. Webster.

[13] See, for example, top of p. 312 of Theory of Elasticity by S. Timoshenko.

[9] Theory of Elasticity, by S. Timoshenko, Engineering Societies Monographs, first edition 1934, McGraw-Hill Book Co., Inc. (See pp. 336–337.)

In general, the expression for a stress is a tensor[14] and is not therefore harmonic. However, the expression for a stress may consist of several parts, one or more of which may be harmonic, and it is legitimate to evaluate these by expanding in zonal harmonics. This, no doubt, was Carothers' procedure although he does not indicate it.

In the present consideration, the circle is not one of complete symmetry as Love has shown and in the absence of any general expression for $\frac{S}{p}$, known to be harmonic or otherwise, there is no sound mathematical basis for the following procedure which is to assume that the general expression for $\frac{S}{p}$ is harmonic and to pass from equations 7 and 8 to series of Legendrian polynomials. It is also assumed that the circle is symmetrical and this is not quite true.

If now the loaded circular area is considered as analogous to an electrically charged disk in potential theory, then one could substitute R for z in equations 8 and 9 and introduce Legendrian coefficients,[15] thereby expanding equation 7 in zonal harmonics. When this is done, equation 8 becomes

$$\frac{S}{p}=\frac{1}{4}+\frac{5}{4}\left[\frac{R}{a}P_1(\cos\theta)-\frac{1\times3}{2}\frac{R^3}{a^3}P_3(\cos\theta)+\frac{1\times3\times5}{2\times4}\frac{R^5}{a^5}P_5(\cos\theta)-\frac{1\times3\times5\times7}{2\times4\times6}\frac{R^7}{a^7}P_7(\cos\theta)\right]-\frac{1}{4}\left[\frac{R^3}{a^3}P_3(\cos\theta)-\frac{1\times3}{2}\frac{R^5}{a^5}P_5(\cos\theta)+\frac{1\times3\times5}{2\times4}\frac{R^7}{a^7}P_7(\cos\theta)\right]\text{-------------}(10)$$

if all powers of $\frac{R}{a}$ beyond the seventh are neglected, equation 9 becomes

$$\frac{S}{p}=\frac{1}{4}+\frac{5}{4}\left[\frac{a^2}{R^2}P_1(\cos\theta)-\frac{1\times3}{2}\frac{a^4}{R^4}P_3(\cos\theta)+\frac{1\times3\times5}{2\times4}\frac{a^6}{R^6}P_5(\cos\theta)\right]-\frac{1}{4}\left[1-\frac{1\times3}{2}\frac{a^2}{R^2}P_1(\cos\theta)-\frac{1\times3\times5}{2\times4}\frac{a^4}{R^4}P_3(\cos\theta)+\frac{1\times3\times5\times7}{2\times4\times6}\frac{a^6}{R^6}P_5(\cos\theta)\right]\text{--}(11)$$

neglecting all powers of $\frac{R}{a}$ beyond the sixth.

EXAMPLES OF APPROXIMATE METHODS GIVEN

The terms, P_1 (cos θ), P_3 (cos θ), etc., are the Legendrian coefficients and numerical values for these coefficients, corresponding to different values of θ, figure 2, may be found in various mathematical treatises. For R less than a and $\theta=0$, equation 10 becomes equation 8 and for R greater than a and $\theta=0$, equation 11 reduces to 9.

It should be emphasized now that aside from the geometric similarity, that is, a circle and an axis of symmetry in both cases, the problem of potential at a point due to a charged disk and the problem of shearing stresses beneath a loaded circular area have nothing in common.

In table 1, for $\frac{r}{a}=0.184$, $B=37°$ and $\frac{r_1}{r_2}=\sin 50°$, it is found that $\theta=16°40'$ and $R=0.6416\ a$. The coefficients, P_1 (cos θ), P_3 (cos θ) corresponding to $\theta=16°40'$ are

$$P_1(\cos\theta)=0.9579$$
$$P_3(\cos\theta)=0.7609$$
$$P_5(\cos\theta)=0.4573$$
$$P_7(\cos\theta)=0.1207$$

Substituting these values in equation 10,

$$\frac{S}{p}=\frac{1}{4}+\frac{5}{4}(0.6141-0.3015+0.0930-0.0118)-\frac{1}{4}(0.2010-0.0744+0.0101)=0.709,$$

which compares favorably with Love's value in table 1 of 0.690.

The series in this case converges rapidly. By using Love's numerical values for p_r, p_z and s_{rz} $\left(\mu=\frac{1}{4}\right)$ and equation 6 one finds for $\frac{R}{a}=0.56$ and $\theta=45°$, that $\frac{S}{p}=0.66$. However, by equation 10 the corresponding value for $\frac{S}{p}$ is 0.75, which is 14 percent higher than Love's value. Again, for $\frac{R}{a}=0.19$ and $\theta=45°$, from Love's tables and equation 6, $\frac{S}{p}=0.42$, which is exactly the same as the value obtained from equation 10. Jürgenson[16] has computed various values for $\frac{S}{p}$, using Carothers' tables for p_r, p_z, and s_{rz} and finds for example, that for $R=2a$ and $\theta=45°$, $\frac{S}{p}=0.25$. The corresponding value found by an expression similar to equation 11 is 0.27. Love's value for $\frac{S}{p}$ for $\frac{r}{a}=0.446$ and $\frac{z}{a}=0.536$ is 0.695 (table 1), whereas equation 10 gives a value of 0.716 which is close to Love's values. For θ equal to 90° and $\frac{R}{a}$ less than 1, the value of $\frac{S}{p}$ is 0.25 by equation 10, which is Love's value in this region. Equations 10 and 11 do not fit the boundary condition for $\theta=90°$ and R greater than a.

In all other cases it may be said that for θ greater than 45° and for $\frac{R}{a}$ or $\frac{a}{R}$ greater than 2/3, the values for $\frac{S}{p}$ computed from equations 10 and 11 deviate by at least 15 percent (in many cases much more) from Love's numerical values. For all other positions of the point Q (fig. 2) the agreement is generally within 15 percent. Carothers states that for values of $\frac{R}{a}$ or $\frac{a}{R}$ approaching unity, his functions expanded in zonal harmonics were "unsatisfactory" and he gives no clue as to an alternative procedure in this case.

