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D. M. BEACH, *Editor*

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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

In This Issue

	Page
Application of the Results of Research to the Structural Design of Concrete Pavements .	83

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APPLICATION OF THE RESULTS OF RESEARCH TO THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS[1]

Reported by E. F. KELLEY, Chief, Division of Tests, Public Roads Administration

DURING the past 20 years many studies have been made of the various factors that influence the structural performance of concrete pavement slabs and the numerous reports of these investigations are scattered through the technical literature. Most of these reports, of necessity, are highly technical and the mass of data presented and the detailed descriptions that are included, both as a matter of record and in order that the reader might have confidence in the validity of the results, frequently tend to obscure the value and importance of the conclusions.

In addition, individual reports frequently cover but a single phase of a given subject and are useful only when considered in connection with the available reports dealing with the remaining phases of the same subject. The net result of this situation is that many facts that have been well established by research are little appreciated and too frequently are given scant consideration in the practical design of pavements. It is the purpose of this paper to bring together under one head and make available for the practical use of the designing engineer the important facts that have been developed thus far in research work relating to the structural design of concrete pavements.

In the field of bridges and buildings the basic principles of design have become so well established that, to many engineers, the term "structural design" conveys the idea of a rather exact and accurate mathematical procedure to be followed in proportioning the several parts of a structure. No such presumed accuracy exists in connection with the structural design of concrete pavements.

From the standpoint of stress analysis the concrete pavement is a highly complex structure. It is supported by soil whose physical properties vary appreciably at different locations, at different points in the same general location, and even at different times at the same point. It is subjected to the action of external forces produced by the wheels of vehicles and the magnitude of these forces and their effect on pavement stresses are influenced by a number of variables. In addition, it is constantly subjected to high internal stresses produced by changes in temperature and moisture. Much has been learned concerning the influence of the different variables on pavement stresses but a great deal of additional research is still needed. However, on the basis of available information, reasonable assumptions of sufficient accuracy can be made to insure a pavement structure that will function in a satisfactory manner.

Structural design, in general, is distinguished by the use of conservative unit stresses which, for structural steel, are well below the elastic limit and, for concrete, well below the ultimate strength. This results in the so-called factor of safety which is depended upon to provide for all the unknown conditions for which it is impossible to make definite provision. In contrast to this the current designs of concrete pavements are generally such that the factor of safety, if any, is so small as to be almost negligible.

The maximum combined stresses due to external loads and to temperature in pavement slabs of the dimensions commonly used will very frequently be found to be so close to the ultimate strength of the concrete that there is little or no margin left to provide for unknown or unforseen conditions. In making this statement there is no intention to imply any general criticism of present practice since the present standards of design have proven reasonably adequate. When the need for the great mileage of existing pavements and the fact that structural failures of these pavements do not generally endanger human life are considered, it seems probable that any significant increase in cost to provide a margin of safety comparable to that provided in bridges, could not have been justified from the economic standpoint. However, it is important to recognize that the low or negligible factor of safety that is provided in designing concrete pavements makes it highly desirable to be somewhat conservative in assuming design values for the different variables that must be considered.

IMPACT REACTION DEPENDENT ON FOUR VARIABLES

Wheel loads and impact.—Neglecting the unpredictable forces caused by localized differential heaving or subsidence of the subgrade soil, the external forces that create stress in the pavement slab are produced by vehicles. Naturally, the heavier vehicles are the more important.

One of the earlier investigations (*1*)[2] developed the important fact that for heavy vehicles of the usual type, that is, four- or six-wheel trucks or trailers, the critical stress developed in a concrete pavement, when the axle spacing is in excess of about 3 feet, is primarily a function of the wheel load and not a function of the gross load on the vehicle or the axle spacing. By means of his theoretical analysis, Westergaard (*2*) subsequently arrived at the same conclusion and this has been confirmed by later tests (*3*). This finding, which permits attention to be confined to wheel loads rather than gross loads, greatly simplifies a problem already sufficiently complicated.

The magnitude of the vertical force exerted on a pavement by the wheel of a moving vehicle may be considered to be the sum of the static weight of the loaded wheel and the additional impact or dynamic force created by the movement of the wheel over the irregularities that exist in the pavement surface. The researches of the Bureau of Public Roads have demonstrated conclusively that the impact reaction of a moving wheel is sufficiently in excess of the static wheel load to make it an important factor in pavement design.

The impact reaction of a moving wheel depends upon four major variables—wheel load, vehicle speed, tire

[1] Paper presented at the annual meeting of the American Concrete Institute, March 1939. Because of its length, this report will be presented in two issues of PUBLIC ROADS. The second installment will appear in the August issue.

[2] Italic figures in parentheses refer to the bibliography, p. 102.

equipment, and road roughness (*4*). Other variables exert some influence but, in general, these four are the important ones. An increase in wheel load or pavement roughness; a decrease in the cushioning qualities of the tires; and, within limits, an increase in vehicle speed; all result in increased impact reactions.

The tests that have been made have amply demonstrated the fact that the magnitude of the impact reaction is a function of the wheel load. Also, these tests have brought out important facts, not previously known, regarding the relation between wheel load and the impact reaction that it produces. In bridge design it is customary to express impact as a percentage of the static live load. Therefore it is important to observe that while the total impact reactions of the wheels of motor vehicles increase with increase in wheel load, the percentage of impact, or the ratio of the dynamic increment to the static load, actually decreases as the wheel load is increased. This fact may be attributed largely to the relative effects of sprung and unsprung weights, and to the relation between size of tire and its cushioning properties.

The force which the wheel of a vehicle delivers to the road surface is made up of two component forces. One of these is caused by the unsprung weight on the wheel (that is, the weight of the parts not supported by the springs), and the other is caused by the spring pressure on the axle at the instant of impact. The part of the total impact reaction caused by the unsprung weight is, in general, considerably greater than the part caused by the sprung weight. However, the ratio of unsprung weight to total weight is not a constant but decreases as the total or gross weight is increased. Also, as the wheel load is increased the tire size is increased and with it the ability of the tire to minimize the effect of surface irregularities. The result is that for a given condition of road roughness an increase in wheel load is not accompanied by a corresponding percentage increase in the dynamic component of the impact reaction.

The magnitude of the impact force is greatly dependent on the type and condition of the tire equipment. Solid, cushion, and pneumatic tires, in the order named, produce impact reactions of decreasing magnitude. The tests that developed this information were made at a time when rubber tires of the solid and cushion types were commonly used. Fortunately, these types are no longer in general use. The relatively few solid tires that are now used must be operated at such low speeds that, in comparison with the pneumatic tires used on high-speed trucks and busses, they need be given no consideration from the standpoint of impact. Therefore attention may be confined to pneumatic tires.

With respect to pneumatic tires it has been found (*5*) that, other conditions being the same, the dynamic increment of the impact reaction of high-pressure and balloon tires is closely proportional to their inflation pressures. Therefore, it follows that for a given wheel load the impact reaction created by low-pressure balloon tires is appreciably less than that caused by high-pressure tires. From the standpoint of pavement protection the balloon tire offers the additional important advantage that it applies the load to the pavement over a larger area of contact, a condition that results in a lower slab stress. This relation will be discussed in detail later.

INTENSITY OF IMPACT DECREASES AS FREQUENCY OF OCCURRENCE INCREASES

Another fact with respect to the effect of tire equipment is that dual tires generally give somewhat higher impact reactions than do single tires of the same type and same load capacity. The difference is a variable which, from the practical standpoint, may safely be ignored since the increased stress in a concrete pavement slab resulting from the greater impact effect of dual tires may generally be expected to be more than offset by the reduction in stress resulting from their greater area of load application. For example, if it be assumed that a certain wheel load on dual high-pressure tires produces an impact reaction of 10,000 pounds then the minimum reaction that may reasonably be expected from the same load on a single high-pressure tire of comparable capacity would be of the order of 9,000 pounds. With reasonable assumptions as to area of tire contact and other variables the computed stresses, by the original Westergaard analysis (*2*), for loads applied at the interior of a 6-inch slab, are about 330 pounds per square inch for the 9,000-pound load on the single tire and about 315 pounds per square inch for the 10,000-pound load on the dual tires.

