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#### Abstract

The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.


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# APPLICATION OF THE RESULTS OF RESEARCH TO THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS 

Reported by E. F. KELLEY, Chief, Division of Tests, Public Roads Administration

Shape of cross section of slab. - Two types of cross section of the pavement slab are in general use; the cross section of uniform thickness, and the cross section in which the edges of the slab are thicker than the central portion. An appreciable number of State highway departments use slabs of uniform thickness but the majority use the thickened-edge design.

Since the thickened-edge pavement design is used so extensively at the present time, the history of its development is of interest.

So far as is known, the thickened-edge section in essentially its present form was first utilized by the California Highway Commission, as an alternate to a section of uniform thickness, in the construction of concrete bases. In this design the edge depth of the slab was 2 inches greater than the interior depth, the slab thickness being reduced from the edge depth to the interior depth at a uniform rate in the outer 18 inches of pavement width. This alternate design is shown in the May 1, 1913, issue of the California Highway Bulletin and it is shown subsequently in the first and second biennial reports of the California Highway Commission (Dec. 31, 1918, and Dec. 31, 1920). In the biennial report for 1921-22 (Nov. 1, 1922) the thickened-edge cross section appears as a standard rather than an alternate design.

According to T. E. Stanton ${ }^{4}$ the alternate thickenededge section was officially adopted in November 1912, for base construction and was used for this purpose from time to time until 1921 after which it was made standard for all concrete pavement construction.

In 1920 Maricopa County, Ariz., undertook a very extensive paving program and on November 12 of that year construction was started on a contract involving 141 miles of concrete pavement, all with thickened edges (35). ${ }^{\text {b }}$ The design provided for a uniform interior thickness of either 5 or 6 inches and an edge thickness 3 inches greater than the interior thickness. The edge thickness was reduced to the interior thickness at a uniform rate in a distance of 2 feet. Thus the section was identical with that which is used today by a number of States and was similar to that now used by a majority of the States. The stated purpose of the design was to "strengthen the edge and at the same time permit simple construction of the subgrade" and to secure "a paving slab with a more uniform resisting strength" (36).

The Pittsburg Test Road at Pittsburg, Calif., was built during the summer of 1921. Traffic tests were begun that year and were finally discontinued in July 1922. The test road contained one thickened-edge section, similar to the 9-6-9-inch section used pre-

[^0]viously in Maricopa County, and in the final report (37), issued January 1, 1923, this section was given the highest rating of any of the sections included in the investigation.

The sections of the Bates Road (21) that were built in 1920 and 1921 did not include any thickened-edge design. However, sections of this design were built in the fall of 1922 and were subjected to traffic tests during 1923. The results corroborated the earlier findings of the Pittsburg tests that thickening the edges of a relatively thin pavement slab greatly increases its resistance to concentrations of heary wheel loads.
In general, two types of thickened-edge cross sections are used. In one, the upper and lower boundaries of the section are parabolic curves so arranged that the thickness gradually increases from a minimum at the center to a maximum at the edge, the edge thickness being from 2 to 3 inches greater than the center thickness. The second type, which is used by a majority of the State highway departments, is the same as that used originally by the California Highway Commission. The central portion of the slab is of uniform thickness and the edge thickness exceeds this by 2 to 3 inches. The edge section is a trapezoid, the edge thickening taking place at a uniform rate over the outer 2 to 4 feet of slab width. In the Arlington tests (17) it has been found that with this type of cross section the greatest uniformity of load stresses throughout the section may be obtained.

Another type of thickened-edge section that is used to a considerable extent is the lip-curb design. In this design a low curb of approximately wedge shape is formed along the edge of the slab. The base of the curb is generally about 12 inches wide and the height is about 3 inches. When such a curb is superimposed on a slab of uniform thickness the stress diagram for loads is very similar to that for slabs of the conventional thickenededge type in which the edge thickening is on the underside of the slab (17). However, the lip-curb design is not used primarily to strengthen the slab edge but rather as a drainage measure to prevent erosion of the road shoulders by storm water.

## EFFECT OF LOAD STRESSES ON SLAB DESIGN DISCUSSED

Use of stress analysis in design.-In introducing the discussion of the application of stress analysis to the design of pavement slabs it is well to emphasize that one of the basic assumptions of the Westergaard analyses, both for load stresses and temperature warping stresses, is that the thickness of the slab is uniform. The equations for edge stress and corner stress are not directly applicable to slabs of thickened-edge design.
With respect to interior stresses the situation is somewhat different. In the Arlington tests (17) it was found that in slabs of uniform thickness the critical stress under a load in the interior of the slab was practically
the same from the center of the slab to a point about $2 \frac{1}{2}$ feet from the edge. A similar condition was found to exist, over an even greater portion of the slab width, in thickened-edge slabs in which the edge thickness was reduced to a uniform interior thickness in a short distance and at a uniform rate. Therefore, it appears appropriate to use the equation for interior load stress both for slabs of uniform thickness and for those with thickened edges since, in the latter case, the maximum interior stresses are not affected appreciably by the edge thickening. Although test data are not available, considerations of similar character lead to the conclusion that it will be approximately correct to consider interior warping stresses in a slab of uniform thickness to be the same as in a thickened-edge slab in which the interior portion is of equal uniform thickness.

In applying stress analysis to the design of slabs of uniform thickness, curves similar to those of figure 9 may be used to determine the thickness required to resist load stresses. For example, assume that it is desired to determine the required thickness of a slab having a modulus of rupture of 700 pounds per square inch for load A, an 8,000-pound wheel equipped with high-pressure pneumatic tires. If the conservative working unit stress of 350 pounds per square inch is used, figure 9 shows that the required thicknesses for the interior, corner and edge are approximately 6.2 inches, 9 inches, and 8.6 inches, respectively. These figures indicate that if the allowable unit stress is to be limited to 350 pounds per square inch the slab should have a uniform thickness of 9 inches. However, the load stresses will not be equal in the several portions of the slab. The indicated stresses at the interior, corner, and edge of this 9 -inch slab are approximately 190,350 , and 330 pounds per square inch, respectively. On the other hand, if a less conservative unit stress is used, say 400 pounds per square inch, then the required thickness of slab, as determined by the corner stress, is approximately 8.3 inches. In this case the computed load stresses at the interior, corner, and edge of the slab are approximately 220,400 , and 370 pounds per square inch, respectively.

In the Arlington tests (17) it has been found that the thickened-edge cross section gives the nearest approach to a design that is balanced for load stresses; that is, one in which the stresses in a cross section of the slab are approximately equal for all positions of the load. It has also been found that the section which most nearly accomplishes this is of uniform thickness in the interior and has an edge thickness about 1.67 times the interior thickness, the edge thickness being reduced to the interior thickness at a uniform rate over a distance of 2 to $2 \frac{1}{2}$ feet.

At present, the only means of applying stress analysis to the design of thickened-edge slabs is to determine the interior thickness in the same manner as for slabs of uniform depth and to determine the edge thickness by the empirical relation between edge and center thickness that has been indicated by the Arlington tests.
On the basis of the same assumptions that have been made for the slabs of uniform thickness, the interior thickness required to resist load A in a thickened-edge slab is indicated to be approximately 6.2 inches if the allowable unit stress is 350 pounds per square inch and 5.7 inches if the allowable unit stress is 400 pounds per square inch. Since these dimensions are based on Westergaard's original analysis rather than on the
modified analysis of interior stresses, it will be sufficiently accurate to use interior thicknesses of 6 inches and 5.5 inches, respectively.

Multiplying these figures by 1.67 gives an edge thickness of 10 inches for the first design and 9.2 inches for the second. The data obtained in the Arlington tests indicate that the load stresses in the edge and interior of the $10-6-10$-inch cross section will be approximately balanced and equal to about 350 pounds per square inch and that the edge and interior load stresses in the 9.2-5.5-9.2-inch cross section will be approximately balanced and equal to about 400 pounds per square inch.

Permissible unit stresses.-Before discussing the design of pavement slabs to resist the combined stresses due to load and temperature warping it is desirable to consider the factors that should influence the selection of permissible maximum unit stresses. Most of these factors have been mentioned in the previous discussion.

As has been stated, consideration of the available data concerning the fatigue limit of concrete has led to the rather general practice of assuming about 50 percent of the ultimate flexural strength as a safe value of the unit stress to be used in designing pavements to resist wheel loads. In general the probable strength of paving concrete at ages greater than 28 days is not definitely known and therefore the design stress has usually been based on the 28 -day strength. Since concrete of the character used in pavements may be expected to have a flexural strength at 28 days of from 600 to 700 pounds per square inch, the customary design stress has been of the order of 300 to 350 pounds per square inch.

FOR COMBINED STRESSES, ALLOWABLE STRESS MAY EXCEED 400 POUNDS PER SQUARE INCH
As applied to load stresses this practice is a conservative one and the considerations that lead to this conclusion are:

1. The possibility that the fatigue limit of concrete, for the loading conditions that obtain in pavements, is greater than 50 percent of the ultimate strength.
2. The possibility that the stresses in pavement slabs caused by impact forces are less than those caused by static loads of the same magnitude.
3. The fact that concrete increases in strength with age and the probability that by the time the pavement has been subjected to enough repetitions of stress due to maximum wheel loads to require consideration of the fatigue limit, the concrete will have attained a strength appreciably in excess of its strength at 28 days.

The numerous investigations that have been made indicate that the rate at which concrete increases in strength after the age of 28 days is a variable that depends on several factors. The averages of the results obtained in a number of these investigations give values of the moduli of rupture at the age of 1 year that exceed the average moduli at the age of 28 days by amounts ranging from about 20 to 45 percent. Since these are average figures it is apparent that under some conditions the 1 -year strength will exceed the 28 -day strength by less than 20 percent.

It must be recognized that, for a given concrete, the 1 -year strength cannot be predicted with any certainty from test results obtained at 28 days. However, when all the factors are considered, it does not seem unreasonable to believe that in general there may be
expected a minimum increase in strength between the ages of 28 days and 1 year of the general order of 20 percent.

If the practice of limiting load stresses to about 50 percent of the 28 -day strength of the concrete is a conservative one, then the same practice would certainly be unduly conservative if applied to the design of slabs proportioned to resist the combined stresses due to load and temperature warping. The additional considerations that lead to this conclusion have been discussed previously and are:
4. The fact that vehicles having maximum wheel loads constitute a small percentage of the traffic on most roads. The occurrence of maximum stress due to load is therefore relatively infrequent and the occurrence of maximum load stress in combination with maximum warping stress is much less frequent. This is particularly true in those localities where the movement of heavy trucks is principally at night when the warping stresses that are of consequence are generally such that the combined stresses are less than the load stresses.
5. The fact that the unknown stresses due to moisture warping appear to reduce, rather than to increase, the maximum stresses due to temperature warping.

On the basis of present knowledge the five factors that have been mentioned cannot be definitely evaluated. However, when all of them are considered, it does not appear unreasonable to conclude that, when the design is based on combined stresses due to load and temperature, the safe allowable unit stress is in excess of 400 pounds per square inch and may be as high as 500 pounds per square inch.

Design of cross section for combined load and tem-perature-warping stresses.-A consideration of slab design on the basis of combined load and warping stresses leads to the conclusion that there must be either an increase in permissible unit stresses even beyond the limits that have been suggested or an acknowledgment that current practice with respect to joint spacing in nonreinforced concrete slabs is incorrect.