---

[14] See for example, From Determinant to Tensor, by W. F. Sheppard, Oxford at the Clarendon Press, 1923.

[15] See for example, Fourier Series and Spherical Harmonics, by W. E. Byerly, Ginn & Co., 1893.

[16] The Application of Theories of Elasticity and Plasticity to Foundation Problems, by Leo Jürgenson, Journal of the Boston Society of Civil Engineers, vol. 21, No. 3, July 1934.

In any case, equations 10 and 11 have very definite limitations in this problem although they tend to give values that are in approximate agreement with those of Love and Carothers at various points.

Functions of solid angles are for the most part of interest only to the mathematician. A practicing engineer has more confidence in numerical values if he can check them by a simple graphical method.

The following simple device is suggested by the author for this purpose. Its use is very limited. The method is as follows:

It is desired, for example, to know the value of $\frac{S}{p}$ at the point where $\theta=45°$ and $R=\frac{2a}{3}$, the value of μ being taken as 0.45 (Carothers' assumed value). In figure 3, AB is the diameter of the loaded circular area on the horizontal surface and OZ is its axis of symmetry. The radial lines AQ, QB, and $OQ=R$ are drawn. The line MN is drawn through O making an angle θ with AB. The projection of AB on MN is $A'B'$, the minor axis of an ellipse. This ellipse is in the plane that is passed through MN perpendicularly to the plane of the paper. Its major axis is equal to AB which passes through the point O and is perpendicular to MN.

The length $A'O$ is $a \cos \theta=\frac{1}{2}$ the minor axis, and $a=AO$ is $\frac{1}{2}$ the major axis. Then the area of the ellipse formed by projecting the circle in the horizontal plane on the plane MN is, $\pi a\,(a \cos \theta)=\pi a^2 \cos \theta$.

Let $b=$ the radius of the equivalent circle, that is, the radius of the circle having an area of $\pi a^2 \cos \theta$. Then $\pi b^2=\pi a^2 \cos \theta$, or $b=a\sqrt{\cos \theta}$. Now the ellipse is replaced with the circle of radius $=b=a\sqrt{\cos \theta}$. A uniform pressure, p, perpendicular to the plane MN and distributed over the circle of radius $a\sqrt{\cos \theta}$ will produce approximately the same stress, $s_{\text{max.}}$, at Q as is caused by the same pressure, p, distributed over the circular area, AB, in the horizontal plane, provided θ is less than 45°. For $\theta=0$, it is observed that p_R becomes p_z.

For the general case, the value of $s_{\text{max.}}$ when the point Q is on the axis of symmetry is obtained from the expression,

$$s_{\text{max.}}=\frac{p}{2}\left[\frac{1-2\mu}{2}+(1+\mu)\frac{R}{(b^2+R^2)^{1/2}}-\frac{3}{2}\frac{R^3}{(b^2+R^2)^{3/2}}\right] \text{--}(12)$$

where $b=a\sqrt{\cos \theta}=$ radius of the equivalent circle

For $R=\frac{2}{3}a$, $\theta=45°$, $\cos \theta=\frac{1}{\sqrt{2}}$, and $\mu=0.45$,

$$\text{at point Q } s_{\text{max.}}=\frac{p}{2}\left[0.05+\frac{2\times 1.45}{3\left(\frac{1}{\sqrt{2}}+\frac{4}{9}\right)^{1/2}}-\frac{\frac{3}{2}\times\frac{8}{27}}{\left(\frac{1}{\sqrt{2}}+\frac{4}{9}\right)^{3/2}}\right]=0.296p.$$

Then $\frac{S}{p}=\frac{2s_{\text{max.}}}{p}=0.592$. Carothers obtains the value, $\frac{S}{p}=0.53$ for this point (table 2). The divergence from Carothers' value in this case is likely indicative of the error involved in transforming the elliptic area into one that is circular.

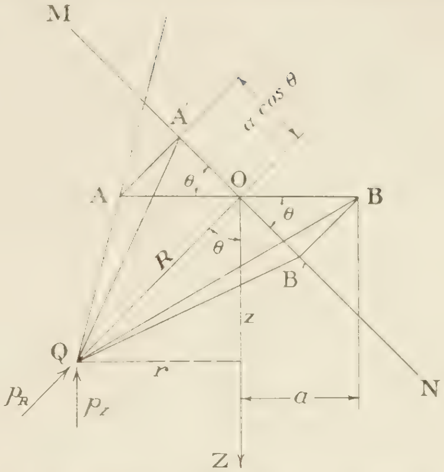

Figure 3.—The Projection of the Loaded Circular Area, AB, in the Horizontal Plane on the Plane MN is an Ellipse With Minor Axis A'B' and Major Axis AB.

METHOD OF KRYNINE CONSIDERED

In addition to approximate methods, such as the foregoing, there are other means of checking precise numerical values, such as those published by Love and Carothers. One such means is the photoelastic method. However, at present there has been but little progress in the analyses of three-dimensional problems of stress distribution by photoelastic devices. R. Weller[17] has suggested that it is possible to use the polarization caused by the scattering of light within a "cloudy" or opaque model in place of the usual analyzer in photoelastic investigations and that this method of polarization enables one to make analyses of three-dimensional stress systems very conveniently. An older method was to cool the model under load from an elevated temperature to room temperature and then analyze it subsequently after slicing it into plates. This is an enormously complicated procedure.

D. P. Krynine[18] has described his device, called a stereogoniometer, which offers much promise in such studies and which has the advantages of simplicity and low cost. This device makes use of the principle of projecting areas from a plane to a spherical surface rather than from one plane to another plane as was done in the preceding example. By this procedure, the stress, p_z, is found in terms of the area of the projection of the loaded surface on the sphere and the volume of the space bounded by this projected area, its projection in turn on the horizontal plane including the load, and the perpendicular lines joining the perimeters of the two projected areas. The solid angle at a point below the loaded area is equal to the projected area on the sphere divided by the square of the radius of the sphere.