When a wheel runs over an obstruction, such as an inclined plane or a rectangular block, two types of vertical impact reactions are developed. One is caused by shock as the wheel strikes the obstruction and the other is caused by the drop of the wheel from the obstruction to the pavement. In the earlier investigations involving pneumatic tires operated over artificial obstructions at speeds up to about 55 miles per hour (*5*), it was found that the shock reactions increased approximately in direct proportion to speed. It was also found that drop reactions reached maximum values at relatively low speeds, of the order of 25 to 35 miles per hour, and that these were not exceeded by the shock reactions except at speeds of the order of 50 miles per hour. In a subsequent investigation (*6*) involving only balloon tires, it was found that the use of artificial obstructions resulted in maximum drop impacts at speeds of from 20 to 40 miles per hour and that these were not exceeded by shock impacts at speeds up to 70 miles per hour.

From these tests with artificial obstructions it might be concluded that the effect of speed on impact reactions is not important for speeds in excess of 40 miles per hour. However, such a conclusion would require some modification as a result of the tests (*6*) that have been made to determine impact reactions resulting from the natural roughness of road surfaces. These tests were made at 28 locations where the natural roughness was as severe as would permit the safe operation of a heavy vehicle at high speed. In each of these 28 locations the shape of the curve of impact reaction versus speed was different depending on the characteristics of the particular roughness condition.

In some cases the maximum impacts were observed at relatively low speeds but in the majority of cases the impact reactions showed a general tendency to increase with increases in speed up to the maximum of 70 miles per hour. However, this statement applies to individual locations. When all the maximum impact reactions were plotted against speed it was found that a general maximum was reached at about 50 miles per hour and that this remained constant up to 70 miles per hour, the maximum speed attained in the tests (fig. 24, Public Roads, Nov. 1932). Therefore, it seems reasonable to conclude that the effect of speed on impact reaction may be neglected for speeds in excess of 50 miles per hour.

Two investigations have been made to determine the effect of conditions of general road roughness on the magnitude of impact reactions (*6*, *7*). This is in contrast to·the study of extreme conditions of roughness already described. In these tests, roads of various degrees of roughness, as determined by the relative roughness indicator (*8*), were selected for study and the test vehicles with different wheel loads and different tire equipments were operated over them at various speeds.

It was found that, other conditions being the same, there was a rather definite relation between the magnitude of the impact reaction and the frequency of its occurrence. Of the great number of impacts that may occur on a given section of road, those of the greatest magnitude occur only a few times while those of lesser intensity occur a greater number of times and the intensity decreases as the frequency of occurrence increases. For example, in the tests with a motor bus equipped with balloon tires and operated at a speed of 40 miles per hour over a very rough concrete road, it was found that the impact factors (ratio of total impact reaction to static wheel load) for frequencies of 1, 40, 80, and 100 times per mile were approximately 2.20, 1.65, 1.55, and 1.50, respectively. However, the magnitude of the impact factor for a given frequency becomes less as the roughness of the pavement decreases. The impact factors for the same vehicle as described above, operated at the same speed of 40 miles per hour over a smooth concrete pavement, were approximately 1.25 and 1.18 for frequencies of 1 and 100 per mile, respectively.

It is immediately apparent from this relation between frequency and magnitude of impact factors that, from the standpoint of pavement design, it is necessary to select some reasonable frequency and to compute dynamic loads on the basis of the impact factor corresponding to this frequency. Designing a pavement for a maximum load that may occur only once per mile would certainly be open to serious question and it is necessary to select an impact force that occurs with sufficient frequency to be of practical importance. A frequency of 100 per mile, corresponding to the maximum impact reaction that may be expected to occur on an average of once every 50 feet, is suggested as a reasonable assumption.

The existing data do not permit the evaluation, from any single series of tests, of all the variables that have been discussed. However, some of the variables have been studied in each series of tests and it is possible, by interpolation and extrapolation, to combine the data in the reports that have been mentioned (*4*, *5*, *6*, *7*) so as to give impact factors that are in agreement with our present knowledge of the subject and which are sufficiently accurate for purposes of design. Such impact factors for a range of static loads on wheels equipped with dual high-pressure and balloon tires, a speed of 50 miles per hour on a pavement having a reasonable degree of smoothness (neither extremely rough nor extremely smooth), and a frequency of 100 per mile, are given in table 1.

The pavements on which impact-frequency studies were made were rated with respect to degree of roughness with the relative roughness indicator (*8*) and it is interesting to observe that, with minor exceptions, the order of rating would have been the same had they been rated for roughness by means of the impact-frequency curves. In other words, the roughness indicator gave a qualitative measure of the characteristics of the pavement surface that determine the magnitude of impact. However, while the roughness indicator is a useful instrument, it is not one of precision. As it has commonly been used the motor vehicle on which it is mounted becomes an integral part of the instrument and the results are reproducible only with the same car operated under the same conditions. Therefore, while a given instrument mounted on a given car gives a qualitative measure of the relative roughness of different road surfaces, it is not possible to express these results in absolute figures.

Table 1.—*Impact factors and total impact-road reactions*

Speed—50 miles per hour.
Frequency—100 per mile.
Condition of pavement surface—reasonably smooth.

Static wheel load, pounds	Dual high-pressure tires		Dual balloon tires	
	Impact factor	Total impact reaction	Impact factor	Total impact reaction
		Pounds		*Pounds*
4,000	2.05	8,200	1.70	6,800
5,000	1.80	9,000	1.54	7,700
6,000	1.67	10,000	1.43	8,600
7,000	1.56	10,900	1.37	9,600
8,000	1.48	11,800	1.31	10,500
9,000	1.41	12,700	1.27	11,400
10,000	1.36	13,600	1.24	12,400

The tests that form the basis for the data given in table 1 were made on pavements that appeared to represent reasonable average conditions of surface roughness, intermediate between extremely smooth and extremely rough surfaces. A more precise definition cannot be given. On account of this variable and the others that affect the magnitude of the impact reactions, the data given in table 1 can be considered only as approximate. They represent the best estimate that can be made, on the basis of existing data, of the maximum impact reactions, important with respect to design, that can reasonably be expected to occur as the result of the normal operation of the heavier motor vehicles. The digit in the second decimal place in the figures for impact factors is without significance. It is included merely for the purpose of making the impact factors agree with the total impact reactions which are given to the nearest hundred pounds.

IMPACT FACTOR USED SHOULD BE INDEPENDENT OF POSITION OF LOAD

As will be shown later, in a concrete pavement slab of uniform thickness the magnitude of the critical stress is greatly influenced by the position of the wheel load; that is, whether it is near an edge, a corner, or in the center of the slab. Since the higher impact reactions will be produced at the points where the surface irregularities are greatest, it follows that higher impact reactions may be expected in the vicinity of transverse joints and cracks than in the interior of the slab. In view of this consideration Bradbury (*9*) has suggested that a higher allowance for impact be made in the computation of stresses at transverse joints than in other portions of the slab. However, in plain (nonreinforced) pavements transverse open cracks are quite likely to develop at random, except in very short slabs, and thereby create a roughness condition similar to that at formed joints. When this takes place in a thickened-edge slab a condition of weakness is created at the broken edge of the slab along the crack that makes it desirable to overdesign rather than underdesign the thickness of the pavement.

Also when a truck wheel leaves the edge of the pavement and then rolls back on the slab from a shoulder that frequently is not at the same elevation, an impact reaction of considerable magnitude may be developed. These considerations lead to the conclusion that nice distinctions with respect to the position of the load on the pavement are unwarranted and that the same impact factor should be used irrespective of the position of the load.

DESIGN STRESS EQUAL TO 50 PERCENT OF ULTIMATE STRENGTH IS CONSERVATIVE

Fatigue limit of concrete.—Concrete, like other structural materials, will fail under repeated loads at unit stresses which are much less than the ultimate strength as determined by the stress at failure produced by one application of static load. The stress at which failure takes place under a very large number of loadings is known as the fatigue limit or the endurance limit and, for concrete, it is expressed as a percentage of the ultimate strength.

Investigations of the fatigue limit in flexure under static load (*10*, *11*, *12*) have shown that concrete may be subjected to an almost unlimited number of applications of a stress equal to about 55 percent of its ultimate strength without danger of failure. A similar study of the fatigue limit of concrete under impact loads (*13*) gave similar results although the maximum number of load applications was only about 83,000 as compared with the one or more million that are usually considered desirable in fatigue studies. From this study it was concluded that, with respect to fatigue, the behavior of concrete may be assumed to be very similar under both static and impact loads and that the same fatigue limit is applicable to both.