In the previous discussion it has been shown that, for the assumed conditions, a slab of 9 -inch uniform thickness is required if the unit load stress is limited to 350 pounds per square inch and that the thickness should be about 8.3 inches if the unit load stress is limited to 400 pounds per square inch. The combined interior and edge stresses (from figures 15 and 16) in these same slabs are shown in table 14. It will be observed that the edge stresses are always greater than the interior stresses; that in a 30 -foot slab the edge stresses are equal to or greater than 600 pounds per square inch; that in a 15 -foot slab they exceed 500 pounds per square inch except when the slab
Table 14.-Combined edge and interior stresses in slabs 10 feet wide and of uniform thickness ${ }^{1}$

| Depth of slab (inches) | Position | Length of sl: b |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 30 feet |  | 15 feet |  | 10 feet |  |
|  |  | $k=100$ | $k=300$ | $k=100$ | $k=300$ | $k=100$ | $k=300$ |
|  | $\left\{\begin{array}{l}\text { Interior }-\ldots . . . \\ \text { Edge......... } \\ \text { Interior...... } \\ \text { Edge......... }\end{array}\right.$ | $\begin{gathered} \text { Lb. per } \\ \text { sq. in. } \\ 570 \\ 650 \\ 570 \\ 660 \end{gathered}$ | $\begin{gathered} \text { Lb. per } \\ \text { sq. in. } \\ 500 \\ 600 \\ 500 \\ 600 \end{gathered}$ | $\begin{gathered} \text { Lb. per } \\ \text { sq. in. } \\ 380 \\ 460 \\ 420 \\ 510 \end{gathered}$ | $\begin{aligned} & \text { Lb. pet } \\ & \text { Eq. in. } \\ & 480 \\ & 530 \\ & 500 \\ & 560 \end{aligned}$ | $\begin{gathered} \text { Lb. per } \\ \text { Eq. in. } \\ 250 \\ 330 \\ 290 \\ 380 \end{gathered}$ | $\begin{aligned} & \text { Lb. per } \\ & \text { sq. in. } \\ & 320 \\ & 370 \\ & 350 \\ & 410 \end{aligned}$ |

[^1]

Figure 18.-Maximum Stress Diagrams for Combined Load and Warping Stresses for Two Typical Cross Sections; Slab Length 20 Feet; Based on Data From the Arlington Tests. Double Hatched Area Shows the Small Reduction Applied to the Observed Load Stress Values to Correct for the Effect of Warping.
thickness is 9 inches and $k=100$; and that it is not until the slab length is reduced to 10 feet that the edge stresses are reduced to values equal to or less than about 400 pounds per square inch.
Since, as has been stated, only the interior stresses can be computed in a thickened-edge slab, it is necessary to depend on the data from the Arlington tests for information concerning balanced design of cross section for slabs with thickened edges. Figure 18 shows such data for a 6 -inch uniform section and a 9-6-9-inch section, the load stresses in both being the stresses observed under a load of 8,000 pounds and the slab length being 20 feet.
ASSUMPTIONS NECESSARY IN APPLYING WESTERGAARD ANALYSIS TO THICKENED-EDGE SLABS
In the 6-inch uniform-thickness slab the observed load stresses of figure 18 are somewhat less than the computed stresses shown in figures 15 and 16. This is to be expected since the loads are not the same. However, the observed warping stresses of figure 18 are greater than the computed warping stresses of figures 15 and 16 even for a slab length of 30 feet. The net result is that the observed combined stresses in the 6 -inch slab, 20 feet long, of figure 18 are of about the same order of magnitude as the average values, for $k=$ 100 and $k=300$, of the computed combined stresses in the 6 -inch slab, 30 feet long, of figures 15 and 16. This is merely a demonstration of the fact that observed stresses are of the same order of magnitude as the maximum stresses obtained by theoretical analysis.

The real importance of figure 18 lies in the fact that, from the standpoint both of maximum stress and of
uniformity of stress, there is no significant difference between the thickened-edge section and the section of uniform thickness. The maximum combined stresses are approximately the same for both slabs and the stress diagrams are of approximately the same shape. Therefore, it may be concluded that for long slabs ( 20 feet or more) there is no particular advantage, from the standpoint of combined stresses at the edge and interior, of thickening the slab edges. This conclusion does not apply to the slab corners where the load stresses are greatly reduced by edge thickening and where the combined stresses do not exceed the load stresses by any great amount. With respect to short slabs (length about 10 feet) a further analysis is necessary before a conclusion can be reached.

As has already been pointed out, the Westergaard analyses for load and warping stresses do not apply to slabs with thickened edges. Therefore there is no exact analytical method available on which to base a comparison of maximum combined stresses in short slabs of uniform thickness with those in slabs with thickened edges. However, by making certain assumptions, which the data from the Arlington tests appear to justify, it is possible to make an approximate computation of stresses in thickened-edge slabs for comparison with stresses, computed by the Westergaard analyses, in slabs of uniform thickness. These assumptions are as follows:

1. That the Westergaard analyses for load and warping stresses are applicable to the interior of thickenededge slabs in which the interior portion of the slab is of uniform thickness.
2. That when the edge thickness of a thickened-edge slab is 1.67 times the interior thickness the maximum load stress at the edge is approximately the same as the maximum interior load stress.
These two assumptions have been discussed previously.
3. That the edge-warping stress in a thickened-edge slab is approximately the same as the edge-warping stress in a slab having a uniform thickness equal to the edge thickness of the thickened-edge slab.
In the Arlington tests ( $(16)$, table 4$)$ it was found that the average observed warping stresses in the edges of slabs 20 feet long and of uniform thickness were not much greater in a 9 -inch slab than in a 6 -inch slab. This result is not in accord with theory and cannot be fully explained. However, the average edge-warping stresses in a 9-6-9-inch section exceeded the average edge stresses in a slab of 6 -inch uniform thickness by about 30 percent.
By using the same assumptions that have been used previously in the computation of warping stresses, it may be shown that in a slab 20 feet long the edgewarping stresses in a 6 -inch slab of uniform thickness are approximately 240 pounds per square inch both for $k=100$ and $k=300$ and that the edge stresses in a 9 -inch slab of uniform thickness are approximately 290 pounds per square inch for $k=100$ and 360 pounds per square inch for $k=300$. The average value of 325 pounds per square inch for the 9 -inch slab exceeds the average value of 240 pounds per square inch for the 6 -inch slab by about 35 percent.

The average computed stress and the average observed stress in the 6-inch slab of uniform thickness are of about the same order of magnitude. The same is true of the computed stress in the 9-inch slab of uniform thickness as compared with the average observed stress
in the 9-6-9-inch section. Also the ratio of the computed edge stress in a 9 -inch slab to that in a 6 -inch slab is approximately the same as the ratio of the observed stress in the edge of the 9-6-9-inch section to that in the edge of the 6 -inch section. Therefore, it appears that it is a reasonable approximation to assume that in a thickened-edge slab the edge warping stress is of the same order of magnitude as in a uniform-thickness slab having the same edge depth.

Approximate interior and edge stresses, computed on the basis of these three assumptions, are shown in table 15 for three thickened-edge sections. Also shown in this table are the stresses in slabs of uniform thickness that are approximately comparable, with respect to maximum stress, with the thickened-edge designs. The three pairs of cross sections are designed for maximum combined stresses of approximately 500,425 , and 350 pounds per square inch.
Table 15.-Combined stresses in thickened-edge slabs and slabs of uniform thickness; for slabs 10 feet wide and 10 feet long ${ }^{1}$

${ }^{1}$ Assumptions with respect to load and other variables same as in figs. 15 and 16 .
THICKENED-EDGE SLAB HAS NO MARKED SUPERIORITY OVER UNIFORM-THICKNESS SLAB
It will be observed that in all cases, for slabs of this length, the maximum combined stress is less when $k=$ 100 than when $k=300$. The difference is not great in any case and, since the value of the subgrade modulus cannot be predetermined, it is considered reasonable to average the stresses for the two subgrade conditions. On the basis of these average stresses the 9-6-9-inch thickened-edge section is comparable with the section
of 7.1 -inch uniform thickness; the 10-6.8-10-inch section may be compared with the 8 -inch uniform section; and the 11.2-7.8-11.2-inch section may be compared with the 9 -inch uniform section.

Since these pairs of slabs are comparable with respect to stress they may also be compared on the basis of probable cost. In making this comparison the depth of the thickened-edge slabs will be assumed to be increased at a uniform rate from the interior thickness to the edge thickness in the outer 2 feet of slab width. Then in a mile of 20 -foot parement the amount of concrete required by the slabs of uniform thickness exceeds that required by the comparable thickened-edge slabs by approximately 260,290 and 280 cubic yards, respectively, for the slabs having uniform thicknesses of $7.1,8$, and 9 inches. When consideration is given to the additional expense involved in the construction of thickened-edge slabs, such as shaping the subgrade, shaping joint fillers, the more expensive side forms that are required, and the expense of strengthening the edges of transverse joints, it appears that there is no great difference in cost between the thickened-edge slab and the slab of uniform thickness.

In the above comparison of thickened-edge and uni-form-thickness slabs no consideration has been given to stresses due to corner loading. There are two reasons for this, the first being the very practical one that there is no accurate method available for computing either the load stresses or the wapping stresses in the corner of a thickened-edge slab.

The second reason is that in slabs of uniform thickness the corner stresses will not exceed the edge stresses except at transverse joints not provided with loadtransfer devices and at transverse cracks in nonreinforced pavements. For the uniform-thickness slabs shown in table 15 the average maximum combined corner stresses (average for $k=100$ and $k=300$ ) are 530,445 , and 375 pounds per square inch, respectively, for the $7.1-, 8$-, and 9 -inch slabs. These corner stresses exceed the comparable edge stresses by a maximum of 40 pounds per square inch. As will be shown later, any of the common types of load-transfer devices used in transverse joints may be expected to reduce corner stresses by much greater amounts than this and therefore the neglect of corner stresses in slabs of uniform thickness will not result in any overstress at transverse cracks or joints in properly reinforced slabs in which the joints are provided with some means for load transfer. The overstresses that may occur at free transverse joints or at transverse cracks in nonreinforced pavements are so small as to be negligible.

While no figures can be produced to support the argument, it is believed that the same reasoning is applicable to thickened-edge slabs and that the designs of table 15 are truly comparable even though they cannot be compared on the basis of corner stresses.

On the basis of the foregoing discussion it is concluded that, when pavement slabs are designed for wheel loads such as are commonly permitted by regulatory laws and when the combined stresses due to load and temperature warping are kept within safe limits, the thickened-edge cross section has no marked advantage over the cross section of uniform thickness.

Edge strengthening at free transverse joints.-When a free transverse joint is introduced in a thickened-edge slab, or when a transverse crack develops in a thickenededge slab that is not reinforced, a condition of relative weakness is created at the edges of the joint or crack.

This is because the central portion of the joint or crack has the same thickness as the interior of the slab but is subjected to the higher stresses which are associated with edge leading.

In table 16 are shown the maximum combined stresses at the interior, the longitudinal edge and the edge of a free transverse joint in each of the three thickenededge slabs that have already been shown in table 15 . However, in table 16 the slabs are assumed to be 30 feet long instead of 10 feet as in table 15.
In table 15, for slabs 10 feet long, the maximum stresses were shown to be approximately 500 pounds per square inch for the 9-6-9-inch section, 425 pounds per square inch for the $10-6.8-10$-inch section and 350 pounds per square inch for the 11.2-7.8-11.2-inch section. It will be noted at once, from table 16, that increasing the slab length from 10 to 30 feet has increased the stresses in the 9-6-9-inch section from a maximum of 500 pounds per square inch to 600 pounds per square inch in the interior and 760 pounds per square inch in the longitudinal edge. It will also be noted that the stresses at the interior and edge of the two heavier slabs are almost as large as in the 9-6-9-inch section. Thus, as has already been shown, the magnitude of combined interior and edge stresses in slabs as long as 30 feet is not greatly affected by variations in the depth of the slab.

Table 16.-Combined stresses in thickened-edge slabs having a width of 10 feet and a length of 30 feet ${ }^{1}$


|  | 10-6.8-10-inch section |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Interior |  | Edge |  | Edge of free transverse joint |  |
|  | $k=100$ | , $k=300$ | $k=100$ | $k=300$ | $k=100$ | $k=300$ |
| Load stress. | Lb. per <br> sq. in. 300 | Lb. per <br> sq. in. 260 | Lb. per <br> sq. in. 370 | Lb. per <br> sq. in. 320 | Lb. per sq. in. 440 | Lb. per <br> sq. in. |
| Warping stress -- | 290 590 | $\begin{aligned} & 300 \\ & 560 \end{aligned}$ | 400 770 | 400 720 | 80 520 | $\begin{aligned} & 160 \\ & 530 \end{aligned}$ |
| Average.. | 575 |  | 745 |  | 525 |  |


|  | 11.2-7.8-11.2-inch section |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Interior |  | Edge |  | Edge of free transverse joint |  |
|  | $k=100$ | $k=300$ | $k=100$ | $k=300$ | $k=100$ | $k=300$ |
| Load stress $\qquad$ Warping stress . . Combined stress <br> Average $\qquad$ | Lb. per <br> sq. in. 240 330 570 | $\begin{aligned} & \text { Lb. per } \\ & \text { sq. in. } \\ & 210 \\ & 330 \\ & 540 \end{aligned}$ | Lb. per <br> sq. in. 310 <br> 440 <br> 750 | Lb. per <br> sq. in. 270 450 720 | Lb. per <br> sq. in. 360 60 420 | Lb. per <br> sq. in. <br> 300 <br> 140 <br> 440 |
|  | 555 |  | 735 |  | 430 |  |

[^2]
## edges of transverse joints must be strengthened

Table 16 shows that in these 30 -foot slabs the stress at the edge of a free transverse joint is approximately equal to or less than the stress in the interior of the slab. This condition might be considered as evidence that there is no necessity for strengthening the edges of transverse joints in thickened-edge pavements. However, the figures presented indicate that combined edge stresses of the order of 750 pounds per square inch may be expected in slabs of this length and it may be anticipated that stresses of this magnitude will eventually result in the formation of transverse cracks. When these cracks develop, the slab length will be reduced and the combined stresses at the interior and edge will also be reduced but the reduction in slab length will have no effect on the combined stress at the edge of free transverse joints. The joint stresses are then likely to be much higher than the edge and interior stresses and should be reduced, by edge strengthening, to safe values and to values which are not excessive as compared with the stresses in other portions of the slab.