It is possible to compute the solid angle w, without this device, as follows: On the axis of symmetry, the expression for the solid angle, w is

$$w=2\pi\left(1-\frac{z}{R}\right) \text{-------------}(13)$$

---

[17] A New Method for Photoelasticity in Three Dimensions, by R. Weller. Letter to the Editor, Journal of Applied Physics, vol. 10, No. 4, Apr. 1939.

[18] Stresses (especially shears) Under a Foundation by D. P. Krynine, Proc. of the Highway Research Board, vol. 18, pt. II, 1938.

where $R=(a^2+z^2)^{1/2}$. The general expression for w (see equation 3) is harmonic and it is mathematically legitimate to expand it in zonal harmonics. By so doing, one obtains the two expressions,

$$w=2\pi-2\pi\left[\frac{R}{a}P_1\ (\cos\theta)-\frac{1}{2}\left(\frac{R}{a}\right)^3P_3\ (\cos\theta)\right.$$
$$\left.+\frac{3}{8}\left(\frac{R}{a}\right)^5P_5\ (\cos\theta)-\frac{15}{48}\left(\frac{R}{a}\right)^7P_7\ (\cos\theta)+,\ -,\ \text{etc.}\right]\text{--}(14)$$

for R less than a, and

$$w=2\pi-2\pi\left[1-\frac{1}{2}\left(\frac{a}{R}\right)^2P_1\ (\cos\theta)+\frac{3}{8}\left(\frac{a}{R}\right)^4P_3\ (\cos\theta)\right.$$
$$\left.-\frac{15}{48}\left(\frac{a}{R}\right)^6P_5\ (\cos\theta)+\frac{105}{384}\left(\frac{a}{R}\right)^8P_7\ (\cos\theta)-,\ +,\ \text{etc.}\right]\text{-}(15)$$

for R greater than a.

For $\frac{R}{a}=\frac{2}{3}$ and $\theta=45°$, one obtains $w=1.07\pi$ from equation 14. Then substituting this value in the expression which is valid only for $\mu=\frac{1}{2}$,

$$p_z=\frac{3p}{2\pi}w\cos^2\theta\text{---------------}(16)$$

it is found that $p_z=0.803p$. The value for p_z at $R=\frac{2}{3}a$ and $\theta=45°$ is $0.858p$ when $\mu=0.45$, according to Carothers.

Similarly, for $R=2a$ and $\theta=45°$, $w=0.195\pi$ from equation 15 and on substitution in equation 16, $p_z=0.146p$. The corresponding value obtained by Carothers for $\mu=0.45$ is $0.158p$. The differential of equation 16 was proposed by D. P. Krynine.[19] The stereogoniometer is a device that performs an exact mechanical integration of the differential of equation 16, θ and w being both variable in the integration.

Equation 16 does not satisfy the boundary conditions, $\theta=0$ and $\theta=90°$. For a uniformly loaded circular area it gives results which are approximately correct for θ greater than 30° and less than 60°, R being greater than zero. On the axis of symmetry,

$$p_z=p\left[1-\frac{z^3}{(a^2+z^2)^{3/2}}\right]\text{------------}(17)$$

If θ is taken as the average angle between the vertical direction and the radial distance R drawn from any element of loaded surface to the point Q, considering all of the elements of the loaded area, the computations by equation 16 become exact. The analytical procedure of obtaining this average θ is a very difficult one. For R greater than $\frac{3}{2}a$, equation 16 together with equations 14, 15, and 17 give values for p_z that do not diverge more than 15 percent (in most cases much less) from Carothers' values when θ does not exceed 60°.

The purpose of the foregoing procedure is to indicate that in substance, at least, the mechanical method used by Krynine is correct. Its outstanding advantage is that in its use the evaluation of the stress, p_z, can be accomplished for any contour of uniformly loaded area. It is not necessary that there be axial symmetry.

LOVE'S CONCLUSIONS APPLIED TO THE CASE OF A WHEEL LOAD

The precise analytical method of obtaining p_z at a point under a uniformly loaded circular area is indicated by equation 5 which is Carothers expression for this stress.

Since

$$w=\int\int\frac{z}{R_1^{\,3}}dx\ dy,$$

Then

$$\frac{\partial w}{\partial z}=\int\int\left(\frac{1}{R_1^{\,3}}-\frac{3z^2}{R_1^{\,5}}\right)dx\ dy\text{-----------------}(18)$$

and

$$z\frac{\partial w}{\partial z}=\int\int\frac{z}{R_1^{\,3}}dx\ dy-3\int\int\frac{z^2}{R_1^{\,2}}\left(\frac{z}{R_1^{\,3}}\right)dx\ dy\text{---}(19)$$

It is possible to evaluate these two integrals at any point Q. The first integral is w, known to be harmonic, and it may be computed at any desired point by means of equations 14 and 15. The second integral may be simplified by expanding $\frac{z}{R_1^{\,3}}$ in Legendrian polynomials since $\nabla^2\left(\frac{z}{R_1^{\,3}}\right)=0$. With w and $z\frac{\partial w}{\partial z}$ thus evaluated, the stress, p_z, at any point, Q, is computed from equation 5.

For a uniform pressure produced by a wheel load on the nonrigid surface, the distribution of vertical pressure, p_z, over the area of subgrade bounded by the radial line R is far from uniform as Hawthorne[20] assumed in his analysis.

Love has shown that for any given value of μ, there is a certain value of R, expressed as some multiple of a, for which $s_{\max.}$ has a greatest value. Thus for $\theta=45°$, the greatest value of $s_{\max.}$ on the radial line is at any point where $R=0.73a$. Similarly for $\theta=90°$, the greatest value of $s_{\max.}$ is at points where $R=a$, that is, at points just under the perimeter. If the points of "partial maxima stress difference" on all of the radial lines for all values of θ are connected, the locus of such points is found to be a "basin-shaped surface of revolution about the axis of the circle." It passes through the circle and lies between two segments of spheres, which have their centers on the axis of symmetry and pass through the circle (fig. 4). These two spheres cut the axis of symmetry at depths equal to $0.620a$ and at $0.712a$ when $\mu=\frac{1}{4}$. According to Love, it is reasonable to conclude that the foundation under a round pillar would be most likely to give way at points on such a basin-shaped surface and that it would be nearly as likely to give way at one point of this surface as at any other. The values given in table 1 are for points on the basin-shaped surface.