On the basis of these investigations it has become rather general practice to assume about 50 percent of the ultimate flexural strength as a safe value of the working stress for use in designing pavements to resist wheel loads. However, the fatigue limit of the order of 50 percent of the ultimate flexural strength of the concrete has been established by tests in which the load applications were repeated at relatively short time intervals, as many as 40 per minute in tests in which the loads were applied without shock. In contrast to this, under normal conditions of traffic the heavy wheel loads that produce maximum stress are applied to the pavement slab at relatively long time intervals.

Hatt concluded (*11*) that the fatigue limit is about the same for beams under continuous fatigue loading as for those under fatigue loading with short rest periods. This is based on tests in which the stress cycles were at the rate of 10 per minute and in which the rest periods were not between individual load applications but were at intervals of several hundred or several thousand stress cycles. It is by no means certain that the fatigue limit might not be considerably different, and possibly higher, for stresses applied at time intervals corresponding to those which occur between successive applications of heavy wheel loads to a pavement under traffic.

It is a well-known fact that stresses above the fatigue limit cause progressive inelastic deformation and final failure. However, the relation between intensity of stress above the fatigue limit and the number of repetitions of this stress that will cause failure is not well established even for rapid repetitions of stress. For less frequent repetitions nothing is known concerning it.

On the majority of highways the heavier vehicles constitute a small percentage of the total traffic and therefore the occurrence of maximum load stresses is relatively infrequent. It appears therefore that the present practice of assuming the design stress to be approximately 50 percent of the ultimate strength o' the concrete is a conservative one insofar as the stresses due to maximum wheel loads are concerned. In view of the possibility that the fatigue limit for these infrequent repetitions of stress may be higher than is indicated by available data, this practice may introduce some factor of safety of unknown magnitude.

However, the limitation of the design stress to 50 percent of the ultimate strength is believed to be unduly conservative when the pavement slab is designed for the combined effect of stresses due to load and those due to temperature warping since, as will be shown later, the maximum combined stresses due to load and temperature occur only in the daytime during the spring and summer months. It is apparent, therefore, that the frequency of occurrence of maximum load stresses in combination with maximum temperature stresses is considerably less than the frequency of passage of the truck wheels that produce maximum load stresses. This is particularly true on those highways where the movement of heavy trucks is principally at night.

In attempting to establish safe unit stresses for use in the design of concrete pavement slabs several factors in addition to fatigue should be considered and these will be discussed later. It is sufficient here to point out that the many uncertainties regarding the fatigue characteristics of concrete render of doubtful value any refinements in the use of existing data.

STATIC LOAD STRESSES MAY EXCEED IMPACT LOAD STRESSES

Static stress versus impact stress.—With respect to the relative stress effects of static and impact loads, exhaustive tests by the Bureau of Public Roads (as yet unpublished) have shown that static and impact forces of the same magnitude, applied through rubber-tired truck wheels, produce approximately equal strains in concrete cantilever beams that are free to deflect. The procedure followed in making these tests has been described (*14*). However, it does not follow from this that the same relationship will exist in a concrete pavement slab resting on a subgrade. In fact, there is some evidence to indicate that it may not.

A very limited series of exploratory tests of the effect of impact loads on pavement slabs has indicated the possibility that the stresses due to impact loads may be somewhat less than those due to static loads and that the difference between the two may not be the same in all portions of the slab. Any differences of this character that may exist undoubtedly result from the complex interrelation between pavement slab and subgrade and from the difference in time duration of the load application. The maximum impact reaction due to a wheel load is effective only for a small fraction of a second while static loads must be applied to the pavement for several minutes before an equilibrium of load and strain is obtained.

In the Arlington tests [3] it was found that in a pavement slab the time duration of the load application had a very important influence on the observed fiber deformation. From the time a static load was fully applied to the slab the observed fiber deformations increased at a fairly uniform rate for a period of several minutes before equilibrium was reached. The increase in deformation during this period amounted to as much as 15 percent. As a result (*15*), in all the studies of the effect of static loads, the loads were held constant for a period of 5 minutes after application before deformation measurements were made. The measured strains were therefore larger than would be caused by the momentary application of loads of the same magnitude.

However, even if significant differences are eventually found to exist between static and impact stresses in a pavement slab, there are no means for evaluating them at this time and therefore the assumption must be made that impact forces create the same stresses as static forces of the same magnitude. It appears that this is a safe practice and one which may introduce some factor of safety that at present is unknown.

Mathematical analysis of stress.—In 1919 Goldbeck (*20*) suggested approximate formulas for computing the stresses in concrete pavement slabs under certain assumed conditions of loading and subgrade support. Among these approximate formulas is one which has since become generally known as the "corner formula". This be expressed in the form

$$\sigma_c=\frac{3P}{h^2} \qquad (1)$$

where σ_c=maximum tensile stress, in pounds per square inch, in a diagonal direction in the top of the slab near a rectangular corner;

P=load, in pounds, applied at a point at the corner;

h=depth of slab in inches.

This simple formula is derived on the assumption that the load is applied at a point at the extreme corner of the slab; that the corner receives no support from the subgrade and acts as a simple cantilever; and that the fiber stresses in the slab are uniform on any section at right angles to a line bisecting the corner angle.

Some years later, in the analysis of the data from the Bates Road tests (*21*), it was found that there was a reasonably good agreement between the wheel loads that caused corner failure and loads computed by the corner formula. However, it is now quite definitely known that the corner formula gives stresses considerably higher than the actual stresses in pavement slabs, even under extreme conditions of warping. The agreement between computed loads and measured loads in the Bates Road report may be explained by the fact that the latter were static wheel loads while the loads that actually caused corner failures were the impact reactions due to these wheel loads. In view of the fact that the truck wheels were equipped with solid rubber tires, the impact loads were undoubtedly considerably higher than the static wheel loads.

In 1925 the analysis by Westergaard (*2*) made available for the first time a logical and scientific basis for evaluating the stresses in concrete pavements. This analysis concerns itself with the determination of maximum stresses in slabs of uniform thickness resulting from the following three conditions of loading:

1. Load applied close to the rectangular corner of a large slab.
2. Load applied in the interior of a large slab at a considerable distance from the edges.
3. Load applied at the edge of the slab at a considerable distance from any corner.

[3] The term "Arlington tests" will be used to designate the investigation of concrete pavement design made by the Bureau of Public Roads at the Arlington Experiment Farm and described in reports listed in the bibliography (*15, 16, 17, 18, 19*).

WESTERGAARD EQUATIONS GIVEN

The anlysis involves the following important assumptions:

1. That the concrete slab acts as a homogeneous, isotropic, elastic solid in equilibrium.
2. That the reactions of the subgrade are vertical only and that they are proportional to the deflections of the slab.
3. That the reaction of the subgrade per unit of area at any given point is equal to a constant, k, multiplied by the deflection at that point. The constant, k, is termed the "modulus of subgrade reaction" or "subgrade modulus" and is assumed to be constant at each point, independent of the deflections, and to be the same at all points within the area under consideration.
4. That the thickness of the slab is uniform.
5. That the load at the interior and at the corner of the slab are distributed uniformly over a circular area of contact. For the corner loading, the circumference of this circular area is tangent to the edges of the slab.
6. That the load at the edge of the slab is distributed uniformly over a semicircular area of contact, the center of the circle being on the edge of the slab.

For the three positions of load, the analysis results in equations which may be expressed as follows:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \qquad (2)$$

$$\sigma_i=0.275(1+\mu)\frac{P}{h^2}\log_{10}\left(\frac{Eh^3}{kb^4}\right) \qquad (3)$$

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-0.71\right] \qquad (4)$$

in which

P=load, in pounds;

σ_c=maximum tensile stress in pounds per square inch at the top of the slab, in a direction parallel to the bisector of the corner angle, due to a load P at the corner;

σ_i=maximum tensile stress in pounds per square inch at the bottom of the slab directly under the load P, when P is at a point in the interior of the slab at a considerable distance from the edges;

σ_e=maximum tensile stress in pounds per square inch at the bottom of the slab directly under the load P at the edge, and in a direction parallel to the edge;

h=thickness of the concrete slab, in inches;

μ=Poisson's ratio for concrete;

E=modulus of elasticity of the concrete, in pounds per square inch;

k=subgrade modulus, in pounds per cubic inch;

a=radius of area of load contact, in inches. The area is circular in the case of corner and interior loads and semicircular for edge loads;

b=radius of equivalent distribution of pressure

$$b=\sqrt{1.6a^2+h^2}-0.675h \text{ when } a<1.724h \text{-----} \quad (5)$$

$$b=a \text{ when } a>1.724h$$

Values of b for various values of a and h are given in table 2.