If the initial design of the slab is to be balanced so that the stresses are approximately the same in all portions of the slab, then it is necessary to reduce the slab length to about 10 feet. In order to have a balanced design it will then be necessary to strengthen the joint edges sufficiently to reduce the joint stresses from 615,525 and 430 pounds per square inch, as shown in table 16 , to 500,425 and 350 pounds per square inch, respectively, the maximum values of the edge and interior stresses shown in table 15.

Thus far the discussion has been confined to combined stresses due to load and temperature but the question of the edge strengthening at joints should also involve a consideration of load stresses only, since maximum load stresses occur much more frequently than do maximum combined stresses due to load and temperature. If the average load stresses at transverse joints of table 16 (average for $k=100$ and $k=300$ ) are compared with the average interior load stresses in table 15 it is found that the load stresses at the edges of free transverse joints exceed the interior load stresses by 105 to 140 pounds per square inch. Thus edge strengthening at the transverse joints is required if the stresses due to load are not to be more severe at joints than at the interior of the slab.

Still another reason for strengthening the edges of transverse joints is the fact, already pointed out, that wheel loads may be expected to develop higher impact reactions in the vicinity of transverse joints than in other portions of the slab.

The discussion that has been presented indicates quite definitely that, when the interior of a thickenededge slab is designed to resist either load stresses or combined stresses due to load and temperature, a condition of relative weakness will be created at the transverse joints if the edges of the joints are not strengthened.

When pavement slabs of uniform thickness are adequately designed to resist edge stresses, no edge strengthening at transverse joints or cracks is necessary. When the thickened-edge design is used the edges of joints may be strengthened by methods which will be described later. But, when a transverse crack develops in a thickened-edge pavement that is not reinforced there is developed a condition of weakness for which there is no remedy and which may eventually lead to complete failure. This possibility may be avoided by
proper design and there are two methods of design available. The first, applicable to nonreinforced pavements, requires the use of a joint spacing of the general order of 10 feet. It is probable that the expense of edge strengthening for so many joints as would be required by this design would lead to the abandonment of the thickened-edge section or the adoption of the second, or alternate, method.

The second method is to use properly designed steel reinforcement. Reinforced slabs can safely be made of any length consistent with the economical use of reinforcement suitably designed to prevent the formation of open cracks. If the design of the reinforcement is such that the stresses to which it is subjected cause either rupture or excessive elongation at the cracks which inevitably will develop, then the edge weakness at cracks will not have been remedied. However, if the reinforcement is adequate to hold the edges of the fractured slab in close contact, the crack will tend to act as a hinged joint thereby relieving the warping stresses at the edge and interior; and the interlocking of the irregular surfaces of fracture may be expected to furnish the required edge strengthening along the crack.

Longitudinal and lateral expansion and contraction.The preceding discussion of stresses due to changes in temperature and moisture content has dealt entirely with warping stresses due to a temperature or moisture gradient between the top and bottom of the slab. It is now necessary to consider general increases or decreases in temperature and moisture that are effective throughout the depth of the slab and which tend to cause corresponding changes in its horizontal dimensions.

If the slab were perfectly free to move, changes in volume would take place without restraint and no stress would be created. However, the subgrade offers considerable resistance to the horizontal movement of the slab. If the slab is attempting to contract as the result of a drop in temperature or a lowering of the moisture content, the subgrade resistance creates tensile stress. If the slab is attempting to expand, the subgrade resistance creates compressive stress. The magnitude of the tensile stress is dependent on the length of slab that is free to contract and the magnitude of the compressive stress is dependent on the distance between free expansion joints.

It has been amply demonstrated by experience that, in pavements not provided with transverse joints, both tensile and compressive failures develop. The tensile failures are evidenced by transverse cracking and the compressive failures by "blow-ups".

## COMPRESSIVE FAILURES DUE PRIMARILY TO COLUMN ACTION

It is apparent from the discussion of temperature warping that many of the transverse cracks that develop in long slabs are due to warping stress but theoretical analysis indicates definitely that some of them are due to contraction of the slab as a whole. For example, assume a pavement slab of such length that the subgrade resistance is sufficient to prevent any movement of the slab in the vicinity of its mid-length. If the concrete has a modulus of elasticity of $5,000,000$ pounds per square inch and a thermal coefficient of 0.000005 per degree Fahrenheit, a drop in temperature of only $20^{\circ} \mathrm{F}$. will create a tensile stress of 500 pounds per square inch, which exceeds by a considerable amount the probable tensile strength of the concrete.


In the same slab a rise in temperature as great as $100^{\circ} \mathrm{F}$. would create a compressive stress of only 2,500 pounds per square inch. A direct compressive stress of this magnitude should cause no distress in concrete of the quality commonly used in parements. Also, such a large change in temperature generally can be expected to take place only over a relatively long period of time and therefore it may be expected that the indicated stress will be reduced somewhat by the plastic flow of the concrete. However, the slab undoubtedly acts to some extent as a long column and its ultimate strength as a column is considerably less than its compressive strength as measured by tests on short specimens. It is believed that compressive failures are due primarily to column action rather than to direct compression and observations of pavement failures support this conclusion. Also, to the compressive stress caused by a rise in temperature must be added the unknown stresses caused by the slow "growth" of the slab that takes place over long periods of time. This growth, and the fact that changes in moisture content probably do not increase compressive stresses, will be discussed later.

Neither the magnitude of the compressive stress that may be dereloped in a long slab nor the stress to which it may safely be subjected are known. It is probable that both are variables depending on conditions. However, it is definitely known from experience that compressive failures may be expected in long slabs. The fact that these usually do not occur until the parement is several years old is an indication that the slow growth of the concrete with age is a contributing factor.

All the facts point definitely to the conclusion that, if failures are to be avoided, joints must be provided in concrete pavements to reduce to safe values the stresses due to expansion and contraction.

Spacing and width of expansion joints.-Theoretically, the spacing of expansion joints should be dependent on the allowable compressive stress in the concrete and on the maximum compressive stress created by the expansion of the slab. However, in practice the maximum spacing of joints is influenced primarily by the desirability of using a rather narrow joint opening. The practice of the various States is not uniform but, in
general, expansion joints are spaced at intervals not greater than 100 feet and, for this spacing, joint openings are usually either $\frac{3 / 4}{4}$ inch or 1 inch wide.

Open transverse cracks may be expected to derelop in nonreinforced slabs of this length and usually it is not considered economical to provide sufficient longitudinal reinforcement to prevent the formation of such cracks. Therefore, it is customary to introduce contraction joints at intervals between the expansion joints and it is convenient to make the spacing of expansion joints some multiple of the spacing of contraction joints.

In general it may be assumed that concrete pavements will be built during periods when the temperature is not more than $60^{\circ} \mathrm{F}$. below the maximum temperature to be expected. In concrete of the character that has been assumed, a rise in temperature of $60^{\circ} \mathrm{F}$. will cause an increase of approximately $3 / 8$ inch in the length of a slab 100 feet long. In a slab of this length the expansion will be restrained to some extent by the subgrade resistance and cause some reduction, probably negligible, in this computed movement of the slab ends. Also after the concrete has been placed there will be some reduction in slab length as a result of contraction due to moisture loss. Thus it might be concluded that a $3_{4}^{3 /}$-inch joint opening would be more than ample.

However, there are two other factors that hare an influence on the required joint opening. If intermediate contraction joints, or open cracks that may have developed, are not maintained in such a manner as to exclude all foreign material, the joints or cracks will gradually become filled with incompressible soil material. This action operates to increase the length of the slab and results in a reduction in the effective width of the expansion joint.

## SUBGRADE RESISTANCE AFFECTS SPACING OF CONTRACTION JOINTS

Also, in arriving at a decision as to the required width of joint opening, consideration should be given to the gradual increase in length, or "growth," of the slab that takes place over long periods of time. Figure 19 presents data obtained in the Arlington tests showing the annual variations in pavement length caused by changes
other than temperature. The data cover the period from September 1930 to February 1938. The graph indicates that there is an annual cyclic variation in length caused br variations in moisture content and that the parement slabs were longest (for a given temperature) during the winter and shortest during the summer. This would indicate that, in climates similar to that of Washington, D. C., the compressive stresses developed by high summer temperatures may be relieved somewhat by contraction due to loss of moisture and that the same action may result in some slight reduction in the width of joint opening theoretically required to provide for increase in slab length due to increase in temperature.

Howerer, figure 19 also shows that, since the summer of 1932 , there has been a definite, progressive yearly increase in the length of the pavement. In the summer of 1937 the length of the pavement exceeded its length during the summer of 1931 by approximately 0.0002 inch per inch. It is not known how long this growth will continue or at what rate. Neither is it known if the same degree of growth would take place in other concrete under other climatic conditions. However, it is known that all concrete has a tendency to increase permanently in volume in the presence of moisture.

The permanent increase in slab length that has taken place in the Arlington tests in a period of 6 years amounts to approximately $\frac{1 / 4}{4}$ inch per 100 feet. The sum of this increased length and the computed expansion due to a temperature rise of $60^{\circ} \mathrm{F}$. equals approximately $5 / 3 \mathrm{inch}$. This indicates rather definitely that a provision for expansion of $3 / 4$ inch per 100 feet is not excessive. It may even prove to be inadequate, particularly in view of the fact that a certain portion of the joint width is frequently occupied by incompressible joint filler.

Subgrade resistance.-The required spacing of transverse contraction joints in concrete parements is dependent on the allowable tensile stress in the parement and on the subgrade resistance which prevents its free contraction.

Included in the investigations by the Bureau of Public Roads have been three studies undertaken to determine the probable magnitude of the resistance offered by the subgrade to the horizontal movement of a concrete slab $(16,38,39)$. In all these investigations slabs of concrete, cast on prepared subgrades of various characteristics, were displaced horizontally over small distances and the relation between the horizontal force required to produce movement and the weight of the slab was determined. This relation is known as the coefficient of subgrade resistance. Of necessity the slabs used in all of these tests were of relatively small size as compared with pavement slabs. These studies have revealed the following facts:

1. The coefficient of subgrade resistance is not a constant but increases with increasing displacement of the slab until a maximum value is reached. This maximum corresponds to the force required to produce free sliding.
2. The resistance to movement on a very wet subgrade, which is not frozen, is less than on a dry or damp subgrade.
3. The resistance is much greater on a frozen subgrade than on one which is not frozen. This fact is probably not of great importance, at least in climates similar to that of Washington, D. C. The temperature observations made in connection with the Arlington tests showed relatively small changes in average concrete temperature during periods of cold weather. This suggests that the morements due to contraction during
cold periods may be so small that the stresses in the parement will not be increased to an important degree by a frozen subgrade.
4. For each of the first few successive applications of a given horizontal force, in repeated tests on the same slab, there is a reduction in the coefficient of resistance until an approximately constant value is reached. This indicates that the subgrade resistance may be greater for the first movement of a newly constructed pavement than it is at later ages when the concrete has expanded and contracted a number of times.
5. When a slab is subjected to a horizontal thrusting force a part of the resistance developed is due to the elastic or semielastic action of the soil. If the thrusting force is removed, even after a considerable period of time, there is a partial return of the slab to its original position.
6. The thrusting force is not directly proportional to the weight of the slab and it appears that this is due to the resistance to deformation of the subgrade. It has been concluded (16) that the subgrade resistance is composed of two elements: A resistance caused by the deformation of the soil; and a resistance that approximates that of simple sliding friction. While data are a vailable only for the one soil involved in the Arlington tests, it seems probable that the relative magnitude of the two components of the subgrade resistance will vary with different subgrade soils.

## LIMITED DATA AVAILABLE ON RELATION BETWEEN THRESTING FORCE AND SLAB DISPLACEMENT

In tables 17 and 18 are given values of the coefficient of subgrade resistance obtained in the first investigation by the Bureau of Public Roads (38) and in the Arlington tests (16), respectively. Both tables show the increase in the coefficient of resistance with an increase in the displacement of the slab. In addition, table 18 shows that, because of the resistance of the subgrade to deformation, the coefficient is not directly proportional to the weight of the slab but increases as the thickness of slab decreases.