A practical consideration follows. If the supporting subgrade is of questionable supporting power, such as medium or soft clay, then considering all possible values of μ and the uncertainty of the effect of contact area where pavement and subgrade meet, it would be on the side of safety to have the thickness of the more resistant flexible pavement at least equal to a, the radius of the equivalent circle. This precaution would tend to confine the dangerous surface to the surfacing

---

[19] Shearing Stresses Under a Spread Foundation, by D. P. Krynine, Proceedings of the Eighteenth Annual Meeting of the Highway Research Board, pt. II, 1938.

[20] A Method of Designing Nonrigid Highway Surfaces, by George Edward Hawthorne, Bulletin No. 83, Engineering Experiment Station, University of Washington, Aug. 1935.

material. But it is again necessary to emphasize the fact that since the flexible pavement and the subgrade are very different materials, all such conclusions must be considered as only generally indicative and not strictly true in a quantitative sense.

The greatest values of $s_{\text{max.}}$ may be computed on the axis of symmetry from equation 12 for different values of μ. Table 3 contains such values and the points at which they are found are defined in terms of $\frac{z}{a}$. It is interesting to note from this table that the shearing stress does not vary greatly as μ varies from 0.25 to 0.50.

TABLE 3.—*Maximum values of* $\frac{s_{\text{max.}}}{p}$ *on the axis of symmetry for different values of* μ

μ	$\frac{s_{\text{max.}}}{p}$	$\frac{z}{a}$ at point of $s_{\text{max.}}$
0. 25	0. 34	0. 62
. 30	. 33	. 64
. 40	. 31	. 67
. 45	. 30	. 69
. 50	. 29	. 71

Benkelman [5] has rightfully emphasized our lack of information concerning conditions at the boundary plane of contact between the flexible surface and the supporting medium. It is possible that this information may be obtained by intelligent and careful experimentation, making use of adequate theory in all such experimental procedures.

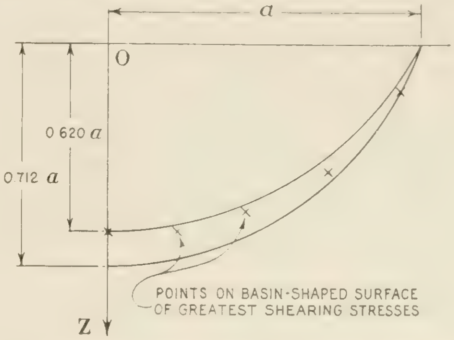

FIGURE 4.—POINTS AT WHICH THE STRESSED MATERIAL UNDER A UNIFORMLY LOADED CIRCULAR AREA WOULD BE MOST LIKELY TO FAIL.

CONCLUSIONS

On the basis of Love's complete solution of the problem of stresses under a uniformly loaded circular area, it is indicated that the greatest shearing stresses may be confined within the flexible pavement and not reach the subgrade if the thickness of the flexible pavement is no less than the radius of a circle having an area equal to the plane of contact between pneumatic tire and flexible pavement.

The conclusions reached by Carothers are in good agreement with those reached independently by Love, but the method used by Carothers requires a more complete presentation.

In the case of a uniform load on a circular area the variation in shearing stress as Poisson's ratio varies from 0.25 to 0.50 is relatively small.

---

[5] Present Knowledge of the Design of Flexible Pavements, by A. C. Benkelman, PUBLIC ROADS, vol. 18, No. 11, Jan. 1938.

(Continued from page 227)

One driver waited until the clear distance ahead decreased from 1,900 feet to less than 600 feet and then started the passing maneuver. He was fortunate, and completed the passing before an oncoming car came into view.

The horizontal scale of figure 11 shows the clear distance ahead that the driver of each passing vehicle had at the time the maneuver was completed. For all the passings shown below the heavy horizontal line, this distance represents the clearance between the vehicle that had completed the passing and the first approaching vehicle in the opposing lane of traffic. For the passings above the heavy line, no oncoming vehicles were in view, so the horizontal scale represents the sight distances when the maneuvers were completed. In this figure, as in figure 10, the passings that are cross-hatched represent maneuvers that could have started at the point of maximum sight distance. In those that are not cross-hatched, the driver had to wait for an oncoming vehicle before encroaching on the left lane.

Figure 10 showed that an oncoming vehicle was in view at the start of the passing in less than 5 percent of the maneuvers. Before the return was made to the right lane, there was an oncoming vehicle in view in about 30 percent of the maneuvers. There is a marked drop in the number of maneuvers that were completed with either a sight distance of less than 300 feet or a clearance from the oncoming vehicle of less than 300 feet. Two of the five passings that were completed with an oncoming vehicle less than 200 feet away could have been started when the maximum sight distance was available. The clearances for the vehicles involved in the two maneuvers below 100 feet were 34 and 49 feet. At the time these vehicles started to return to their right-hand lane, the distances to the oncoming vehicles were 876 and 228 feet respectively. The first vehicle could have started the maneuver when the sight distance was 1,900 feet but did not enter the left lane until the sight distance had decreased to 1,675 feet. The second vehicle had to wait for an oncoming vehicle to pass and started the maneuver with a sight distance of 625 feet.

By similar analyses of the passings that occur on different sections, it will be possible to determine the relative effectiveness of different alinements in providing for passing requirements.

Up to the present time, the analyses dealt primarily with the passings that actually occurred on the study sections. Of equal importance are the passings that the drivers wanted to make but did not attempt because they felt that the available sight distances or the "holes" in the opposing traffic were not of sufficient length to complete the passing maneuvers in safety. No attempt has been made to take this particular information from the records, but it is believed that it can be obtained. So far, it has been possible to obtain all the factors that have seemed important in a study of passing distances.

From the rather meager results that have been presented it can be seen that these studies provide, for the first time, accurate information on what actually takes place in a stream of moving traffic. They are certain to provide extremely valuable information regarding the causes of accidents even though none may actually occur during the studies.