Value of Poisson's ratio.—If an isotropic, elastic material is subjected to stress in one direction a unit deformation is produced in the direction of the force and, in addition, a smaller deformation is produced in the direction perpendicular to the force. The relation between these two deformations, expressed as the ratio of the smaller to the larger, is known as Poisson's ratio. It appears in the Westergaard equations and therefore a value must be assigned to it.

The results of several investigations to determine the magnitude of Poisson's ratio are available (*22*, *23*, *24*). The general conclusion from these investigations is that there is no definite relationship between the strength of concrete and Poisson's ratio. With respect to other variables, such as age, the trends are not very definite and the conclusions reached by different investigators are not always in agreement. It is apparent that Poisson's ratio for a given concrete cannot be foretold and that for purposes of design it is necessary to select some reasonable and safe value.

TABLE 2.—*Values of b for various values of a and h, computed by equation 5*

Ratio a/h	Values of b in inches for different values of h in inches								
	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$	$h=9$	$h=10$	$h=11$	$h=12$
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*
0	1.30	1.63	1.95	2.28	2.60	2.93	3.25	3.58	3.90
.1	1.33	1.66	2.00	2.33	2.66	3.00	3.33	3.66	4.00
.2	1.43	1.78	2.14	2.50	2.85	3.21	3.57	3.92	4.28
.3	1.58	1.97	2.37	2.76	3.16	3.55	3.95	4.34	4.73
.4	1.78	2.23	2.67	3.12	3.57	4.01	4.46	4.90	5.35
.5	2.03	2.54	3.05	3.56	4.07	4.57	5.08	5.59	6.10
.6	2.32	2.90	3.48	4.06	4.64	5.22	5.80	6.38	6.96
.7	2.64	3.30	3.96	4.62	5.29	5.95	6.61	7.27	7.93
.8	2.99	3.74	4.49	5.23	5.98	6.73	7.48	8.22	8.97
.9	3.36	4.20	5.04	5.88	6.72	7.56	8.40	9.24	10.08
1.0	3.75	4.69	5.62	6.56	7.50	8.44	9.37	10.31	11.25
1.1	4.15	5.19	6.23	7.27	8.31	9.35	10.38	11.42	12.46
1.2	4.57	5.71	6.86	8.00	9.14	10.28	11.43	12.57	13.71
1.3	5.00	6.25	7.50	8.75	10.00	11.25	12.50	13.75	14.99
1.4	5.43	6.79	8.15	9.51	10.87	12.23	13.59	14.95	16.30
1.5	5.88	7.35	8.82	10.29	11.76	13.23	14.70	16.17	17.64
1.6	6.33	7.91	9.49	11.08	12.66	14.24	15.82	17.41	18.99
1.7	6.79	8.48	10.18	11.88	13.57	15.27	16.97	18.66	20.36
1.724 [1]	6.90	8.62	10.34	12.07	13.79	15.52	17.24	18.96	20.69

[1] When a/h is greater than 1.724, $b=a$

The digest by Richart and Roy (*22*) shows values of Poisson's ratio, obtained by several investigators and involving a number of variables, ranging from 0.08 to 0.28. Koenitzer (*24*) reports about 250 values for a range of conditions, of which the minimum is 0.08, the maximum is 0.40, and the average is 0.18. Approximately 20 percent of the values reported by Koenitzer do not exceed 0.15, 78 percent do not exceed 0.20 and 95 percent do not exceed 0.25.

If it be assumed, on the basis of these data, that a range of Poisson's ratio to be reasonably expected is from 0.10 to 0.20 and an average figure of 0.15 is assumed for design purposes, then the maximum error in computed stresses within this range will be plus or minus 4.3 percent for interior stresses and plus or minus 2.5 percent for edge stresses. The effect of Poisson's ratio on corner stresses is negligible. Even if Poisson's ratio happens to have the rather high value of 0.25 the error involved in assuming it equal to 0.15 will be only 8.7 percent for interior stresses and 5 percent for edge stresses, the effect on corner stresses still being negligible. It appears, therefore, that the general practice, first suggested by Westergaard, of assuming for the purpose of pavement design that Poisson's ratio is equal to 0.15, is an entirely reasonable one, and that value will be used hereafter in this paper.

In addition to the quantities that appear directly in the three stress equations, there is the radius of relative stiffness, l, which is defined by the equation

$$l=\sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}} \text{------------------} \quad (6)$$

Values of l for various values of E, h, and k are given in table 3.

Westergaard has expressed equation 2 in terms of l, as follows:

Corner loading

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right] \text{---------------} \quad (7)$$

and Bradbury (*9*) has shown that, when $\mu=0.15$, equations 3 and 4 may be expressed in the form:

Interior loading

$$\sigma_i=0.31625\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+1.0693\right] \text{----} \quad (8)$$

Edge loading

$$\sigma_e=0.57185\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+0.3593\right] \text{----} \quad (9)$$

NEW FORMULA FOR CORNER STRESSES IN AGREEMENT WITH TEST RESULTS

Modified equations for corner loading.—If, in equation 2, for corner loading, the radius of contact area, a, is assumed equal to zero then the influence of the subgrade modulus, k, and the modulus of elasticity, E, are eliminated and the equation reduces to the corner formula

$$\sigma_c=\frac{3P}{h^2} \text{---------------------------} \quad (1)$$

TABLE 3.—*Radius of relative stiffness, l, computed by equation 6* $\mu=0.15$

Modulus of elasticity of concrete E	Subgrade modulus k	Radius of relative stiffness, l, in inches for different values of h, in inches								
		$h=4$	$h=5$	$h=6$	$h=7$	$h=8$	$h=9$	$h=10$	$h=11$	$h=12$
Lb. per sq. in.	*Lb. per cu. in.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*	*In.*
3,000,000	50	23.9	28.3	32.4	36.4	40.2	43.9	47.6	51.1	54.5
	100	20.1	23.8	27.3	30.6	33.8	37.0	40.0	43.0	45.9
	150	18.2	21.5	24.6	27.7	30.6	33.4	36.1	38.8	41.4
	200	16.9	20.0	22.9	25.7	28.4	31.1	33.6	36.1	38.6
	300	15.3	18.1	20.7	23.3	25.7	28.1	30.4	32.6	34.8
	400	14.2	16.8	19.3	21.6	23.9	26.1	28.3	30.4	32.4
4,000,000	50	25.7	30.4	34.8	39.1	43.2	47.2	51.1	54.9	58.6
	100	21.6	25.6	29.3	32.9	36.4	39.7	43.0	46.2	49.3
	150	19.5	23.1	26.5	29.7	32.8	35.9	38.8	41.7	44.5
	200	18.2	21.5	24.6	27.7	30.6	33.4	36.1	38.8	41.4
	300	16.4	19.4	22.3	25.0	27.6	30.2	32.7	35.1	37.4
	400	15.3	18.1	20.7	23.3	25.7	28.1	30.4	32.6	34.8
5,000,000	50	27.2	32.1	36.8	41.4	45.7	49.9	54.0	58.0	62.0
	100	22.9	27.0	31.0	34.8	38.4	42.0	45.4	48.8	52.1
	150	20.7	24.4	28.0	31.4	34.7	37.9	41.1	44.1	47.1
	200	19.2	22.7	26.0	29.2	32.3	35.3	38.2	41.0	43.8
	300	17.4	20.5	23.5	26.4	29.2	31.9	34.5	37.1	39.6
	400	16.2	19.1	21.9	24.6	27.2	29.7	32.1	34.5	36.8
6,000,000	50	28.4	33.6	38.6	43.3	47.8	52.3	56.6	60.7	64.8
	100	23.9	28.3	32.4	36.4	40.2	43.9	47.6	51.1	54.5
	150	21.6	25.6	29.3	32.9	36.4	39.7	43.0	46.2	49.3
	200	20.1	23.8	27.3	30.6	33.8	37.0	40.0	43.0	45.9
	300	18.2	21.5	24.6	27.7	30.6	33.4	36.1	38.8	41.4
	400	16.9	20.0	22.9	25.7	28.4	31.1	33.6	36.1	38.6

The derivation of the corner formula (equation 1), involves two assumptions, of which one is manifestly incorrect and the other is very questionable. When the radius of contact area is zero the load is assumed to be concentrated at a point at the extreme corner of the slab. This is an impossible condition since a rubber-tired wheel distributes its load over an area of contact of appreciable size. The second assumption is, in effect, that when a load is applied to the corner of a slab which is warped upward the effect of subgrade support is completely eliminated. The combination of these two assumptions results in computed stresses that are much higher than have been observed in carefully conducted tests.