Table 17.-Coefficients of subgrade resistance for concrete slabs of 6 -inch thickness on various kinds of bases in damp but firm condition ${ }^{1}$

| Kind of hase | Coefficients of resistance for displacements of- |  |  |
| :---: | :---: | :---: | :---: |
|  | 0.001 inch | 0.01 inch | 0.05 inch |
| Level clay | 0.55 | 1.30 | 2. 07 |
| Uneren clay | . 5.7 | 1. 29 | 2.07 |
| Loam | 34 | 1. 18 | 2.07 |
| Level sand. | . 69 | 1. 24 | 1. 38 |
| 3 -inch gravel | . 52 | 1. 10 | 1. 26 |
| 3 -4-inch crushed stone | . 44 | . 92 | 1. 09 |
| 3 -inch crushed stone. | 1. 84 | 1. 78 | 2.18 |

${ }^{1}$ Data from table 1, p. 20, ptiblic ROaDs, July 1924
TABLE 18.-Coefficients of subgrade resistance for concrete slabs of different thichnesses on a silt loam soil (class A-4) ${ }^{1}$

| Slah thickness (inches) | Coefficients of resistance for displacerrents of- |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.01 inch | 0.12 inch | 0.03 inch | 0.104 inch | 0.07 inch | 0.10 inch $^{2}$ |
| 8. | 0.8 | 1.2 | 1. 5 | 1.8 | 2.1 | 2. 2 |
| ${ }^{4}$. | 9 | 1.3 | 1.6 | 20 | 24 | 2. 5 |
| 4. | 1.1 | 1.5 | 1.8 | 2. 2 | 2.4 | 3.1 |
| 2 | 1. 3 | 1.7 | 2.1 | 2. 5 | 3.3 | 3. 5 |

[^3]

Figure 20.-Comparison of Actula and Approximate Curves Showing Relation Between Coefficient of Subgrade Resistance and Horizontal Displacement.

Stresses due to contraction. ${ }^{5}$ - When a pavement slab contracts, the total forces developed by the resistance of a uniform subgrade will be equal and opposite in each half of the slab and theoretically no movement will take place at the center line. The total displacement due to contraction then increases at a nearly uniform rate from zero at the center line to a maximum at the end of the slab. Since the subgrade resistance varies with the displacement it is apparent that an accurate analysis of slab stress should take account of the subgrade resistance corresponding to the total displacement of each increment of slab length.

Utilizing the data obtained in the tests with the 6 -inch slab of table 18 , such a method of analysis is illustrated in the report of the Arlington tests (16), the stresses being those due to an assumed change in temperature of $100^{\circ} \mathrm{F}$. As will be shown later this temperature change is excessive when applied to the computation of stresses in slabs provided with joints at reasonable intervals but the principles of the analysis are correct.

An exact analysis of this character requires the use of test data showing the relation between thrusting

[^4]force and slab displacement and therefore is applicable only when such data are available. However, if it may be assumed that the general shape of the forcedisplacement curve will be similar under all conditions, then a simple approximate method of analysis may be developed for general use. The available data are limited and it is recognized that the relation between thrusting force and slab displacement may be different at different locations, depending largely on the character of the subgrade. However, the approximate method that will be presented gives results that appear to be reasonable and it is believed that its use will not involve any serious errors.
The solid curves of figure 20 show the force-displacement relation, as developed in the Arlington tests, for slabs of four thicknesses. The curves are the same as those of figure 20, PUBLIC ROADS, November 1933. The dotted lines represent an approximation of the actual force-displacement relation. The curved portion of each dotted line is a parabola, with vertex at the origin, passing through the point having an ordinate equal to the maximum coefficient of subgrade resistance which, in these tests, was developed at a displacement of approximately 0.10 inch , and having an abscissa equal to a displacement of 0.06 inch. In comparison with these test results the approximate force-


Figure 21.-Approximate Variation in Value of the Coefficient of Subgrade Resistance From the Center to the End of a Pavement Slab.
displacement curves are conservative since, in general, they give values of the subgrade coefficient that are greater than the test values.

At a given distance from the center of a pavement slab, a given drop in temperature will result in a certain movement due to contraction and, theoretically, the subgrade resistance which is developed should be that corresponding to this movement. At the center of the slab the movement and the corresponding resistance are zero. As the distance from the center of the slab is gradually increased the contraction movement, due to a given drop in temperature, and the corresponding coefficient of subgrade resistance are also gradually increased until, if the slab is long enough, a point is reached at which the subgrade coefficient reaches a maximum and constant value. An average value of this variable subgrade coefficient may be determined and, for the computation of the maximum contraction stress at the center of the slab, this average value may be considered as applied over the entire length of slab.

## maximum contraction stresses occur during a period of CONTINUOUSLY FALLING TEMPERATURE

On the assumption that the force-displacement relation is as shown by the dotted lines of figure 20, figure 21 shows the variation in the value of the coefficient of subgrade resistance along the length of a pavement slab. In this figure, $X$ equals the distance from the center of the slab to the point where the transition from the parabolic variation to a constant value occurs. Case $I$ is that in which the distance $X$ is less than half the slab length and Case II is that in which $X$ is greater than half the slab length.

The distance $X$, in feet, is determined by the equation

$$
\begin{equation*}
X=\frac{D}{12 T e} \tag{21}
\end{equation*}
$$

in which
$D$ =assumed minimum displacement, in inches, at which the maximum value of the coefficient of subgrade resistance is developed;
$T=$ the temperature drop, in degrees F .;
$e=$ thermal coefficient of contraction per degree.
$D$ has already been assumed as 0.06 inch and if, as in previous examples, $e$ is assumed equal to 0.000005 , then

$$
\begin{equation*}
X=\frac{1,000}{T}(\text { feet }) \tag{22}
\end{equation*}
$$

The equations for the average value of the coefficient of suhgrade resistance are as follows:

Case I, $X$ less than $\frac{L}{2}$

$$
\begin{equation*}
C_{a}=C_{m}\left(1-\frac{2 X}{3 L}\right) \tag{23}
\end{equation*}
$$

Case II, $X$ greater than $\frac{L}{2}$

$$
\begin{equation*}
C_{a}=\frac{2 C_{m}}{3} \sqrt{\frac{L}{2 X}} \tag{24}
\end{equation*}
$$

in which
$C_{a}=$ average value of the coefficient of subgrade resistance;
$C_{m}=$ maximum value of the coefficient of subgrade resistance;
$L=$ free length of slab, in feet, for computation of longitudinal forces and free width of slab, in feet, for computation of transverse forces.
With respect to the type of resistance to slab movement that is offered by the subgrade, it appears that subgrades may be divided into two general classes: those which have some elasticity, such as the subgrades involved in the Arlington tests, and those which have no elasticity as, for example, sand.

When a pavement slab on a partially elastic subgrade contracts as a result of a decrease in temperature, the tensile stress that is created may be considered as being developed in three successive increments. The first increment of stress is due to the resistance of the subgrade to elastic deformation, the second is due to the resistance to inelastic deformation, and the third is due to the resistance developed by sliding friction. If the slab displacement is small, only the resistance to elastic deformation may be developed, but large displacements will develop all three increments of stress. If the subgrade has no elasticity the stress developed is due only to the resistance to inelastic deformation and to frictional resistance.

When the temperature has reached a minimum the slab ceases to shorten and, since the movement ceases, the stress due to inplastic deformation and frictional resistance is immediately reduced to zero. In the case of the semielastic subgrade, that portion of the stress caused by resistance to elastic deformation remains in the slab until it is relieved by expansion due to an increase in temperature. As the temperature gradually increases from the minimum, the tensile stress created by the resistance to elastic deformation is gradually reduced and is completely relieved when the temperature reaches its initial level.

If the temperature does not return to its initial upper level, a residual tensile stress remains in the slab. The total stress in the slab, after another drop in temperature equal to that which occurred during the first cycle, may therefore be somewhat greater than that which was developed during the first cycle. Also, if the slab length is such that large changes in temperature produce small displacements, the resistance of the subgrade to elastic deformation may not be exceeded until there have occurred several cycles of temperature change during which the level of the minimum temperature has decreased.

It is apparent from this discussion that the maximum contraction stress in a pavement slab is not dependent on the annual change in temperature. Rather it is dependent on the subgrade resistance that can be de-
veloped during a single period of continuously falling temperature or, at most, during a relatively few cycles of temperature change in which the general level of the minimum temperatures is decreasing. Since many subgrade soils are not elastic and since the degree of elasticity that has been observed is rather small, it is believed that the changes in slab temperature that take place during successive cycles are of considerably less importance than the drop in slab temperature which may take place during any one day.

## MAXIMUM DAILY RANGE IN AVERAGE SLAB TEMPERATURE ASSUMED AS $40^{\circ} \mathrm{F}$.

The daily change in average slab temperature is dependent on the daily change in air temperature and the relation between the two is influenced by the season of the year and by the particular climatic conditions that happen to obtain when the comparison is made.
In the Arlington tests it was found that, in general, the maximum daily change in the average temperature of the slab was considerably less during the cold months of the year than during the warm months. However, there were numerous occasions during the winter when the daily change in air temperature was as great as during the summer. Therefore, the lower daily change in slab temperature during the winter may be attributed to a lesser absorption of solar heat, since during this period the rays of the sun strike the pavement at a relatively low angle of incidence. This is a matter of importance when the attempt is made, on the basis of daily changes in air temperature, to establish for design purposes the maximum daily change in slab temperature.

Unpublished data obtained in the Arlington tests during the period from April to September, inclusive. on a number of selected days when the change in average slab temperature was relatively high, show that the daily change in the average temperature of a 6 -inch slab was generally less than the daily change in air temperature. However, in a number of cases the difference was so small as to be negligible and in a few cases the change in slab temperature exceeded the change in air temperature by as much as $5^{\circ} \mathrm{F}$. The maximum observed daily change in the average temperature of a 6 -inch slab was $32^{\circ} \mathrm{F}$. on a day when the change in air temperature was $47^{\circ} \mathrm{F}$. Very little information is available concerning the relation between slab temperature and air temperature in slabs having a thickness greater than 6 inches. Apparently the daily change in the average temperature of thick slabs is always less than in thin ones and the few data that are available from the Arlington tests indicate that the daily range in average temperature in a 9 -inch slab is about 80 percent of that in a 6 -inch slab.

In table 19 are given the maximum ranges in air temperature that occurred during the years 1936 to 1938, inclusive, at selected cities in the United States. Excluding a few extremely high values that were observed during the winter months, it will be seen that a maximum daily range in air temperature of the order of $45^{\circ} \mathrm{F}$. is of rather general occurrence except along the Pacific Coast, in some of the southern States, and in certain areas in the northeastern States. In the light of these data and the preceding discussion it is concluded that it will be conservative to assume, for general use in the United States, a maximum daily range in average slab temperature of $40^{\circ} \mathrm{F}$. and that the climatic conditions in certain areas justify the use of a somewhat lower value.

Table 19.-Greatest daily range in air temperature for selected cities, 1936 to 1998, inclusive ${ }^{1}$

| City | Greatest daily temperature range for year |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1936 |  | 1937 |  | 1938 | Aver age |
|  | ${ }^{\circ} \mathrm{F}$. | Month | ${ }^{\circ} \mathrm{F}$. | Month | ${ }^{\circ} \mathrm{F}$. | Month | ${ }^{\circ} \mathrm{F}$. |
| Seattle, Wash | 31 | Aug | 34 | Selst | 33 | Feb_ | 33 |
| Portland, Oreg | 37 | Apr | 36 | May | 36 | Sept | 36 |
| San Francisco, Cali | 32 | Sept.-Oct. | 34 | Sept | 35 | Sept | 34 |
| Los Angeles, Calif | 39 | Oct.-..... | 32 | Oct | 31 | Aug | 34 |
| Reno, Nev...... | 44 | July-Aug.- | 45 | July | 42 | Sept. 0 et | 44 |
| Phoenix, Ariz | 42 | June-OctNov. | 43 | May | 43 | Apr. | 43 |
| Salt Lake City, Utah | 46 | May. | 44 | Aug.-Sept | 41 | July | 44 |
| Helena, Mont | 58 | Dec | 45 | Feb. | 48 | Dec | 50 |
| Bismarck, N. Dak | 46 | Oct | 52 | Jan | 45 | Aug | 48 |
| Denver, Colo | 60 | Feb | 41 | Apr | 48 | Jan | 50 |
| Albuquerque, N. Me | 47 | A pr | 45 | Apr | 49 | Mar. | 47 |
| Omaha, Nebr | 46 | Apr | 44 | Oct | 46 | Feb--Oct | 45 |
| St. Louis, Mo | 48 | Jau | 44 | Jan | 45 | Jan. | 46 |
| Chicago, 111. | 47 | Apr | 39 | Apr | 39 | Apr | 42 |
| Indianapolis, Ind | 49 | Jan | 39 | Feb | 35 | Mar | 41 |
| Washington, D. C | 44 | Mar | 45 | Apr | 37 | May | 42 |
| Rochester, N. Y | 37 | May June | 35 | Jan. | 36 | Feb | 36 |
| Portland, Maine | 42 | May .-...- | 36 | May | 40 | Apr | 39 |
| Little Rock, Ark | 40 | Apr | 38 | Jan. | 37 | Jan. | 38 |
| Atlanta, Ga | 44 | Apr | 42 | Seit | 37 | Mar | 41 |
| Houston, Tex | 41 | Nov | 40 | Jan | 33 | Jan | 38 |
| Mobile, Ala | 31 | Feb.-July Oct--Dee. | 32 | Mar | 34 | Nov. | 32 |
| Miami, Fla | 25 | Mar.-Nov | 27 | Dec. | 27 | Oct | 26 |