1940 CENSUS TO PROVIDE HOUSING DATA VALUABLE TO HIGHWAY OFFICIALS

A comprehensive picture of housing and home ownership in the United States will be compiled from information to be collected by the U. S. Bureau of the Census in April when it conducts the Sixteenth Decennial Census. In response to a schedule of questions bearing on the type of structure, equipment, and ownership, data will be obtained for each of the approximately 35,000,000 dwellings throughout the country.

The data obtained will be useful to highway officials in planning street improvements, and to city planning officials in determining the need for extending transportation and communication systems, police and fire protection, schools, etc.

STATUS OF FEDERAL-AID HIGHWAY PROJECTS

AS OF JANUARY 31, 1940

STATE	COMPLETED DURING CURRENT FISCAL YEAR			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS *
	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	
Alabama	$6,239,830	$3,017,347	256.0	$4,624,781	$2,290,828	135.7	$1,828,891	$907,935	76.2	$3,902,598
Arizona	1,892,271	1,347,963	94.4	1,563,725	1,105,821	84.4	306,802	166,000	9.3	1,955,773
Arkansas	4,930,534	3,918,101	226.3	1,304,697	763,616	52.4	533,240	266,420	22.6	2,235,343
California	5,075,272	2,762,835	79.1	4,849,633	2,561,931	93.7	2,419,380	1,232,100	34.3	5,474,649
Colorado	3,815,457	2,119,520	89.3	1,561,772	875,606	39.9	152,739	50,000	4.7	3,636,447
Connecticut	802,205	395,926	10.0	1,520,108	758,035	15.0	1,119,652	546,326	9.4	1,460,136
Delaware	723,874	343,734	24.8	1,254,241	625,331	13.7				1,587,136
Florida	2,302,305	1,148,305	9.1	3,540,941	1,770,246	83.0	1,837,112	916,682	70.8	2,669,218
Georgia	3,897,580	1,900,267	218.7	5,071,268	2,535,634	255.0	3,156,303	1,578,152	106.3	7,196,533
Idaho	2,208,564	1,309,785	113.9	874,808	534,106	63.3	148,839	85,628	14.6	2,397,342
Illinois	8,737,645	4,353,142	195.7	4,938,202	2,467,112	85.9	3,469,550	1,734,775	76.1	5,758,549
Indiana	3,464,564	1,719,790	70.5	5,163,611	2,575,399	108.7	854,566	427,126	13.9	4,365,628
Iowa	3,929,715	1,826,856	190.7	3,599,669	1,597,988	107.9	970,207	459,300	43.4	3,194,554
Kansas	3,494,784	1,722,647	175.9	2,026,992	1,012,617	117.4	2,987,960	1,490,460	128.2	6,640,717
Kentucky	3,075,985	1,525,964	105.7	2,031,951	1,014,419	37.5	1,080,721	540,360	24.8	4,723,641
Louisiana	716,918	355,250	23.9	12,109,126	3,103,437	46.2	1,842,820	903,707	51.2	3,665,704
Maine	2,183,320	1,070,865	52.4	717,470	358,735	16.7	13,240	6,620	.6	1,321,220
Maryland	2,712,006	1,311,115	35.8	1,584,275	783,325	30.9	512,000	216,500	6.8	2,547,054
Massachusetts	3,134,614	1,564,618	25.1	731,370	364,841	5.7	1,524,431	759,083	12.1	3,918,372
Michigan	4,780,068	2,344,648	115.4	2,611,400	1,217,700	89.2	3,684,000	1,842,000	109.4	4,425,898
Minnesota	5,243,257	2,554,330	365.0	4,065,867	2,012,974	180.6	860,977	429,918	45.7	6,178,874
Mississippi	5,269,300	1,766,960	243.8	5,095,158	2,197,345	169.5	3,067,550	1,310,070	166.7	2,864,930
Missouri	3,239,051	1,615,728	139.3	3,417,706	1,680,606	105.5	3,895,737	1,540,129	112.3	6,939,732
Montana	3,281,001	1,855,632	189.8	2,118,450	1,201,259	108.8	1,261,452	715,494	107.5	5,226,496
Nebraska	4,114,859	2,045,294	327.7	3,736,128	1,812,062	364.9	2,561,899	1,192,568	305.7	4,312,607
Nevada	1,113,680	953,000	53.6	1,125,247	966,181	52.6				2,076,542
New Hampshire	821,381	404,233	27.3	726,443	356,095	15.5	59,293	29,647		1,498,999
New Jersey	829,670	405,711	7.0	4,599,878	2,298,989	36.4	1,692,720	846,360	13.8	2,299,140
New Mexico	1,986,434	1,216,018	134.7	1,047,592	660,345	78.3	785,762	490,394	33.7	2,318,901
New York	8,635,200	4,258,235	167.9	11,997,632	5,736,851	168.1	1,360,990	595,495	21.4	5,734,057
North Carolina	5,693,660	2,837,970	351.8	3,875,405	1,939,447	174.4	914,360	451,050	61.8	3,545,615
North Dakota	266,707	141,907	41.7	1,250,475	670,099	78.0	2,177,590	1,167,133	238.4	5,121,688
Ohio	5,943,447	2,912,343	84.3	7,002,692	3,477,972	57.5	9,508,921	4,694,325	91.6	6,002,222
Oklahoma	1,895,113	1,002,012	97.9	2,630,917	1,395,333	71.7	2,918,010	1,551,841	108.8	5,065,064
Oregon	2,456,341	1,470,097	105.8	3,308,178	1,786,472	96.2	623,985	364,180	39.8	2,365,373
Pennsylvania	9,665,392	4,782,885	113.7	4,812,541	2,264,071	43.6	4,183,322	2,045,285	43.8	6,910,317
Rhode Island	601,970	300,865	7.8	901,224	449,656	8.9	119,150	59,575	1.3	1,394,417
South Carolina	2,165,870	978,200	81.9	1,562,934	701,209	67.5	831,589	385,698	70.1	3,235,702
South Dakota	3,482,838	1,917,656	329.1	2,623,960	1,491,060	307.3	1,681,580	926,010	237.5	4,286,762
Tennessee	3,666,788	1,748,336	88.5	2,592,162	1,296,081	49.9	2,526,516	1,263,258	50.2	4,882,163
Texas	11,226,920	5,513,521	618.6	8,903,875	4,418,278	424.5	4,869,202	2,312,281	199.0	9,095,207
Utah	2,262,717	1,627,522	98.1	625,885	453,575	46.8	182,755	127,010	4.6	1,862,322
Vermont	736,369	352,575	18.4	711,393	355,507	22.8	129,374	64,665	4.6	857,308
Virginia	2,300,410	1,146,828	76.9	2,329,061	1,109,477	52.3	939,990	469,995	23.2	2,710,918
Washington	2,188,160	1,119,757	38.4	3,299,282	1,628,309	27.0	566,967	297,000	7.9	2,044,244
West Virginia	1,905,151	1,008,304	50.0	1,680,476	836,881	47.7	949,941	469,166	20.3	2,900,082
Wisconsin	5,226,831	2,570,449	187.9	4,978,600	2,443,880	153.0	99,163	46,805	2.6	4,419,522
Wyoming	1,494,690	918,924	141.3	1,263,852	796,581	134.8	694,202	426,821	61.1	1,489,568
District of Columbia	373,200	186,600	2.5	233,724	116,862	2.4	326,286	141,188	2.4	603,475
Hawaii	360,498	176,232	4.4	786,412	379,777	13.1	601,757	299,199	9.9	1,597,567
Puerto Rico	706,072	351,850	14.1	1,306,457	646,785	25.3	65,005	32,220	.2	904,565
TOTALS	167,270,488	86,198,652	6,321.9	156,289,646	75,896,746	4,667.1	78,898,498	38,873,954	2,900.6	183,820,891