When the corner of the slab is warped upward there may be a complete lack of subgrade support immediately beneath the corner and to this extent the original Westergaard analysis (equations 2 or 7), which involves the assumption of uniform subgrade support, is incorrect. Westergaard has recognized this and has suggested a modification of the analysis which takes account of this condition (*25*). This modification involves assumptions as to the reduction in subgrade support which cannot be readily evaluated at the present time. However, it does recognize the fact, which is corroborated by test data, that while there may be no contact between slab and subgrade immediately beneath a corner load, nevertheless the subgrade support in the vicinity of the corner is effective in reducing the maximum stress by a considerable percentage below that computed by the corner formula.

In a somewhat limited but carefully conducted series of tests on large slabs under laboratory conditions, Spangler and Lightburn (*26*, *27*) observed corner stresses appreciably greater than those computed by the Westergaard equation.

As a result of these observations Bradbury (*9*) has suggested the modified equation

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a}{l}\right)^{0.6}\right] \qquad (10)$$

In effect this equation represents the assumption that the subgrade modulus in the vicinity of the corner is only one-fourth of the modulus that is effective under the other portions of the slab.

In the Arlington tests (*19*), in which the slabs were exposed to normal weather conditions, it has been found that in the daytime, when the corner is warped downward and has contact with the subgrade, there is very good agreement between observed stresses and those computed by the Westergaard formula (equation 7). However, at night, when the corner is warped upward, the observed stresses, while lower than those given by the corner formula, are much higher than those computed either by the Westergaard equation or by Bradbury's formula (equation 10).

Westergaard has shown that for the conditions assumed in his analysis the maximum corner stress occurs at a distance from the corner, measured along the diagonal bisector of the corner angle, equal to X_1 where

$$X_1=2\sqrt[4]{2}\,\sqrt{al}$$

In the Arlington tests it has been found that when the slab is warped upward the maximum stress occurs at a distance from the corner several inches greater than the computed value of X_1. It has also been found that observed stresses are in good agreement with stresses computed by the equation

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right] \qquad (11)$$

It will be observed that this equation has the same general form as the Westergaard formula (equation 7) and Bradbury's formula (equation 10). However, it is purely empirical and has no theoretical background. Its only virtue is its algebraic simplicity and the fact that it gives results that are in reasonably good agreement with a considerable number of tests on pavement slabs exposed to normal fluctuations of temperature and moisture. Its use is suggested pending the time when more exact information may be available.

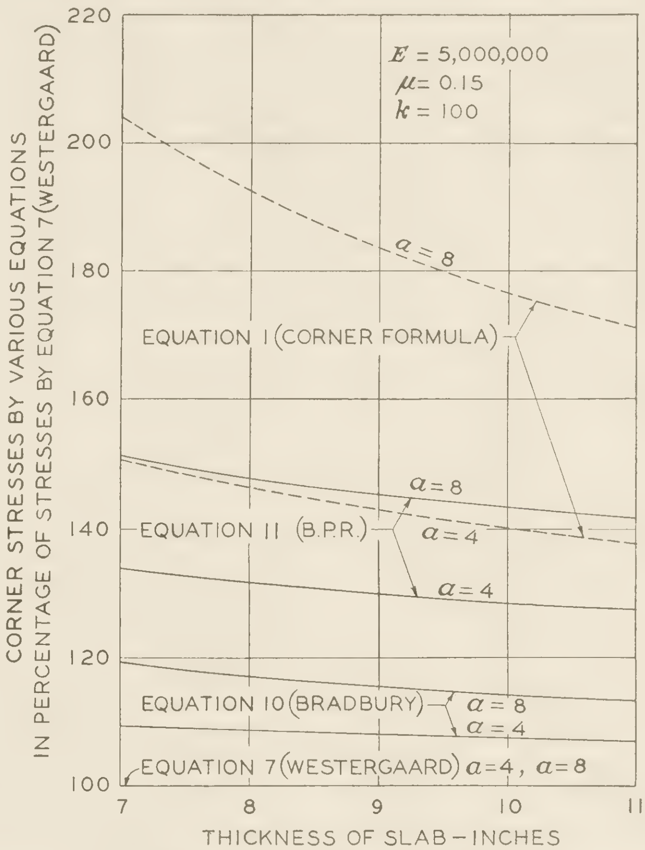

Figure 1.—Comparison of Corner Stresses Computed by Various Equations.

A comparison of the results given by equations 1, 7, 10, and 11 is shown in figure 1. For the range of conditions assumed, the corner stresses computed by Westergaard's formula (equation 7) are exceeded by those computed by Bradbury's formula (equation 10) by 7 to 20 percent, by those computed by equation 11 by 27 to 51 percent, and by those computed by the corner formula, equation 1, by 38 to 104 percent.

MODIFIED EQUATIONS FOR INTERIOR AND EDGE LOADING GIVEN

Modified equations for interior loading.—Early in the Arlington tests it was found that the observed stresses due to loads in the interior of the slab were not as great as those computed by equation 3 and as a result Westergaard modified his original analysis (*28*). The modified equation for stress due to interior loading is

$$\sigma_i=0.275\,(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2 Z\right] \qquad (12)$$

in which

L = maximum value of the radius of the circular area, with center at the point of load application, within which a redistribution of subgrade reactions is made;

Z=ratio of reduction of the maximum deflection.

Westergaard has stated that, under actual conditions, Z may be expected to vary between 0 and 0.39. When $Z=0$, equation 12 reduces to equation 3. He has also suggested as a reasonable assumption that $L=5l$. It is immediately apparent that the values assigned to Z and L and the relation of these values to each other have a major effect on the computed stresses. Moreover, reasonably exact values can be developed only from the data obtained in tests of large slabs.

As an approximation Bradbury (*9*) has suggested that an average value of $Z=0.20$ be assumed and this, and the further assumption that $L=5l$ and $\mu=0.15$, leads to the equation:

$$\sigma_i=0.31625\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+0.6330\right] \quad (13)$$

For the conditions which obtained in the Arlington tests, values of $L=1.75l$ and $Z=0.05$ were quite well established and these values, with $\mu=0.15$, lead to the equation

$$\sigma_i=0.31625\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+0.1788\right] \quad (14)$$

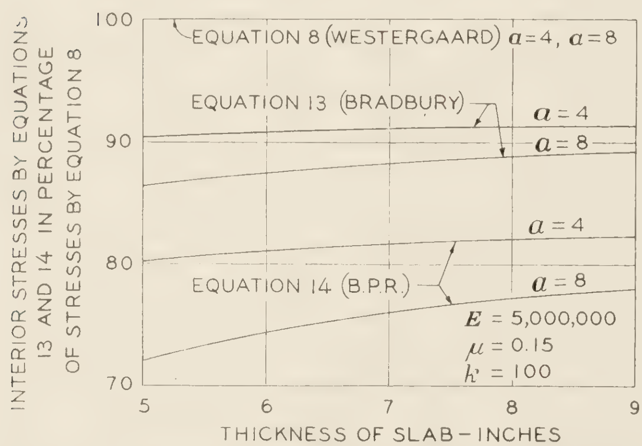

Figure 2.—Comparison of Interior Stresses Computed by Various Equations.

A comparison of the results given by equations 8, 13, and 14, is shown in figure 2. For the range of conditions assumed, the interior stresses computed by equation 14 are from 72 to 82 percent, and those computed by equation 13 are from 86 to 91 percent, of those computed by Westergaard's original formula (equation 8).

The reduction of interior stresses, as expressed by equation 12, is dependent on the characteristics of the subgrade and the slab and the complex reaction between them. Equation 14 is representative of what may be expected under the conditions obtaining in the Arlington tests but these were concerned with only one type of subgrade and one class of concrete. In view of this it is believed that equation 14, with its rather large stress reductions, is not suitable for general use as representative of average conditions. In the light of present knowledge it will be conservative, and not uneconomical, to continue to use the results given by the original Westergaard analysis, equation 8.