Data obtained from the U. S. Weather Bureau.
Having established a basis for computing the value of the average coefficient of subgrade resistance, an analysis may be made to determine the maximum contraction stress in a pavement slab.
For a slab without reinforcement the maximum contraction stress is given by the equation

$$
\begin{equation*}
\sigma_{s}=\frac{W L C_{a}}{24 h}- \tag{25}
\end{equation*}
$$

in which
$\sigma_{s}=$ tensile stress in concrete in pounds per square inch;
$W=$ weight of slab in pounds per square foot;
$L=$ length of slab in feet;
$h=$ depth of slab in inches;
$C_{a}=$ average value of the coefficient of subgrade resistance as determined by equation 23 or equation 24.
For an assumed drop in average slab temperature of $40^{\circ} \mathrm{F}$., the distance $T$ as determined by equation 22 is 25 feet. For a value of $L=100$ feet the calculated value of $C_{a}$ (equation 23) is $0.83 C_{m}$. In table 18 the maximum observed value of the coefficient of subgrade resistance, $C_{m}$, for the 6 -inch slab is shown to be 2.5 . Then for a 6 -inch slab having a length of 100 feet and a weight of 75 pounds per square foot,
$\sigma_{s}=\frac{75 \times 100 \times 0.83 \times 2.5}{24 \times 6}=108$ pounds per square inch.
CONSTRUCTION PRACTICES TO REDUCE SUBGRADE RESISTANCE NOT EFFECTIVE IN REDUCING TRANSVERSE CRACKING
One of the more recent investigations of the tensile strength of concrete (40) indicates that concrete of the quality used in pavements, if thoroughly cured for : period of 28 days, may be expected to have a tensile strength at that age of the order of 200 to 250 pounds per square inch. When the computed contraction stress of 108 pounds per square inch in a slab 100 feet. long is compared with a probable 28-day tensile strength of at least 200 pounds per square inch, it seems very probable that, in pavements provided with transverse joints at reasonable intervals, any transverse cracking,
except that which may occur at very early ages, must be attributed primarily to the effect of warping stresses.

If this is true, it follows that the difference in degree of cracking that is observed in pavements constructed with different aggregates is due not so much to differences in the strength of the concrete as to differences in modulus of elasticity, thermal coefficient of expansion and, possibly, to differences in thermal conductivity that may affect the magnitude of the temperature differentials.
Some evidence of this is found in the records of the old Ohio Post Road which was constructed in 1914 and 1915 (41). In a part of the project the concrete aggregate was gravel and in the remainder it was crushed stone. Samples of concrete were taken from the pavement in 1932 and the compressive and flexural strengths determined. Both the gravel concrete and the crushed-stone concrete had compressive strength of approximately 6,600 pounds per square inch. The modulus of rupture of the specimens of gravel concrete was 1,150 pounds per square inch and that of the specimens of crushed-stone concrete was 1,030 pounds per square inch. Yet, in a given length of pavement, the transverse cracks in the gravel concrete were much more numerous than in the crushed-stone concrete Tests made in recent months indicate that the gravel concrete has a higher modulus of elasticity and higher thermal coefficient of expansion than the stone concrete. On the assumption that the temperature differential is the same in both kinds of concrete, the differences in the values of modulus of elasticity and thermal coefficient are sufficient to account for warping stresses 25 percent higher in the gravel concrete than in the stone concrete.

In the light of the foregoing discussion it also seems very probable that any special construction practices designed to reduce the subgrade resistance, and thereby reduce or eliminate transverse cracking, will not be particularly effective for the purpose. The limited experimental data that are available support this conclusion.

Some years ago it was observed in western Iowa that extensive hair cracking developed during the curing period in concrete pavements constructed on the loess soils that are prevalent in that area and in other portions of the valleys of the Missouri and Mississippi Rivers (42). These loess soils, unless saturated, are highly water absorbent. The hair cracking, which is caused by contraction, was attributed to the rapid drying of the concrete owing to excessive water absorption by the subgrade soil. It was found that a layer of tar paper, placed on the subgrade before the placing of the concrete, was quite effective in preventing this excessive loss of water and in eliminating the formation of hair cracks.

Since the development of the tar-paper subgrade treatment in Iowa it has been used extensively in other States. In some cases it has been used rather generally on all soils without regard to their capacity to absorb water from the concrete and apparently this practice has been influenced somewhat by the belief that the treatment would lower the subgrade resistance sufficiently to have a beneficial effect in the reduction of transverse cracking.

The effect of the tar-paper treatment was studied to a very limited extent in one of the investigations by the Bureau of Public Roads (39). This investigation, made primarily to study methods of curing concrete, involved the construction of a number of long concrete
slabs. Included in these were two slabs, each 6 inches deep, 2 feet wide, and 200 feet long, that were cured in the same manner. The only difference between them was that one was placed on a dry soil and the other was placed on tar paper. The slabs were constructed during the summer of 1926.

In connection with the same investigation a determination was made of the effect of the tar-paper treatment on subgrade resistance. It was found that for small displacements of the test slabs the resistance was about the same for a slab on a dry subgrade as for one on tar paper. However, for displacements of the order of 0.05 inch it was found that the resistance developed by the dry subgade was about twice that which was developed with the tar-paper treatment.

In spite of this difference in subgrade resistance the 200 -foot slab on the dry subgrade contained only 4 transverse cracks at the age of 5 days while at the age of 2 days the 200 -foot slab on tar paper contained 6 transverse cracks. A survey made during the summer of 1938, when the slabs were about 12 years old, showed 11 cracks in the slab built on the dry subgrade and 15 cracks in the slab built on tar paper.

Thus, while the tar-paper treatment of the subgrade is undoubtedly effective for the purpose for which it was originally used, both theory and experiment point to the conclusion that it has no merit as a means for preventing the transverse cracking of pavements.

## STEEL REINFORCEMENT BENEFICIAL IN CONCRETE PAVEMENT

 SLABSUse of steel reinforcement. - It has been pointed out previously that, if detrimental cracking is to be prevented in thickened-edge pavements, the use of steel reinforcement is an alternate to the use of very short slabs with edge strengthening at all transverse joints. It has also been stated that in slabs of uniform thickness, adequately designed to resist edge stresses, no edge strengthening at transverse joints or cracks is required. While this is true, it should not lead to the conclusion that it will necessarily be safe to build long slabs of uniform thickness with the idea that the formation of open transverse cracks will not be detrimental.

In New Jersey (43) and elsewhere it has been observed that, even when the edge strength at transverse joints is adequate, trouble may develop at the joint from other causes unless the two slab ends are connected in such manner that the deflection of each will be approximately equal under the action of heavy wheel loads. In the absence of such a connection between the slab ends it has been found that, under certain conditions of soil and drainage, the end of the slab which is on the side of the joint opposite the approaching wheel load is gradually forced permanently below the level of the adjacent slab. This results in poor riding quality, increased impact reactions, and the eventual development of pavement failure in the vicinity of the joint. While this experience does not appear to be universal, it suggests that, at least under some conditions, the use of steel reinforcement in long slabs of uniform thickness may be beneficial in preventing the faulting that might otherwise develop at transverse cracks.

Design of reinforcement.-For a reinforced slab the same assumptions that are used in the derivation of equation 25 leads to the equation

$$
\begin{equation*}
A=\frac{W L C_{a}}{2 f_{0}}- \tag{26}
\end{equation*}
$$

in which $W$ and $C_{a}$ are the same as in equation 25 , and $L=$ distance in feet between free joints (spacing of free transverse joints for computing longitudinal steel, and spacing of free longitudinal joints for computing transverse steel);
$A_{s}=$ effective cross-sectional area of steel in square inches per foot of slab width;
$f_{s}=$ allowable unit tensile stress in the reinforcement, in pounds per square inch.
If the steel reinforcement is to maintain in a tightly closed condition the warping cracks that will develop, it is necessary to limit its elongation at cracks to a very small amount. The total elongation of steel subjected to tensile stress is dependent on the length that is free to elongate. The reinforcement in a concrete pavement initially is in bond with the concrete and, when a crack forms, the bond is destroyed over a certain length of steel. This length is then free to elongate under the stress induced by the subgrade resistance. However, the length over which the bond is destroyed is not known and, therefore, it is impossible to compute accurately the total elongation corresponding to a given stress. This, in turn, makes it impossible to determine with accuracy the maximuin allowable stress in the steel that will insure the maintenance of tightly closed cracks.
It is common practice to base the design of steel members on an allowable unit stress which is considerably less than the yield point of the steel. This is to minimize the possibility of elastic failure due to the occurrence of unforeseen stresses greater than those used in design. The practice is a logical one to follow but, in the case of slab reinforcement, the maximum permissible elongation should also be considered.

Slab reinforcement should be designed to limit the maximum width of cracks that may develop to a small dimension. But the crack width is dependent on the elongation of a certain length of steel and this elongation is in turn dependent, not on the strength of the steel, but on its modulus of elasticity and the unit stress to which it is subjected. Since all grades of reinforcing steel have approximately the same modulus of elasticity, it follows that the elongation in a given length is independent of the grade and varies only with the unit stress. Therefore, in the determination of a safe allowable unit stress, consideration should be given both to the yield point and to the maximum permissible elongation. However, as has been stated, the elongation corresponding to a given unit stress cannot be determined because the length of reinforcement that is free to elongate is not known. In addition, nothing definite is known concerning the maximum width of crack that can be permitted without the development of edge weakness.

In view of these considerations the best that can be done, until more information becomes available, is to select maximum allowable unit stresses that appear to be reasonably conservative when considered in relation to the yield point of the steel. Having done this, it is then possible to compute elongations that may he developed under certain assumed conditions.

SAMILECALCUIATION OF AMOUNT OF REINFOR(EMEN'TREQUIREI) IN A PAVEMENT SLAB
The standard specifications of the American Society for Testing Materials require minimum yield points in the various grades of reinforcing steel, as follows:


There is precedent for the use of an allowable working unit stress in steel equal to 50 percent of its minimum allowable yield point and the adoption of this value is suggested, pending the development of the information that is required for a more logical determination. In table 20 are shown computed elongations for the different grades of reinforcing steel, on the basis of this suggested unit stress, for assumed lengths of free elongation of 12, 18, and 24 inches.

The figures of table 20 indicate that if the steel is free to elongate over a length as great as 24 inches, the stresses permitted in the higher-strength steels are likely to result in the formation of open cracks having a width as great as 0.02 inch. On the other hand, the elongation in this length will not greatly exceed 0.01 inch for a unit stress of the order of 16,000 pounds per square inch. The data from the Arlington tests give some indication that an opening of 0.02 inch may result in some reduction in edge strength at a crack in a reinforced slab but the evidence is by no means conclusive.

Table 20.-Elongation of steel reinforcemenl ${ }^{1}$

| Grade of steel | Unit stress 50 percent of yield point | Elongation in a length of- |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 12 inches | 18 inches | 24 inches |
| Structural | Lb. per sq. in. 16, 500 | Inches $0.007$ | Inches <br> 0.010 | Inches |
| Intermediate | 20, 000 | . 008 | . 012 | . 016 |
| Hard and rail steel | 25, 000 | . 010 | . 015 | . 020 |
| Cold-drawn wire. | 28,000 | . 011 | . 017 | . 022 |

${ }^{1}$ Modulus of elasticity of steel $=30,000,000$ pounds per square inch.
Certainly a crack opening of 0.01 inch is less likely to create edge weakness than an opening of 0.02 inch, but the adoption of the lower limitation would require the use of a low unit stress for all grades of steel. This, in turn, would require the use of much greater amounts of steel than are commonly used and, since the necessity for it is not definitely indicated, the adoption of the low unit stresses would hardly be justified at the present time.