* Includes apportionment for Fiscal Year 1941.

STATUS OF FEDERAL-AID SECONDARY OR FEEDER ROAD PROJECTS

AS OF JANUARY 31, 1940

STATE	COMPLETED DURING CURRENT FISCAL YEAR			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS*
	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	
Alabama	$497,335	$242,657	27.8	$742,832	$270,643	21.9	$803,200	$399,650	45.0	$625,001
Arizona	265,371	190,062	33.4	122,287	88,179	16.0	53,263	32,500	2.4	443,612
Arkansas	880,619	744,916	84.4	132,274	117,823	14.0	74,882	47,556	6.3	361,836
California	875,573	467,084	36.1	389,944	212,124	7.4	185,940	106,232	19.5	1,146,047
Colorado	872,052	471,535	31.3	225,564	103,748	5.9	30,502	4,429	1.3	278,665
Connecticut	163,280	67,660	2.9	110,274	54,954	.2	210,228	99,380	3.1	234,223
Delaware	80,623	39,067	17.5	69,537	34,768	7.8				309,475
Florida	548,017	269,750	17.4	355,529	177,307	14.4	113,046	56,523	1.3	563,503
Georgia	315,508	152,194	39.4	184,939	92,470	19.8	447,483	223,742	45.5	1,302,099
Idaho	457,974	252,546	45.1	128,663	78,481	12.8				289,799
Illinois	1,318,851	650,674	79.7	998,500	443,950	56.7	1,011,950	505,975	39.0	689,378
Indiana	892,023	442,650	77.7	299,900	148,746	18.3	229,992	114,950	18.1	946,710
Iowa	303,910	145,468	116.1	1,074,275	487,310	104.0	876,173	413,550	196.7	1,013,712
Kansas	77,730	38,849	43.9	580,564	290,282	9.4	206,496	103,248	16.5	1,537,887
Kentucky	1,096,826	325,230	71.4	552,881	182,184	37.1	561,042	181,418	54.6	432,790
Louisiana	803,271	378,742	67.2	180,205	90,103	15.3	269,567	138,197	22.8	441,101
Maine	431,716	215,038	25.4	68,900	33,244	3.5	8,300	4,150	.5	139,781
Maryland	229,641	111,157	17.7	124,696	50,348	4.8	157,000	58,355	8.8	438,989
Massachusetts	373,212	185,203	9.2	284,559	140,915	5.9	190,910	94,705	4.2	575,040
Michigan	1,281,718	630,991	112.7	818,410	409,205	42.9	710,620	341,379	32.8	808,854
Minnesota	806,551	397,568	104.6	383,659	191,829	35.0	173,348	42,795	12.5	1,430,850
Mississippi	366,500	183,250	40.7	961,822	474,626	52.1	50,530	25,260	4.9	730,903
Missouri	899,069	442,043	133.5	501,373	250,390	44.7	278,174	115,816	41.2	906,928
Montana	835,992	474,167	71.6	168,138	95,304	20.8	266,305	150,656	25.3	915,937
Nebraska	989,792	474,534	205.0	335,420	167,290	41.3	249,941	124,970	44.7	553,753
Nevada	161,442	133,635	25.0	200,928	172.790	25.5				226,136
New Hampshire	61,156	29,708	2.4	83,280	39,815	3.1	68,831	32,155	1.1	191,412
New Jersey	334,990	164,755	12.1	316,520	158,260	12.3	7,610	3,805	.8	667,730
New Mexico	485,092	298.043	42.1	266,824	86.594	13.8	101,564	63,386	13.1	331,083
New York	1,759,039	867,032	90.9	1,666,975	802,543	45.1	553,800	204,974	12.9	781,553
North Carolina	994,230	495,960	94.5	525,193	265,208	48.8	191,430	94,710	14.6	541,618
North Dakota	115,841	61,370	8.3	47,816	25,629		66,953	35,886	2.7	1,066,585
Ohio	697,800	347,590	41.8	665,090	339,320	25.6	2,167,860	1,038,287	72.6	1,107,335
Oklahoma	260,138	133,815	10.5	429,056	228,293	23.4	451,370	240,162	42.4	1,073,643
Oregon	613,273	347,752	67.6	233,493	103.177	27.6	110,460	66,300	18.6	467,534
Pennsylvania	2,087,028	1,028,639	117.1	1,217,672	602,731	41.4	298,100	149,050	10.9	736,020
Rhode Island	93,827	46,890	2.2	81,236	40,618	.2	150,483	75,215	3.4	94,217
South Carolina	562,159	228,890	56.9	141,620	54,567	12.1	264,400	95,550	22.7	408,575
South Dakota	16,243	8,940	4.1	11,056	6,088					1,284,534
Tennessee	811,584	352,928	31.7	146,416	73,208	.7	132,927	66,463	7.9	1,005,907
Texas	2,302,667	1,123,298	256.1	1,072,614	527,895	112.1	325,886	158,935	41.0	1,444,944
Utah	224,185	126,765	38.5	121,120	76,528	8.3				297,598
Vermont	143,680	69,603	5.6	164,292	50,336	6.3	105,500	47,800	3.3	81,849
Virginia	652,124	316,930	65.5	395,980	168,420	12.8	233,720	108,528	20.8	357,021
Washington	588,899	305,893	43.1	274,792	144,118	16.9	97,856	51,200	4.8	402,756
West Virginia	145,150	72,575	8.3	300,115	150,057	15.9	141,360	70,680	7.8	461,938
Wisconsin	897,970	446,696	34.5	321,054	160,190	4.7	379,319	179,040	4.4	825,785
Wyoming	470,702	286,620	26.0	329,482	195,846	23.5	53,943	34,052	15.5	189,322
District of Columbia	98,700	49,350	1.0	18,992	8,996	.2	28,500	14,250	.3	73,654
Hawaii	89,392	44,391	3.8	370,944	185,976	11.0				162,758
Puerto Rico				224,465	109,130	12.8	119,048	57,380	4.3	103,118
TOTALS	30,330,465	15,381,103	2,601.3	19,422,170	9,462,556	1,116.1	13,209,812	6,273,244	972.9	31,501,498