Modified equation for edge loading.—In the Arlington tests it has been found that for what may be considered

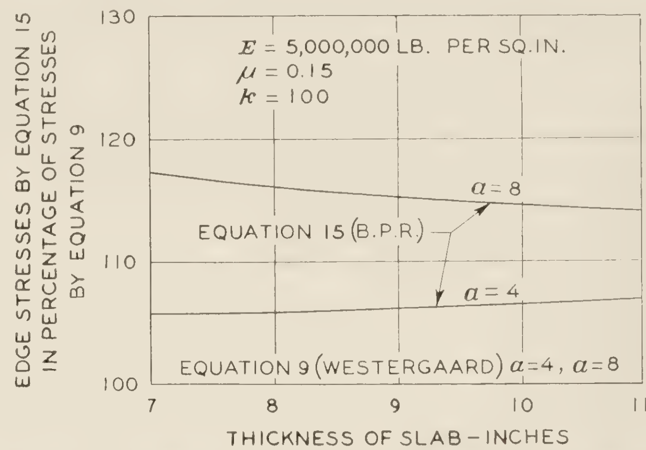

Figure 3.—Comparison of Edge Stresses Computed by Equations 9 and 15.

as average values of a, the radius of contact area, there is good agreement between observed edge stresses and those computed by Westergaard's formula (equation 9) when the slab is in an unwarped condition. For smaller values of a the observed stresses are somewhat less than the theoretical stresses and for larger values of a the observed stresses are somewhat greater than the theoretical stresses. However, the differences are not great and no serious errors will result from the use of equation 9 for the computation of edge stresses in a slab which is not warped. The same equation is also applicable when the edges of the slab are warped downward during the daytime, although in this case the computed stresses may generally be expected to be slightly less than the actual stresses.

When the edges of the slab are warped upward at night the observed load stresses exceed the theoretical stresses, as in the case of corner loading although not to the same extent. It has been found that the observed stresses under the conditions of nighttime warping are in reasonably good agreement with the empirical equation

$$\sigma_e=0.57185\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+\log b\right] \quad (15)$$

A comparison of the results given by equations 9 and 15 is shown in figure 3. For the range of conditions assumed, the edge stresses computed by equation 15 exceed those computed by equation 9 by 6 to 17 percent.

SIMPLIFIED METHOD OF COMPUTING STRESSES PRESENTED

Simplification of Stress Computations.—The equations of Westergaard and the modified equations that have been discussed are simple algebraic expressions but their solution requires a considerable amount of tedious labor. However, Bradbury (*9*) has suggested a simplified method of computation which reduces the determination of stress by means of these equations to a simple slide-rule operation.

He has pointed out that all the equations have the general form,

$$\sigma=\frac{CP}{h^2} \quad (16)$$

in which C is a quantity that may be termed a stress coefficient. The coefficients C_i and C_e, for interior and edge stresses, respectively, are fixed by the ratio l/b; while the coefficient C_c for corner stresses is fixed by the ratio a/l.

Values of stress coefficients are given in tables 4, 5, 6, and 7. Table 4 gives the coefficients for corner loading by the Westergaard equation 7. Table 5 gives coefficients for corner loading by the modified equation 11. Table 6 gives coefficients for interior loading by equation 8. Coefficients for interior loading by equation 13 may be obtained by subtracting 0.138, and those corresponding to equation 14 by subtracting 0.282, from the values given in table 6. Table 7 gives the coefficients for edge loading by equation 9. Table 8 gives a correction factor to be added algebraically to the coefficients of table 7 to obtain the stress coefficients corresponding to equation 15.

TABLE 4.—*Stress coefficients, C_c, for corner loading, computed by equation 7 (Westergaard), $\mu=0.15$*

Ratio a/l	C_c	Ratio a/l	C_c	Ratio a/l	C_c
0	3.000	0.20	1.594	0.40	0.869
0.01	2.767	0.21	1.552	0.41	.837
0.02	2.647	0.22	1.511	0.42	.805
0.03	2.549	0.23	1.471	0.43	.774
0.04	2.465	0.24	1.431	0.44	.743
0.05	2.388	0.25	1.392	0.45	.713
0.06	2.317	0.26	1.354	0.46	.682
0.07	2.251	0.27	1.316	0.47	.652
0.08	2.189	0.28	1.279	0.48	.622
0.09	2.129	0.29	1.243	0.49	.593
0.10	2.072	0.30	1.206	0.50	.563
0.11	2.018	0.31	1.171	0.51	.534
0.12	1.965	0.32	1.136	0.52	.505
0.13	1.914	0.33	1.101	0.53	.477
0.14	1.865	0.34	1.067	0.54	.448
0.15	1.817	0.35	1.033	0.55	.420
0.16	1.770	0.36	.999	0.56	.392
0.17	1.724	0.37	.966	0.57	.364
0.18	1.680	0.38	.933	0.58	.336
0.19	1.636	0.39	.901	0.59	.309
0.20	1.594	0.40	.869	0.60	.282

TABLE 5.—*Stress coefficients, C_c, for corner loading, computed by equation 11 (Bureau of Public Roads) $\mu=0.15$*

Ratio a/l	C_c	Ratio a/l	C_c	Ratio a/l	C_c
0	3.000	0.20	2.341	0.40	1.486
0.01	2.982	0.21	2.301	0.41	1.440
0.02	2.958	0.22	2.261	0.42	1.394
0.03	2.932	0.23	2.221	0.43	1.348
0.04	2.904	0.24	2.180	0.44	1.302
0.05	2.875	0.25	2.138	0.45	1.256
0.06	2.845	0.26	2.097	0.46	1.209
0.07	2.813	0.27	2.055	0.47	1.162
0.08	2.780	0.28	2.013	0.48	1.115
0.09	2.747	0.29	1.971	0.49	1.068
0.10	2.713	0.30	1.928	0.50	1.021
0.11	2.678	0.31	1.885	0.51	.973
0.12	2.643	0.32	1.841	0.52	.925
0.13	2.607	0.33	1.798	0.53	.877
0.14	2.570	0.34	1.754	0.54	.829
0.15	2.533	0.35	1.710	0.55	.781
0.16	2.496	0.36	1.666	0.56	.732
0.17	2.458	0.37	1.621	0.57	.684
0.18	2.419	0.38	1.576	0.58	.635
0.19	2.380	0.39	1.531	0.59	.586
0.20	2.341	0.40	1.486	0.60	.537

The procedure to be followed in using these tables is very simple. By means of the ratio a/h, b is determined, by interpolation if necessary, from table 2, and l is obtained from table 3. Then the ratios a/l and l/b are computed. Using the ratio a/l, the coefficient C_c is obtained from table 4 or table 5. Using the ratio l/b, the coefficient C_i is obtained from table 6 and the coefficient C_e from table 7. To obtain the stress coefficient, C'_e, corresponding to equation 15, the correction factor K_e corresponding to the value of a/h is obtained from table 8 and is added algebraically to the value of C_e obtained from table 7.

Effect of variables on computed stresses.—For a specific pavement design to be used in a specific location it is not possible at present to predetermine, with any degree of precision, the values to be assigned to several of the variables which appear in the stress equations. Therefore it is necessary, both when the design is for a particular project and when it is a general design to be used on a number of projects, to assign reasonable and rather conservative values to these variables. In order to do this it is necessary to have some knowledge of their relative effects on computed stressses.

It is apparent from the equations that the computed stress varies directly with the magnitude of the wheel load. The effect of variations in Poisson's ratio has already been discussed.