It will now be of interest to determine, from the preceding equations, the amount of reinforcement required in a pavement slab. The following assumptions will be made. The pavement is 20 feet wide with a longitudinal joint with bonded tie bars; the transverse joints are 50 feet apart; the slab is 8 inches thick and weighs 100 pounds per square foot; the maximum drop in temperature is $40^{\circ} \mathrm{F}$.; the value of $C_{m}$ (table 18) is 2.2 ; and the reinforcement will be welded wire fabric with an allowable unit stress of 28,000 pounds per square inch.
$X=25$ feet, and for the stress in the longitudinal direction $C_{a}$, as determined either by equation 23 or equation 24, equals $0.67 C_{m}$. By the use of equation 26 it is then found that the required cross-sectional area of longitudinal steel is 0.132 square inch per foot of slab width. For stress in the transverse direction $L=20$ and $C_{a}$, as determined by equation 24 , equals $0.42 C_{m}$. Then the required cross-sectional area of the transverse steel, as determined by equation 26, equals 0.033 square inch per foot of slah width. These requirements may be met by No. 3 -gage longitudinal
wires on 4 -inch centers ( $A_{s}=0.140$ ) and No. 5-gage transverse wires on 12 -inch centers ( $A_{s}=0.034$ ), resulting in a fabric weighing about 63 pounds per 100 square feet. ${ }^{6}$ Similar calculations for a slab 30 feet long indicate that wire fabric weighing about 37 pounds per 100 square feet is required.

In the above examples the transverse steel has been designed on the assumption that $L=20$ feet which, in turn, involves the assumption that the reinforcement is continuous through the longitudinal joint. This is not a usual condition since in common practice tie bars constitute the only reinforcement extending through the longitudinal joint.

When tie bars are used and the transverse reinforcement is interrupted at the longitudinal joint, the maximum tensile stress in the transverse steel is developed at the end of the tie bars and not at the joint. Therefore the effective value of $L$ is less than the width of pavement by an amount equal to the length of the tie bars. Since this is the case, the amount of transverse steel computed as in the foregoing examples is somewhat excessive.

Also, since longitudinal cracks in slabs 10 feet wide are the exception rather than the rule, it is believed to be entirely safe to reduce the transverse reinforcement to the minimum practicable amount. The minimum might be established as No. 6-gage wires at 12 -inch centers. The substitution of No. 6 -gage wire for the No. 5-gage wire would reduce the weight of the fabric by a little less than 2 pounds per 100 square feet.
The above calculations to determine the required amount of reinforcement are for purposes of illustration only. The results should not be considered as necessarily applicable to all conditions.

Since the total cost of transverse joints in a given length of pavement increases as the required amount of steel reinforcement decreases, it is evident that the economical design of reinforced pavements requires consideration of both factors.
joints needed to prevent cracking and to provide for expansion and contraction
Longitudinal and transverse joints.--The need for longitudinal and transverse joints in concrete pavements is demonstrated both by theory and by extensive experience. Longitudinal joints which divide the slab into lanes 10 to 12 feet in width are required to prevent the unsightly and detrimental longitudinal cracks that otherwise may be expected to develop. Transverse expansion joints are required at reasonable intervals, consistent with a rather narrow joint opening, to prevent compressive failures or blow-ups. In nonreinforced pavements, intermediate transverse contraction or warping joints are required at frequent intervals if cracks due to warping stresses are to be eliminated. In reinforced pavements the need for contraction joints is dependent on the spacing of expansion joints. The expansion joints may be placed at the ends of each reinforced slab, in which case no other transverse joints are required, or the distance between expansion joints may be made some multiple of the slab length in which case the intermediate joints are contraction joints.

Joints of numerous types and design are in use but no attempt will be made to describe all of them here. The discussion will be confined to the more common

[^5]types of joints that were investigated in the Arlington tests. These are shown in figure 22.

The devices used to connect adjoining slabs either at transverse or longitudinal joints are required for several purposes. In the case of longitudinal joints in the interior of thickened-edge slabs the joint edges require strengthening and the joint designs shown in figure $22-\mathrm{A}, \mathrm{B}$, and C are frequently used for this purpose. The transverse tie bars are bonded to the concrete and are required to prevent the separation of the slabs and the consequent loss of joint efficiency. The butt joint of figure $22-\mathrm{D}$ and the thickened-edge joint of figure 22 - E are suitable only for the so-called lane-at-a-time construction in which each width of slab is constructed separately. The butt joint may be used in the interior of thickened-edge slabs in which case the bonded tie bars are required to prevent loss of joint efficiency.
The longitudinal butt joint of figure 22-D may also be used in slabs of uniform thickness. In this case, and also in the case of the longitudinal thickened-edge joint of figure $22-\mathrm{E}$, the tie bars are not required for the purpose of edge strengthening but they are needed to prevent the separation of the slabs and the development of an unsightly appearance. The tarred felt shown in the butt and thickened-edge longitudinal joints is desirable to prevent any bond between the concrete in adjacent slabs and also to provide the play in the joint needed to relieve warping stresses.
All of the transverse expansion and contraction joints of figure 22 , with the exception of the thickened-edge joint (fig. 22-G), when used in thickened-edge slabs require the use of dowels or other devices for the purpose of edge strengthening. When these joints are used in pavements of uniform thickness, or when the thickenededge joint is used, the dowels are not needed for edge strengthening but, as has already been indicated, they may be needed under certain conditions to prevent the development of faults at the joints.
Provision for slab movement must be made in transverse joints and, in order that the dowels may be free to move, it is necessary to prevent the formation of a bond between the dowels and the concrete at least on one side of the joint. This is usually accomplished by painting or greasing the dowels, or both. Also, in expansion joints, caps or sleeves are required on one end of each dowel in order to provide space for the movement of the dowel into the slab when the joint closes. These dowel caps are not required in contraction joints.

## ideal longitudinal joint would act as a hinge

Design of tie bars.-The purpose of tie bars is to hold the edges of longitudinal joints in close contact and they may be designed in the same manner as steel reinforcement. For example, in a two-lane pavement the tie bars may be designed by means of equation 26 in which $L$ is taken as the width of pavement. If intermediate grade bars, with an allowable unit stress of 20,000 pounds per square inch, are used in the center joint of the 8 -inch uniform thickness slab for which the steel reinforcement has already been designed, the required area of steel is found to be 0.046 square inch per foot of joint. This requirement may be met by $1 / 2$-inch round bars spaced 51 inches apart.

It should be noted that tie bars designed in this manner are intended only to hold the edges of the joint in close contact and they may not be adequate in all cases to furnish the edge strengthening that is required


TRANSVERSE EXPANSION JOINTS


DOWELLED EXPANSION JOINT
F

TRANSVERSE CONTRACTION JOINTS SMOOTH ROUND DOWELS NOT IN BOND


DUMMY JOINT OR
PLANE OF WEAKNESS
I


Figure 22.-Types of Joints for Concrete Pavements.
in the longitudinal joints of thickened-edge slabs. As will be shown later, the Arlington tests indicate that longitudinal tongue-and-groove joints, provided with $1 / 2$-inch round tie bars spaced 60 inches apart, are quite effective in furnishing the necessary edge strengthening but that in longitudinal joints of the butt and dummy types it would be desirable to increase the size and number of the bars.

The depth of embedment of the tie bars in each slab should be sufficient to develop their strength in bond. The depth of embedment required to accomplish this is dependent on the allowable unit tensile stress in the steel and the allowable unit bond stress, and may be expressed by the equation.

$$
\begin{equation*}
D=\frac{f_{s} d}{4 u} \tag{27}
\end{equation*}
$$

in which
$D=$ depth of embedment in inches;
$f_{s}=$ allowable unit tensile stress in the steel, in pounds per square inch;
$u=$ allowable unit bond stress in pounds per square inch;
$d=$ diameter of a round bar, or side of a square bar, in inches.
The 1937 Progress Report of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete recommends for plain bars a unit bond stress equal to 4 percent of the ultimate compressive strength of the concrete but not to exceed 160 pounds per square
inch, and for deformed bars a unit bond stress equal to 5 percent of the ultimate compressive strength of the concrete but not to exceed 200 pounds per square inch.

For intermediate grade steel with an allowable unit stress of 20,000 pounds per square inch the required depths of embedment for the maximum bond stresses of 160 and 200 pounds per square inch are, respectively, $31 \frac{1 / 4}{4}$ diameters for plain bars and 25 diameters for deformed bars. If deformed bars are used, the maximum bond stress of 200 pounds per square inch would require the total length of a $1 \frac{1}{2}-$ inch round tie bar to be 25 inches. A lower permissible unit bond stress or a higher permissible unit stress in the steel would require the use of longer bars.

The above method for designing tie bars is predicated on the assumption that the joint is of a type that will act as a hinge and will be incapable of developing any appreciable resistance to warping. If the design is such as to permit resisting moments to develop during warping it is not possible to calculate the stresses in the tie bars and even if it were practicable to do so it would not be desirable, in a joint offering high restraint to warping, to introduce sufficient steel to take the warping stresses since this would invite failure in other portions of the slab. The ideal longitudinal joint that acts wholly as a hinge has not yet been developed but by proper attention to the details of design it is possible to effect some reduction in the warping stresses that are caused by restraint in the joint.

In longitudinal joints that contain bonded tie bars
the use of a design that does not permit the development of large resisting moments is desirable not only to reduce transverse warping stresses in the pavement as a whole, but also to reduce compressive stresses in the concrete at the joint and to prevent the tie bars from being overstressed in tension.

If restraint to warping is to be reduced it is necessary to prevent the abutting faces of the joint from being brought into close contact during warping, particularly at the top and bottom of the joint. In the butt joints of figure 22-D and E this may be accomplished by the introduction of a compressible layer of filler material between the slab edges.

The use of filler material throughout the depth of the joint would not be practicable in the dummy joint of figure $22-\mathrm{C}$. In this joint the resistance to downward warping is reduced by the groove in the top of the slab and it would appear that the most practical way to reduce the resistance to upward warping would be to form a similar groove in the bottom of the slab.

In the tongue-and-groove joints of figure $22-\mathrm{A}$ and $B$ the use of a compressible filler for the full depth of joint would be undesirable since it would reduce the ability of the joint to transfer load and to reduce edge stresses. However, strips of filler fastened to the vertical portions of the steel partition plates should be quite effective in reducing joint restraint without greatly reducing joint efficiency.

Even under the most favorable conditions it does not appear probable that restraint to warping will be completely eliminated in any of the types of longitudinal joints now in use and this should be taken into account in determining the length of tie bars. When warping takes place in a pavement it causes rotation of the joint faces, and when the rotation is sufficient to bring the faces into tight contact it develops compression in the concrete and causes the slab edges to separate at the plane of the steel. The tensile stress developed in the steel for a given separation of the joint faces is entirely dependent on the length of steel that is free to elongate.

## EFFICIENCY OF JOINTS DISCUSSED

When a tie bar is in bond a very small rotational movement in the joint may create a very high initial stress in the steel. This may be expected to result in a necking down of the steel until it is ruptured or until the bond is destroyed over a suficient length to permit the bar to elongate the required amount without rupture. It has been observed in pavements that this destruction of the bond actually takes place for a distance of several inches on each side of the joint. As a result Friberg ${ }^{7}$ has suggested that the midsection of tie bars, for a distance of several inches on each side of the joint, be coated with bitumen definitely to break the bond and also to furnish protection against corrosion.

Even if no definite provision is made for breaking the bond in the midsection of the bar it appears very probable that the bond will be destroyed over some minknown length by high stresses produced by warping. Therefore it appears desirable to make some arbitrary increase in the theoretical length of tie bars as computed be equation 27: An additional depth of embedment of at least 6 inches on each side of the joint or an increase of not less than 1 foot in the total length of the har, is suggested.

Efficiency of joints. The efficiency of any joint device used for edge strengthaning is dependent on the

[^6]degree to which it reduces the edge stresses that would otherwise be developed. In the past it has frequently been assumed that the relation between observed maximum deflections of adjacent slab ends under load could be taken as a measure of joint efficiency and that when these deflections were equal the joint was 100 percent efficient.