---

* Includes apportionment for Fiscal Year 1941.

U. S. GOVERNMENT PRINTING OFFICE: 1940

PUBLICATIONS of the PUBLIC ROADS ADMINISTRATION

(Formerly the *BUREAU OF PUBLIC ROADS*)

Any of the following publications may be purchased from the Superintendent of Documents, Government Printing Office, Washington, D. C. As his office is not connected with the Agency and as the Agency does not sell publications, please send no remittance to the Federal Works Agency.

ANNUAL REPORTS

Report of the Chief of the Bureau of Public Roads, 1931. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1933. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1934. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1935. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1936. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1937. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1938 10 cents.

Report of the Chief of the Bureau of Public Roads, 1939. 10 cents.

HOUSE DOCUMENT NO. 462

Part 1 . . . Nonuniformity of State Motor-Vehicle Traffic Laws. 15 cents.

Part 2 . . . Skilled Investigation at the Scene of the Accident Needed to Develop Causes. 10 cents.

Part 3 . . . Inadequacy of State Motor-Vehicle Accident Reporting. 10 cents.

Part 4 . . . Official Inspection of Vehicles. 10 cents.

Part 5 . . . Case Histories of Fatal Highway Accidents. 10 cents.

Part 6 . . . The Accident-Prone Driver. 10 cents.

MISCELLANEOUS PUBLICATIONS

No. 76MP . . The Results of Physical Tests of Road-Building Rock. 25 cents.

No. 191MP . . Roadside Improvement. 10 cents.

No. 272MP . . Construction of Private Driveways. 10 cents.

No. 279MP . . Bibliography on Highway Lighting. 5 cents.

Highway Accidents. 10 cents.

The Taxation of Motor Vehicles in 1932. 35 cents.

Guides to Traffic Safety. 10 cents.

An Economic and Statistical Analysis of Highway-Construction Expenditures. 15 cents.

Highway Bond Calculations. 10 cents.

Transition Curves for Highways. 60 cents.

Highways of History. 25 cents.

DEPARTMENT BULLETINS

No. 1279D . . Rural Highway Mileage, Income, and Expenditures, 1921 and 1922. 15 cents.

No. 1486D . . Highway Bridge Location. 15 cents.

TECHNICAL BULLETINS

No. 55T . . . Highway Bridge Surveys. 20 cents.

No. 265T. . . Electrical Equipment on Movable Bridges. 35 cents.

Single copies of the following publications may be obtained from the Public Roads Administration upon request. They cannot be purchased from the Superintendent of Documents.

MISCELLANEOUS PUBLICATIONS

No. 296MP. . Bibliography on Highway Safety.

House Document No. 272 . . . Toll Roads and Free Roads.

Indexes to PUBLIC ROADS, volumes 6–8 and 10–19, inclusive.

SEPARATE REPRINT FROM THE YEARBOOK

No. 1036Y . . Road Work on Farm Outlets Needs Skill and Right Equipment.

TRANSPORTATION SURVEY REPORTS

Report of a Survey of Transportation on the State Highway System of Ohio (1927).

Report of a Survey of Transportation on the State Highways of Vermont (1927).

Report of a Survey of Transportation on the State Highways of New Hampshire (1927).

Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio (1928).

Report of a Survey of Transportation on the State Highways of Pennsylvania (1928).

Report of a Survey of Traffic on the Federal-Aid Highway Systems of Eleven Western States (1930).

UNIFORM VEHICLE CODE

Act I.—Uniform Motor Vehicle Administration, Registration, Certificate of Title, and Antitheft Act.

Act II.—Uniform Motor Vehicle Operators' and Chauffeurs' License Act.

Act III.—Uniform Motor Vehicle Civil Liability Act.

Act IV.—Uniform Motor Vehicle Safety Responsibility Act.

Act V.—Uniform Act Regulating Traffic on Highways.

Model Traffic Ordinances.

A complete list of the publications of the Public Roads Administration (formerly the *Bureau of Public Roads*), classified according to subject and including the more important articles in PUBLIC ROADS, may be obtained upon request addressed to Public Roads Administration, Willard Bldg., Washington, D. C.