TABLE 6.—*Stress coefficients, C_i, for interior loading,[1] computed by equation 8, $\mu=0.15$*

Ratio l/b	C_i	Ratio l/b	C_i	Ratio l/b	C_i
1.0	0.338	6.0	1.323	11.0	1.656
1.1	.391	6.1	1.332	11.1	1.660
1.2	.438	6.2	1.341	11.2	1.665
1.3	.482	6.3	1.349	11.3	1.670
1.4	.523	6.4	1.358	11.4	1.675
1.5	.561	6.5	1.367	11.5	1.680
1.6	.596	6.6	1.375	11.6	1.685
1.7	.630	6.7	1.383	11.7	1.689
1.8	.661	6.8	1.391	11.8	1.694
1.9	.691	6.9	1.399	11.9	1.699
2.0	.719	7.0	1.407	12.0	1.703
2.1	.746	7.1	1.415	12.1	1.708
2.2	.771	7.2	1.423	12.2	1.712
2.3	.796	7.3	1.430	12.3	1.717
2.4	.819	7.4	1.438	12.4	1.721
2.5	.842	7.5	1.445	12.5	1.726
2.6	.863	7.6	1.452	12.6	1.730
2.7	.884	7.7	1.460	12.7	1.734
2.8	.904	7.8	1.467	12.8	1.739
2.9	.923	7.9	1.474	12.9	1.743
3.0	.942	8.0	1.481	13.0	1.747
3.1	.960	8.1	1.487	13.1	1.752
3.2	.977	8.2	1.494	13.2	1.756
3.3	.994	8.3	1.501	13.3	1.760
3.4	1.010	8.4	1.507	13.4	1.764
3.5	1.026	8.5	1.514	13.5	1.768
3.6	1.042	8.6	1.520	13.6	1.772
3.7	1.057	8.7	1.527	13.7	1.776
3.8	1.072	8.8	1.533	13.8	1.780
3.9	1.086	8.9	1.539	13.9	1.784
4.0	1.100	9.0	1.545	14.0	1.788
4.1	1.113	9.1	1.551	14.1	1.792
4.2	1.127	9.2	1.557	14.2	1.796
4.3	1.140	9.3	1.563	14.3	1.800
4.4	1.152	9.4	1.569	14.4	1.803
4.5	1.164	9.5	1.575	14.5	1.807
4.6	1.177	9.6	1.581	14.6	1.811
4.7	1.188	9.7	1.586	14.7	1.815
4.8	1.200	9.8	1.592	14.8	1.819
4.9	1.211	9.9	1.598	14.9	1.822
5.0	1.222	10.0	1.603		
5.1	1.233	10.1	1.609		
5.2	1.244	10.2	1.614		
5.3	1.254	10.3	1.619		
5.4	1.265	10.4	1.625		
5.5	1.275	10.5	1.630		
5.6	1.285	10.6	1.635		
5.7	1.294	10.7	1.640		
5.8	1.304	10.8	1.645		
5.9	1.313	10.9	1.651		
6.0	1.323	11.0	1.656		

[1] For values of C_i corresponding to equation 13, subtract 0.138 from the values given in this table.
For values of C_i corresponding to equation 14, subtract 0.282 from the values given in this table.

CONSERVATIVE VALUE OF SUBGRADE MODULUS RECOMMENDED

Effect of variations in subgrade modulus, k.—It has been stated repeatedly in the literature that variations in the modulus of subgrade reaction have a minor effect on the computed stresses. The accuracy of this statement appears to depend on the range of conditions that are under consideration and the degree of error in computed stresses that can be tolerated.

Figure 4 shows the effect of variations in subgrade modulus between 50 and 300 pounds per cubic inch on stresses computed for interior, corner, and edge loadings for a reasonable range in values of a, the radius of

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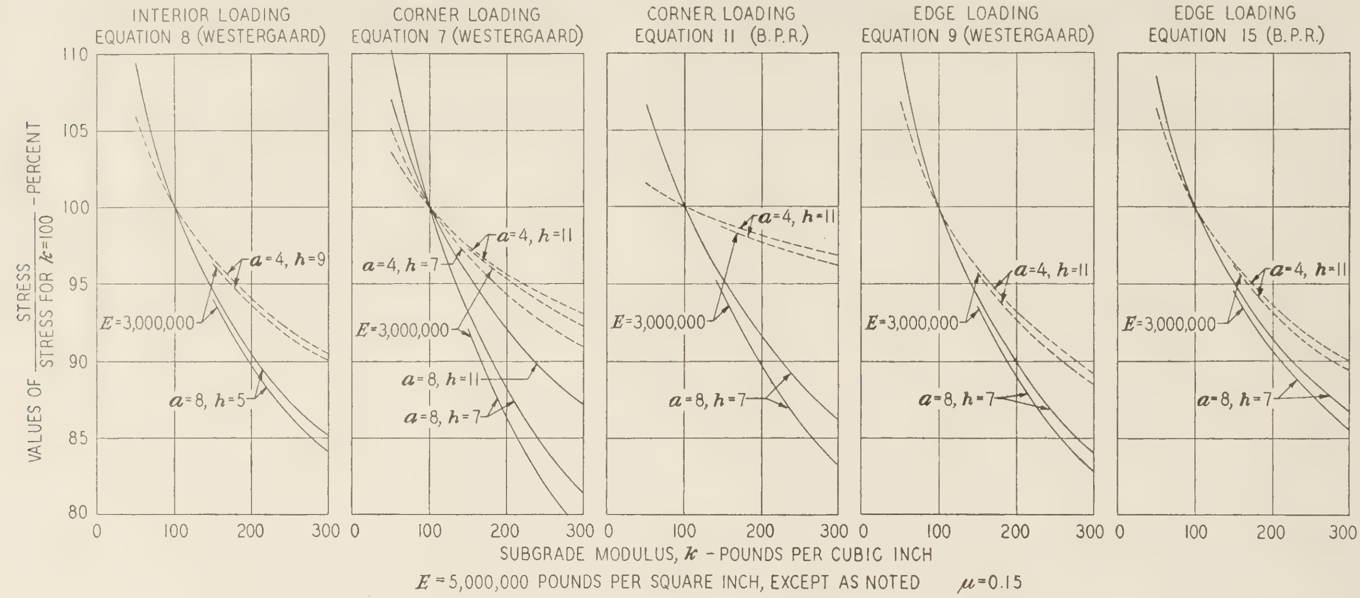

FIGURE 4.—EFFECT ON COMPUTED STRESSES OF VARIATIONS IN SUBGRADE MODULUS, k.

TABLE 7.—*Stress coefficients, C_e, for edge loading, computed by equation 9, $\mu=0.15$*

Ratio l/b	C_e	Ratio l/b	C_e	Ratio l/b	C_e
1.0	0.205	6.0	1.985	11.0	2.588
1.1	.300	6.1	2.002	11.1	2.597
1.2	.387	6.2	2.018	11.2	2.605
1.3	.466	6.3	2.034	11.3	2 614
1.4	.540	6.4	2.050	11.4	2.623
1.5	.608	6.5	2.065	11.5	2.632
1.6	.672	6.6	2.080	11.6	2.640
1.7	.733	6.7	2.095	11.7	2.649
1.8	.789	6.8	2.110	11.8	2.657
1.9	.843	6.9	2.124	11.9	2.666
2.0	.894	7.0	2.139	12.0	2.674
2.1	.943	7.1	2.153	12.1	2.682
2.2	.989	7.2	2.167	12.2	2.690
2.3	1.033	7.3	2.180	12.3	2.699
2.4	1.075	7.4	2.194	12.4	2.707
2.5	1.116	7.5	2.207	12.5	2.715
2.6	1.155	7.6	2.220	12.6	2.722
2.7	1.192	7.7	2.233	12 7	2.730
2.8	1.228	7.8	2.246	12.8	2.738
2.9	1.263	7.9	2.259	12.9	2.746
3.0	1.297	8.0	2.271	13.0	2.753
3.1	1.329	8.1	2.284	13.1	2.761
3.2	1.361	8.2	2.296	13.2	2.769
3.3	1.392	8.3	2.308	13.3	2.776
3.4	1.421	8.4	2.320	13.4	2.784
3.5	1.450	8.5	2.331	13.5	2.791
3.6	1.478	8.6	2.343	13.6	2.798
3.7	1.505	8.7	2.355	13.7	2.806
3.8	1.532	8.8	2.366	13.8	2.813
3.9	1.557	8.9	2.377	13.9	2.820
4.0	1.583	9.0	2.388	14.0	2.827
4.1	1.607	9.1	2.399	14.1	2.834
4.2	1.631	9.2	2.410	14.2	2.841
4.3	1.654	9.3	2.421	14.3	2.848
4.4	1.677	9.4	2.431	14.4	2.855
4.5	1.700	9.5	2.442	14.5	2.862
4.6	1.721	9.6	2.452	14.6	2.869
4.7	1.743	9.7	2.463	14.7	2.876
4.8	1.764	9.8	2.473	14.8	2.882
4.9	1.784	9.9	2.483	14.9	2.889
5.0	1.804	10.0	2.493		
5.1	1.824	10.1	2.503		
5.2	1.843	10.2	2.513		
5.3	1.862	10.3	2.522		
5.4	1.881	10.4	2.532		
5.5	1.899	10.5	2.541		
5.6	1.917	10.6	2.551		
5.7	1.934	10.7	2.560		
5.8	1.952	10.8	2.569		
5.9	1.969	10.9	2.578		
6.0	1.985	11.0	2.588		

contact area, and h, the depth of the slab. All stresses are expressed as percentages of the stresses computed for $k=100$. The curves that are continuous from $k=50$ to $k=300$ are for stresses computed with the modulus of elasticity, E, equal to 5,000,000 pounds per square inch.