The Arlington tests (18) have shown that this assumption is incorrect. It was found, when a load was applied on one side of a joint, that the maximum deflections of the two edges might be identical but that the maximum stress in the loaded edge might be more than twice as great as that in the unloaded edge. As a result, the efficiencies of the joints involved in the Arlington tests were determined by a more logical method of analysis.
This analysis is based on the conception that if the joint fulfills its function perfectly, that is, with an efficiency of 100 percent, the stresses at the joint will not be greater than if the continuity of the slab were not broken. The efficiency of a given joint may then be expressed by the equation

$$
\begin{equation*}
J=100\left(\frac{\sigma_{e}-\sigma_{j}}{\sigma_{e}-\sigma_{i}}\right) \tag{28}
\end{equation*}
$$

in which
$J=$ joint efficiency in percent;
$\sigma_{e} \sigma_{j}$, and $\sigma_{i}$ are the critical stresses due to the application of a given load at the free edge, the joint edge, and the interior, respectively, of a slab of given uniform thickness.
This equation indicates a joint efficiency of zero when the critical stress at the joint equals the critical edge stress and an efficiency of 100 percent when the joint stress equals the interior stress.
Design of dowels.- The first theoretical analysis of the required spacing of dowel bars was that of Westergaard (44). This analysis enables one to compute the effect of dowel spacing on the critical stress in the edge of a joint, when the load is applied midway between two dowels, on the assumption that only the four dowels nearest the load are sufficiently active to require consideration and on the further assumption that the dowels are sufficiently stiff to cause the two joint edges to deflect exactly the same amount at all points. On the basis of his enalysis Westergaard concluded that a dowel spacing of 3 feet is too great to result in any significant reduction in the critical edge stress and that, if the dowels are to be effective for the purpose, the spacing should not exceed about 2 feet.
A more detailed study of dowel spacing, on the basis of the Westergaard analysis, is included in the report of the Arlington tests (18). This study indicated that if rigid dowels are to effect the same stress reduction that would be effected by slab continuity, the spacing must be considerably less than 2 feet.

In considering these indications it should be remembered that they are based on the assumption that the dowels are rigid. Therefore they cannot apply to the small round dowels commonly used except as they may indicate general trends. Also it may be noted that, while increasing the stiffness of dowels will increase their efficiency, it will at the same time increase restraint to longitudinal warping. Dowels that are too stiff may cause more distress in the pavement slab than would result from their complete omission.
The analysis and tests by Friberg ( 45,46 ), which have become available only in recent months, make it possible for the first time to design dowelled joints on a
rational basis. The analysis shows that a maximum joint efficieney can be obtained with round steeldowels of reasonable size only by using much smaller spacings than those indicated by the Westergaard analysis.

## DOWEL IENGTH OF 2 FEET FOUND EXCESSIVE

The analysis and tests by Friberg show that:

1. The lowest joint efficiency occurs when the load is between two dowels.
2. If the dowels are to have their greatest effectiveness in slabs of normal thickness the dowel spacing should not exceed about 12 inches.
3. The efficiency of the dowel decreases as the width of the joint is increased and increases as the diameter of the dowel is increased. For example, Friberg has shown that for a dowel directly under a load the percentage of load transfer of a 1 -inch dowel across a joint in a 7 -inch slab is 29 percent for a $\%$-inch joint and 25 percent for a 1 -inch joint; and that for a $\frac{1}{2}$-inch joint the load transfer of a 34 -inch dowel is 22 percent as compared with 29 percent for a 1 -inch dowel.
On the assumption that the effectiveness of the dowel is such that it will result in a stress relief of 25 percent it is of interest to compute the efficiency of a dowelled joint in a 7 -inch slab. For the 8,000 -pound wheel on dual high-pressure tires that has been used in previous stress computations, the same assumed characteristics of the concrete and a value of $k=100$, the interior load stress in a 7 -inch slab is 290 pounds per square inch and the edge stress at a transverse joint (equation 15) is 490 pounds per square inch. By means of equation 28 it is found that the joint efficiency equals $100\left(\frac{490-0.75 \times 490}{490-290}\right)$, or 61 percent.
4. The length of effective embedment of the dowel in the concrete of each slab need not be greater than 5 inches for $3_{4}$-inch dowels and not greater than 7 inches for 1 -inch dowels. Thus it is indicated that the dowel length of 2 feet, that has been customary, is excessive. It is important to note that when these short lengths of embedment are used the length of dowel cap and the width of joint opening should be considered in determining the required length of dowel.
5. Initial failure at dowels occurs by spalling of the concrete at the face of the joint under loads that may be as much as 50 percent less than the ultimate load sustained by the joint. This initial failure greatly reduces, if it does not completely destroy, the effectiveness of the dowels for stress relief.

Required efficiency of joints and load transfer devices.Theoretically, even with very stiff dowels, the maximum. amount of load transfer at a joint can never equal exactly 50 percent of the load applied on one side of the joint, on account of the eccentricity of the point of load application with respect to the joint. The unavoidable, and also desirable, flexibility of the joint device further reduces the possibility of ever obtaining at a joint a stress reduction of 50 percent. However, such a reduction is not necessarily required in order to obtain a joint efficiency of 100 percent nor is a joint efficiency of 100 percent always required in order to limit joint stresses to safe values.
In the preceding example it has been shown that, for the conditions assumed, a stress reduction of 25 percent results in a joint efficiency of 61 percent. In this example the interior and edge stresses are, respectively 290 and 490 pounds per square inch. If it be assumed that a safe unit stress is 350 pounds per square inch,
then the required joint efficiency equals $100\binom{490-350}{490-290}$. or 70 percent. This joint efficiency would require a stress reduction of $100 \times \frac{140}{490}$ ) or about 29 percent.
The preceding computations of joint efficiency have involved only stresses due to load. In the following examples the combined stresses due to load and temperature warping will be considered. It will be asisumed that the slab is 10 feet wide and 10 feet long, that $k_{i}=100$, and that the load, the temperature differential, and the properties of the concrete are the same as in preceding stress calculations.

## JOINT EFFICIENCY OF 100 PERCENT NOT REQUIRED FOR SAFE

 STRESSESIn a thickened-edge slab having an interior thickness of 7 inches the load stresses at the interior and at the joint edge (equation 9) are, respectively, 290 and 420 pounds per square inch. The interior and edge warping stresses are, respectively, 90 and 70 pounds per square inch. The combined stresses are then 380 pounds per square inch at the interior and 490 pounds per square inch at the edge. The joint efficiency will be computed on the assumption that the joint device used results in a stress reduction at the joint of 25 percent. No joint device can be expected to reduce the transverse warping stresses and therefore the stress reduction applies only to load stress. Reducing by 25 percent the load stress of 420 pounds per square inch and adding to this the warping stress of 70 pounds per square inch gives a value of the combined stress, $\sigma_{j}$, equal to 385 pounds per square inch. The joint efficiency then equals $100\left(\frac{490-385}{490-380}\right)$, or about 95 percent.
It has been shown in table 15 that if the slab length is 10 feet the combined stresses at the edge and interior of a 10-6.8-10-inch thickened-edge slab are well balanced and are limited to approximately 425 pounds per square inch. With $k=100$ the combined interior stress in this slab is 390 pounds per square inch and the combined stress at the edge of a free transverse joint (table 16) is 520 pounds per square inch. If it is desired to limit the combined edge stress to 42.5 pounds per square inch, the required joint efficiency is $100\left(\frac{520-425}{520-390}\right)$, or 73 percent. The load stress at the joint edge is 440 pounds per square inch and therefore the reduction in load stress equals $100 \times \frac{95}{440}$, or about 22 percent. On the other hand, if it were desired to have a joint of 100 percent efficiency it would be necessary to reduce the edge stress from 520 pounds per square inch to 390 pounds per square inch. In this case the required reduction in load stress, or transfer of load, equals $100 \times \frac{130}{440}$, or about 30 percent.
Thus it is seen that a load transfer, or stress reduction of 50 percent is not necessarily required in order to obtain a joint efliciency of 100 percent and that a joint efliciency of 100 percent is not necessarily required in order to limit to safe values the stresses in the joint edge.

Tests of joint efficiency.-In connection with the Arlington tests (18) a great many tests were made on the types of joints included in the investigation to determine their effectiveness in reducing edge stresses due
to load. The results are summarized in tables 21 and 22 , the reported efficiencies having been computed by equation 28.

With respect to the longitudinal joints it may be noted that the measured efficiencies of the two tongue-and-groove joints containing bonded tie bars were relatively high even though the tie bars were only one-half inch in diameter, and were spaced 5 feet apart. It may also be noted that the omission of tie bars from a tongue-and-groove joint reduced its efficiency by about one-third.

Table 21.-Observed efficiency of longitudinal joints (average values for tests at a number of points) ${ }^{1}$

| Type of joint | Designation in fig. 22 | Spacing of tie bars ${ }^{2}$ | Diameter of bars | Joint efflciency |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Inches | Inches | Percent |
| Triangular tongue. | A | 60 |  | 75 |
| Rectangular tongue | B | 60 | 1/2 | 78 |
| Do.-. |  | None |  | 50 |
| Butt. | D | 24 |  | 52 |
| Do. | D | 36 | $3 / 4$ | 42 |
| Do. | D | 48 | $3 / 4$ | 51 |
| Do. | D | 60 | $3 / 4$ | 47 |
| Dummy | C | 60 | $1 / 2$ | 44 |
| Do. |  | None |  | 39 |

: Data from table 11, PUBLIC ROADS, October 1936.
All tie bars in bond.
${ }^{2}$ All tie bars in bond.

TABLE 22.-Observed efficiency of transverse joints (average values for a number of tests) 1

| Type of joint | Des- <br> igna- <br> tion in fig. 22 | Spacing of dowels ${ }^{2}$ | Joint opening | Joint efficiency |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Win ter | Summer | Average (various seasons) | Over dowels |  |
| Dowel_........Do.......Do......Do........ | F | Inches | Inches | Percent | Percent | Percent | Percent | Percent |
|  |  | 36 |  |  |  |  | 46 | 8 |
|  |  | 27 | 12 |  |  |  | 31 | 6 |
|  | F | 27 | $3 / 4$ |  |  |  | 16 | 20 |
|  | F | 18 | $1 / 2$ |  |  |  | 28 | 8 |
|  | F | 18 | $3 / 4$ |  |  |  | 40 | 28 |
| Dummy | I | 18 |  | 71 | 66 |  |  |  |
| Do- |  | None |  | 4 | 41 |  |  |  |
| Dowel plate ${ }^{3}$ | H |  |  |  |  | 59 |  |  |
| Do.--- | H | ------- | 3/4 |  |  | 66 |  |  |

Data from table 10, PUBLIC ROADS, October 1936.
2 All dowels $3 / 4$-inch diameter-not in bond.
${ }^{3}$ Dowel plates 4 inches by $1 / 4$ ineh.
The longitudinal butt joints, which were all in slabs of the same thickness, had much lower average efficiencies than the tongue-and-groove joints in spite of the fact that the tie bars were of larger size and in general were more closely spaced. In the butt joints there is no consistent relation between average joint efficiency and tie-bar spacing. This is contrary to what would be expected and may be at least partially explained by the fact that the figures given are average values from tests in which the loads were applied at a great many different points. It was found in testing these butt joints that there was a rather consistent relation between joint efficiency and the distance from the center of the load to the center of the nearest tie bar. The average observed efficiencies for a load directly over a tie bar and at distances of 18 and 30 inches from it were about 70,45 , and 35 percent, respectively (fig. 35, PUBLIC ROADS, Oct. 1936). This would indicate that tie-bar spacing has an influence on the efficiency of longitudinal butt joints in spite of the lack of evidence in the average values given in table 21.

TESTS INDICATE DOWEL SPACINGS FORMERLY USED ARE EXCESSIVE
The average efficiency of the longitudinal dummy joint with tie bars was of about the same order of magnitude as that of the butt joints and the omission of tie bars reduced the average efficiency by only 5 percent. Both results may seem somewhat surprising, the first because it is so low and the second because it is so high, but here again average values are being considered. In testing these longitudinal dummy joints it was found that for loads at certain positions the indicated efficiency was very high while at other positions it was practically zero. It was also noted frequently that the joint was efficient for a load on one side of it and inefficient when the load was placed directly opposite on the other side of the joint. It seems evident that the measured efficiency of a dummy joint is largely dependent on the form of the fracture, particularly the direction of its slope, directly under the load.

The thickened-edge longitudinal joint shown in figure 22-E was not investigated in the Arlington tests but no tests are necessary to establish its efficiency. This is entirely dependent on the proper proportioning of the edge section in the manner that has already been discussed.

The transverse doweled expansion joints were tested at points directly over the dowels and midway between them, as indicated in table 22. In general the average efficiency was very low for a load between the dowels and, with one exception, was considerably greater for a load directly over a dowel. This investigation was planned in 1930 when the knowledge of the action of joint devices was considerably less than at present. The tests themselves, now supplemented by the analysis by Friberg, have shown that the program was quite inadequate for a thorough investigation of the efficiency of doweled joints. It is rather definitely indicated that the dowel spacings were too great for effective dowel action and analysis of the data is complicated by the fact that the joints were installed in slabs of different thickness. Therefore the results obtained should not be considered as indicative of the best performance of doweled expansion joints that can be expected.

The transverse dummy contraction joints were tested both in summer and winter and the joint with dowels had a high efficiency in both seasons of the year. The joint without dowels had a fair efficiency during the summer when the slabs were in an expanded condition and the width of the crack was small, but the efficiency was negligible in the winter when contraction had taken place and the width of crack was as great as 0.03 inch. Therefore, it appears that even in slabs as short as these ( 20 feet) the interlocking of the fractured faces in a transverse dummy joint cannot be depended upon to provide adequate load transfer when the slabs are in a contracted condition.