STATUS OF FEDERAL-AID GRADE CROSSING PROJECTS

AS OF JANUARY 31, 1940

STATE	COMPLETED DURING CURRENT FISCAL YEAR					UNDER CONSTRUCTION					APPROVED FOR CONSTRUCTION					* BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
	Estimated Total Cost	Federal Aid	NUMBER			Estimated Total Cost	Federal Aid	NUMBER			Estimated Total Cost	Federal Aid	NUMBER			
			Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise			Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise			Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise	
Alabama	$ 930,463	$ 917,059	9	2		$ 362,507	$ 361,884	8			$ 529,100	$ 529,100	3		5	$ 885,316
Arizona						518,061	515,813	4								400,746
Arkansas	184,062	184,017	3			669,853	663,348	3			108,542	108,542	3		9	1,032,917
California	1,156,964	1,156,419	8		1	1,122,970	960,073	6			269,114	269,114	2			1,849,410
Colorado	623,127	612,007	5		14	300,054	300,054	1		4	19,543	15,783			7	901,562
Connecticut						194,055	182,342		1	7	359,115	358,023	4			719,748
Delaware	7,839	7,839	2								96,433	96,433			16	568,269
Florida	428,094	428,094	2			196,653	192,154	2			23,917	23,917			6	1,438,713
Georgia	196,480	196,480	5			538,033	538,033	9	3	1	467,004	467,004	4	4	20	2,397,613
Idaho	204,578	172,785	3			110,538	110,176	1			254,160	223,685	3			486,062
Illinois	2,631,324	2,572,286	17	4	49	1,372,749	1,331,377	5		18	1,075,099	879,230	7		94	2,460,652
Indiana	756,020	756,020	3	1	57	556,928	556,928	3		34	295,372	295,372		2	59	1,452,673
Iowa	749,696	681,831	11		96	485,950	457,100	4		88	458,470	376,516	2		89	1,408,098
Kansas	933,613	933,613	11		5	560,421	560,421	7			450,865	450,865	6		12	1,296,522
Kentucky	596,662	592,361	8	4	5	687,789	687,789	6			321,466	321,466	4		18	855,091
Louisiana	279,995	279,978	3			674,302	620,806	6			627,885	570,153	11			789,393
Maine	331,672	329,136	3	2		206,701	206,701	2	1		924	924			1	437,590
Maryland	94,896	85,789	1		3	184,009	184,009	1		2	523,600	435,807	2	1	10	821,925
Massachusetts	400,519	399,288	2	2		122,047	122,014	2			14,320	14,320	1			2,321,561
Michigan	825,839	823,499	6	2	26	800,810	800,810	4		11	609,430	609,430	3	3	12	1,762,190
Minnesota	506,912	480,951	4	4	12	1,441,986	1,440,892	10	2	5	122,930	122,930		1	2	1,668,077
Mississippi	246,300	246,300	6			491,973	491,973	4			141,700	141,700	2	1		1,123,639
Missouri	402,019	400,714	2	1		1,337,397	1,337,397	9	1		194,902	185,086		4		2,277,234
Montana	850,426	850,426	9			213,154	98,073	3			81,237	79,925	2			624,616
Nebraska	660,200	659,600	19		19	716,695	716,695	6	2	11	158,482	158,482	1		22	930,022
Nevada	196,253	196,253		2	11	25,436	24,534		1	2						248,145
New Hampshire	100,927	100,459	7		1	104,790	104,754	3			91,061	91,061	1			371,461
New Jersey	141,570	141,570	1		1	725,086	725,086	2	3		140,550	140,550	1			1,754,825
New Mexico	75,359	75,149	2		1	187,196	187,196	2			125,614	125,614	1			631,289
New York	1,832,964	1,826,676	5	7		2,148,218	2,051,118	8	9		1,185,033	1,145,053	4	5		3,982,255
North Carolina	1,196,122	1,161,022	6	5	37	474,724	474,724	7	1	11	368,715	368,715	2	1	18	1,293,445
North Dakota	527,681	479,243	7	1		395,927	395,927	5			25,170	25,170			8	904,365
Ohio	541,190	526,190	8	1	2	1,502,813	1,431,861	8	2		627,010	627,010	4		3	3,727,509
Oklahoma	275,789	275,252	4		32	304,525	303,625	3	1	11	550,509	550,509	12		37	2,231,505
Oregon	176,240	174,742	2			134,132	132,837	1								651,272
Pennsylvania	660,961	656,054	1	2		1,876,028	1,669,036	9	2	1	110,177	110,177	2		1	5,918,697
Rhode Island	442,828	442,828	1	3		7,406	7,406									287,266
South Carolina	483,689	450,307	7	4	7	359,387	348,093	3	3		232,175	232,175	1	3	26	1,088,758
South Dakota	331,694	331,694	3	2	17	75,435	75,435	1	1		47,100	47,100	1			1,399,463
Tennessee	283,757	283,757		2	5	589,158	589,158	3	2							1,899,097
Texas	1,666,101	1,634,847	15	3	6	2,312,796	2,221,492	20		5	300,316	284,463	2		15	2,884,462
Utah	220,570	220,416	3		77	152,298	152,298			50	48,460	48,460			17	369,488
Vermont	33,416	28,618			7	206,402	206,402		2							262,541
Virginia	691,979	599,079	7	3	18	206,811	206,811	3			27,561	27,561	1		2	1,394,206
Washington	293,029	291,618	3		13	226,450	226,450	1	3		89,111	89,111	3		1	759,141
West Virginia	175,781	175,781	3			214,770	199,010	6		1	9,400	9,400			3	1,360,732
Wisconsin	883,349	879,905	9		2	858,603	812,506	7	2	9	444,636	422,403	2	1	6	1,378,336
Wyoming	136,165	136,009	1	1	7						85,910	85,910	1			631,210
District of Columbia						366,812	333,268	1								193,303
Hawaii	162,720	160,090	2	1		30,682	30,682	2			236,535	236,525	2		1	261,573
Puerto Rico	49,040	48,840	1			345,312	343,310	8								650,883
TOTALS	25,576,874	25,062,891	240	59	531	27,696,832	26,689,881	209	42	271	11,948,653	11,400,774	100	26	520	67,444,866

* Includes apportionment for Fiscal Year 1941.

O'Brien