The two dowel-plate expansion joints that were tested had efficiencies comparable with the efficiency of the dummy contraction joint with dowels. The figures indicate that a dowel plate of the size investigated is an effective means for bridging the openings in expansion joints but more information is needed regarding the required depth of embedment of the dowel plate in the slab and the required thickness of plate.

The butt contraction joint shown in figure 22-J was not investigated in the Arlington test but its performance should be expected to be much the same as that of the doweled expansion joints, with probably a
somewhat greater efficiency on account of the smaller width of joint opening.

For the thickened-end transverse expansion joint shown in figure $22-\mathrm{G}$ the efficiency observed in the Arlington tests was low since the edge thickness was inadequate. When the edge section is properly designed the edge stress is the same as the interior stress and no edge strengthening or load transfer is required.

In the past the thickened-end type of transverse joint has been criticised on the ground that it offers additional resistance to contraction, with the result that a transverse crack is likely to develop near the junction of the end section with the interior of the slab. No action of this kind has been observed in the Arlington tests. The slabs with thickened ends have expanded and contracted as freely as any of the other slabs tested and no transverse cracks have developed in them in a period of more than 8 years. There is nothing in the results of these tests to indicate that edge thickening cannot be applied to transverse expansion joints with as much success as to the longitudinal edges of the slab.
Very little information of a definite character is available concerning the reported unsatisfactory performance of thickened-end transverse joints. The only reference that has been found is in a 1932 report of a committee of the American Road Builders' Association (47). This report merely states that experience with the thickened-end joint in three States has not been entirely satisfactory; that transverse cracking usually develops near the joint, with subsequent buckling of the slab ends due to expansion and with the further result, in some cases, of complete breakage under the action of traffic.

In contrast to this is the experience of Kent County, Mich. Mr. Otto S. Hess ${ }^{8}$ is authority for the following report of that experience.

## EXPERIENCE SHOWS THICKENED-END SLABS SATISFACTORY

Since 1926 practically all of the concrete pavements built by the Kent County Road Commission have been constructed with thickened-end transverse expansion joints spaced 50 feet apart and with no intermediate contraction joints. The 50 -foot slabs are reinforced with wire fabric or bar mats. The expansion joints are $3 / 4$ inch wide and a premolded joint filler is used. The ends of adjacent slabs are not connected in any manner.

With this design, transverse cracking has been almost eliminated. Not a single transverse crack has been observed in the vicinity of the joints where the endthickening begins. The contention that contraction in a thickened-end slab will cause the ends to ride up on the subgrade and create roughness at joints has not been supported since no difficulty has developed because of vertical movement of the slab ends. The experience of Kent County indicates that if the strength required in joint edges is obtained by thickening the slab ends it is not necessary to connect the slabs with dowels or other devices in order to maintain smooth joints.

The Arlington tests were quite inadequate from the standpoint of a comprehensive study of joint action since the variables included in the program were not of sufficient number or of sufficient range. However, the results obtained, when viewed in the light of the Friberg analysis and the discussion of the required efficiency of joints, indicate that if proper attention is given to the design of both the slab and the joint a

[^7]number of the types of joints in common use can be expected to effect the required stress reduction.

Effect of joints on corner stresses.-An assumption similar to that used in deriving equation 28 , which gives a measure of the efficiency of a joint in reducing edge stress, might be used in developing a measure of the efficiency of a joint in reducing corner stress. For example, it might be assumed that with a joint of 100 percent efficiency the corner stress should be no greater than the stress in the edge of the slab at some distance from the corner. However, it is not necessary to do this and, in some cases, such an assumption would result in an indicated efficiency in excess of 100 percent in joints having no provision whatever for stress reduction.

In a slab of uniform thickness, corner load stresses computed by equation 11 exceed edge load stresses computed by equation 15 , but only by relatively small amounts. In the case of combined stresses in slabs 15 to 30 feet long and ranging in depth from 7 to 10 inches, figures 15 and 17 show that the edge stresses are always greater than the corner stresses. In 10 -foot slabs of these depths the combined corner stresses exceed the combined edge stresses by 50 to 80 pounds per square inch when $k=100$, but when $k=300$ the edge and corner stresses are practically the same. Therefore it appears that in a slab of adequate design there is no great need for stress reduction at the joint corners and that any reduction effected by the joint device will be in the nature of a factor of safety.
In the Arlington tests the difference between the stress at a free corner and that at a joint corner was determined and this stress reduction was expressed as a percentage of the stress at the free corner (table 12, PUBLIC ROADS, October 1936). It was found that the transverse joints (table 22) were about equally effective in reducing corner stress and that the average reduction was about 40 percent. Of the longitudinal joints that could be tested, the butt joint with tie bars spaced 24 inches apart and the dummy joint with tie bars resulted in an average reduction in corner stress of about 50 percent and the dummy joint without tie bars reduced the corner stress by about 40 percent. Thus all the joints tested were quite effective in reducing corner stress although some of them were quite ineffective in reducing edge stress.

## conclusions

The discussion that has been presented leads inevitably to certain conclusions which, if accepted, require a rather drastic revision in some of the accepted ideas concerning the structural design of concrete pavements. These conclusions are open to attack principally on the ground that practical experience in certain localities or under certain conditions does not always support them. This is recognized but it is believed that, for the country as a whole, they are supported by observations of the behavior of pavements in service. The exceptions may be due to a number of causes, an important one being that many concrete pavements are not subjected to loads of the magnitude and frequency for which presumably they were designed.

In other engineering structures, such as bridges and buildings, the absence of failure is not necessarily an evidence of adequate design since structures do not always fail even when dangerously overstressed. The same is true of concrete pavements. It is recognized,
of course, that it would be unreasonable to be as conservative in the design of pavements as in the design of bridges but it should also be recognized that the factor of safety in many pavement designs in current use is negligible.

On the basis of the information presented, concrete pavements may be designed with reasonable assurance that they will be free from structural defects over a long period of time. A lowering of the indicated requirements of design may result in structural failures of varying degrees of importance. The extent to which the possibility of such failures can be tolerated is a matter to be decided on the basis of engineering judgment.

The more important conclusions that are indicated are as follows:

1. The critical load stresses developed in a concrete pavement are primarily dependent on single wheel loads and not on axle loads, axle spacing or the gross weight of vehicle.
2. Impact forces considerably in excess of static wheel loads should be used in the design of pavements. The impact factor (ratio of total impact reaction to static wheel load) is less for balloon tires than for highpressure tires and decreases as the wheel load increases.
3. The stresses in a concrete pavement are approximately the same for an 8,000 -pound wheel load on dual high-pressure tires and for a 9,000 -pound wheel load on dual balloon tires.
4. The stress analyses of Westergaard, with the modifications suggested by the Arlington tests, are suitable for use in the design of concrete pavement slabs and form the only adequate basis for such design.

5 . Since the physical characteristics of the subgrade and of the concrete can never be foretold with certainty it is desirable to be conservative in the selection of values representing these various characteristics for use in design.
6. Warping stresses due to differentials of temperature within the slab may be of the same order of magnitude as the stresses due to heavy wheel loads and therefore require consideration in pavement design.
7. Reasonable assurance of the absence of transverse cracking in concrete pavements can be obtained only by the use of short slabs having lengths not greater than 10 to 15 feet.
8. Transverse cracks in thickened-edge pavements without reinforcement create a weakened condition in the interior of the slab which may be serious. The introduction of properly designed steel reinforcement in long slabs will not completely eliminate transverse cracking but it will reduce or eliminate the detrimental effect of the cracks which may develop.
9. The edges of transverse joints in thickened-edge slabs require strengthening because the central portion
of the joint has the same thickness as the interior of the slab but is subjected to the higher stresses that are associated with edge loading.
10. When the pavement is designed for the combined stresses due to load and temperature it is safe practice to use an allowable unit stress in excess of 50 percent of the 28-day flexural strength of the concrete.
11. When the pavement is designed for maximum legal wheel loads and in such manner that the combined stresses due to load and temperature are limited to safe values and are reasonably well balanced, the thickened-edge section has no great advantage over the section of uniform thickness from the standpoint of over-all cost per mile.
12. Transverse joints are required in concrete pavements to relieve warping stresses due to temperature and also to provide for longitudinal expansion and contraction. Longitudinal joints are required to prevent the longitudinal cracking that usually develops otherwise.
13. If proper attention is given to the design of both the slab and joint, the required edge strengthening at joints in thickened-edge slabs can be obtained with a number of the types of load-transfer devices in common use.
14. The thickened-end transverse expansion joint is indicated, both by tests and experience, to be a highly effective method of providing the edge strengthening that is required at transverse joints in thickened-edge slabs.
15. Longitudinal joints of the tongue-and-groove type appear to be considerably more effective than other types in common use in providing the strengthening that is required in the edges of the longitudinal joints of thickened-edge slabs.

## ACKNOWLEDGMENTS

This paper is essentially a compilation and interpretation of published data and, insofar as practicable, the sources of material are indicated in the bibliography.

The author desires to acknowledge the invaluable advice and assistance given by his associates in the Public Roads Administration: Mr. L. W. Teller, Mr. A. L. Gemeny, Mr. J. A. Buchanan, Mr. E. C. Sutherland, Mr. W. F. Kellermann, Mr. R. J. Lancaster and Mr. A. L. Catudal.

He also desires to express his appreciation of the generous permission granted by Mr. Royall D. Bradbury to appropriate a number of original ideas from his book "Reinforced Concrete Pavements". Special credit is due Mr. Bradbury for originating the simplified methods, used throughout this paper, of computing stresses due to loads and temperature warping. The use of these methods changes a very tedious operation to a very simple one.






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7 No special taxes on motor carriers reported \& Ton-mile and nassenger-mile taxes paid hy motor carriers in lieu of registration fees included in motor-
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3 A pproximately $\$ 84,000$ alloted for use on county roads under State control in North Carolina ineluded in State highway purnoses.
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${ }_{3}$ Ampated for hire (motor-carrier taxes). See tables on pp. 127 to 129 which give distribution of receipts separately. funds and lag between accounts of collecting and expending agencies.
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$\$ 2,407,000$.
${ }^{5}$ Reimbursement to local units of government for amounts spent on roads now on State system.
${ }^{8}$ In States indicated by star $\left(^{*}\right)$ law provides that these funds may also be used for service of local highway
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Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio (1928).
Report of a Survey of Transportation on the State Highways of Pennsylvania (1928).
Report of a Survey of Traffic on the Federal-Aid Highway Systems of Eleven Western States (1930).

## UNIFORM VEHICLE CODE

Act I.-Uniform Motor Vehicle Administration, Registration, Certificate of Title, and Antitheft Act.
Act II.-Uniform Motor Vehicle Operators' and Chauffeurs' License Act.
Act III.-Uniform Motor Vehicle Civil Liability Act.
Act IV.- Uniform Motor Vehicle Safety Responsibility Act.
Act V.-Uniform Act Regulating Traffic on Highways.
Model Traffic Ordinances.

A complete list of the publications of the Public Roads Administration (formerly the Bureau of Public Roads), classified according to subject and including the more important articles in Public Roads, may be obtained upon request addressed to Public Roads Administration, Willard Bldg., Washington, D. C.



[^0]:    ${ }^{4}$ Materials and Research Engineer, Division of Highways, California Department of Public Works.
    a Because of its length, this report is presented in two issues of Public Roads. The first installment appeared in the July issue.
    Italic figures in parenthesis refer to the bibliography, p. 102, of the preceding issue.

[^1]:    ${ }^{1}$ From figs. 15 and 16.

[^2]:    ${ }^{1}$ Assumptions with respect to load and other variables same as in figs. 15, 16 and 17

[^3]:    ${ }^{1}$ Data from table 3, pU'BLIC RoADS, Norember 193.
    ? Displacement of 0.10 inch corresponds to maximum horizontal resisting force that could be developed.

[^4]:    ${ }^{5}$ The original manuscript of this section on stresses due to contraction has been completely rewritten as a result of suggestions made by Mr. R. D. Bradbury, to whom credit is due for the development of the method for computing the average value of the coefficient of subgrade resistance.

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[^5]:    ${ }^{6}$ Gace numbers are those of the Standard Specifications for Cold-Drawn Steel Wire for Concrete Reinforcement of the American Society for Testing Materials,

[^6]:    Beng1 F. Friherg, Research Engineer, Ladede Steel Co., St. Louis, Mo.

[^7]:    ${ }^{8}$ Engineor-Manager, Kent County Rond Commission, Grand Rapids, Mich.