TABLE 8.—*Values of correction factor,*[1] *K.*

Ratio a/h	Values of K_e for different values of h in inches								
	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$	$h=9$	$h=10$	$h=11$	$h=12$
0	−0.140	−0.085	−0.040	−0.001	0.032	0.062	0.087	0.111	0.133
0.1	−.134	−.079	−.034	.005	.038	.067	.093	.117	.139
0.2	−.117	−.062	−.017	.022	.055	.084	.110	.134	.156
0.3	−.092	−.037	.009	.047	.080	.109	.135	.159	.181
0.4	−.062	−.006	.039	.077	.110	.140	.166	.189	.211
0.5	−.029	.026	.071	.110	.143	.172	.198	.222	.244
0.6	.004	.059	.104	.143	.176	.205	.231	.255	.277
0.7	.036	.091	.137	.175	.208	.237	.263	.287	.309
0.8	.067	.122	.167	.206	.239	.268	.294	.318	.339
0.9	.096	.151	.196	.235	.268	.297	.323	.347	.368
1.0	.123	.178	.223	.262	.295	.324	.350	.374	.396
1.1	.148	.204	.249	.287	.320	.350	.376	.399	.421
1.2	.172	.227	.273	.311	.344	.373	.400	.423	.445
1.3	.194	.250	.295	.333	.366	.396	.422	.445	.467
1.4	.215	.270	.316	.354	.387	.416	.443	.466	.488
1.5	.234	.290	.335	.373	.407	.436	.462	.486	.507
1.6	.253	.308	.353	.392	.425	.454	.480	.504	.526
1.7	.270	.326	.371	.409	.442	.471	.498	.521	.543
1.724 [2]	.274	.330	.375	.413	.446	.475	.502	.525	.547

[1] To be added algebraically to the edge coefficient, C_e, obtained from table 7, to obtain the edge coefficient, C_e', corresponding to equation 15_{10}.
[2] When a/h is greater than 1.724, $b=a$ and $K_e=0.57185$ (log 10 $a-0.3593$).

The curves that are only partially complete are for stresses based on a value of E equal to 3,000,000 pounds per square inch. The upper portions of these curves are omitted since they so nearly coincide with the upper portions of the curves for $E=5,000,000$ that their inclusion would detract from the clarity of the charts.

It is evident from these curves that the value of E has no significant influence on the relation between subgrade modulus and stress when, as in this case, stresses are expressed as percentages of a basic stress which is different for each curve. Therefore, the subsequent discussion of the effect on stress of variations in the subgrade modulus will be confined to the curves for $E=5,000,000$.

It will be observed in the second chart from the left in figure 4 that the two curves, one for the minimum value of a in combination with the maximum value of h, and the other for the maximum value of a in combination with the minimum value of h, form an envelope for the curves for all intermediate values of a and h. In order to clarify the presentation, only these envelope curves are shown in the other charts of this and succeeding figures of similar character.

Before discussing figure 4 it will be well to examine the available data regarding observed values of the subgrade modulus. Unfortunately, these data are very meager. It is not known if a value of 50 pounds per cubic inch is the minimum that may be expected but there is reason to believe that the maximum may exceed 300 pounds per cubic inch, at least in some cases. Therefore the range that may be encountered in practice is not known.

In corner-loading tests and working with what may be termed synthetic subgrades, that is, earth subgrades consolidated in the laboratory by tamping, Spangler (*26*) observed in one very stiff clay subgrade (probably very dry) a subgrade modulus of the order of 1,000 pounds per cubic inch. In another test, with a subgrade of more normal characteristics, he observed that the apparent subgrade modulus was reduced by repeated corner loading from about 275 to about 40 pounds per cubic inch.

In still another corner-loading test Spangler and Lightburn (*27*) found that the subgrade modulus was constant at a given point in the slab but varied with the distance of the point from the corner, being about 300 pounds per cubic inch at the corner and about 75 pounds per cubic inch at distances of 4.5 feet from the corner. They concluded, however, that the assumption of a uniform value of the subgrade modulus appears to be justifiable for analytical solutions since stresses computed with a modulus equal to about the average of the two extreme values were in good agreement with observed stresses.

In considering the values of subgrade modulus obtained in the tests by Spangler and Lightburn it is well to remember that the subgrades with which they worked were protected from the weather and were not exposed to natural fluctuations of moisture.

In the Arlington tests the pavement slabs were exposed to the weather but it is necessary to bear in mind that only one subgrade was involved. In these tests the values of the subgrade modulus observed under normal conditions of subgrade support varied from about 170 to about 280 pounds per cubic inch.

These meager data indicate that the subgrade modulus may vary over a rather wide range, the limits of which are unknown; that its value may be affected by repeated loading of the slab; and that, at the same location, it is likely to be different at different times. The development of additional data is hampered by the present lack of any simple method of making the required tests over the wide range of conditions that merit study. The situation makes it highly desirable to be conservative in the selection of values of the modulus for use in stress computations.

Examination of figure 4 shows that variations in subgrade modulus have little effect on stresses computed by the modified equation for corner loading, equation 11, for small values of a and large values of h. The effect of variations in the modulus on interior, corner, and edge stresses computed by the Westergaard equations, on edge stresses computed by equation 15, and on corner stresses by equation 11 for large values of a and small values of h, is very similar.

On the assumption that a range in subgrade modulus from 50 to 300 pounds per cubic inch can reasonably be expected in practice, figure 4 shows that stresses computed on the basis of $k=300$, may be too low by as much as 25 percent if the modulus happens to have a value of 50. On the other hand, stresses computed on the assumption that $k=100$ will be too low by less than 10 percent if k happens to equal 50.

In view of all the uncertainties, a value of the subgrade modulus equal to 100 pounds per cubic inch is suggested as a reasonable figure for general use, pending the development of more exact information than is now available.

VALUE OF E=5 MILLION POUNDS PER SQUARE INCH SUGGESTED FOR GENERAL USE

Effect of variations in modulus of elasticity of concrete.—In contrast to the lack of data concerning the subgrade modulus, there is a wealth of information with respect to the modulus of elasticity of concrete. Numerous investigations have demonstrated that, in general, the modulus of elasticity increases with age, with increase in strength of the concrete, and with increase in temperature; that it may be higher in wet concrete than in dry; and that it is influenced by the character of the aggregate.

Thirty-five reports on the subject, published during the period 1928 to 1938, inclusive, and involving many variables such as type of aggregate, type of cement, water-cement ratio, and age, give values of the modulus of elasticity ranging from about 1,000,000 to 7,000,000 pounds per square inch for concrete ranging in compressive strength from about 1,000 to 7,000 pounds per square inch. For nearly all of the specimens involved in these investigations the ratio of the modulus of elasticity to the compressive strength falls between the values of 650 and 1,500 and a fair average value of this ratio for all the specimens is 1,000. This is in agreement with the building regulations of the American Concrete Institute (*29*) which recommend that for design purposes the modulus of elasticity of concrete be taken as 1,000 times its compressive strength.

For concrete of the character generally used in pavement construction a range in the value of the modulus of elasticity from 3,000,000 to 6,000,000 pounds per square inch may reasonably be expected. Within this range it is believed that the tendency will be for the values to be high rather than low and the use of relatively high values in design is on the side of safety. The concrete used in the Arlington tests, with flexural and compressive strengths at 28 days of 765 and 3,525 pounds per square inch, respectively, is believed to be fairly representative of the average run of paving concrete. The modulus of elasticity of this concrete, as determined by flexure tests of beams, was about 4,500,000 pounds per square inch for air-dry beams and about 5,500,000 pounds per square inch for beams in a moist condition. The same range in values was observed in tests on the pavement slabs themselves, the higher values being obtained in winter and the lower values in summer.

Figure 5 shows the effect of variations in modulus of elasticity between 3,000,000 and 6,000,000 pounds per square inch on stresses computed for interior, corner, and edge loadings for the same range in values of a and h as in figure 4 and for values of $k=100$ and $k=300$. All stresses are expressed as percentages of the stresses computed for $E=5{,}000{,}000$. It may be concluded from these curves that variations in the modulus of elasticity between 3,000,000 and 6,000,000 pounds per square inch do not have a major influence on computed stresses and that the effect of these variations is not

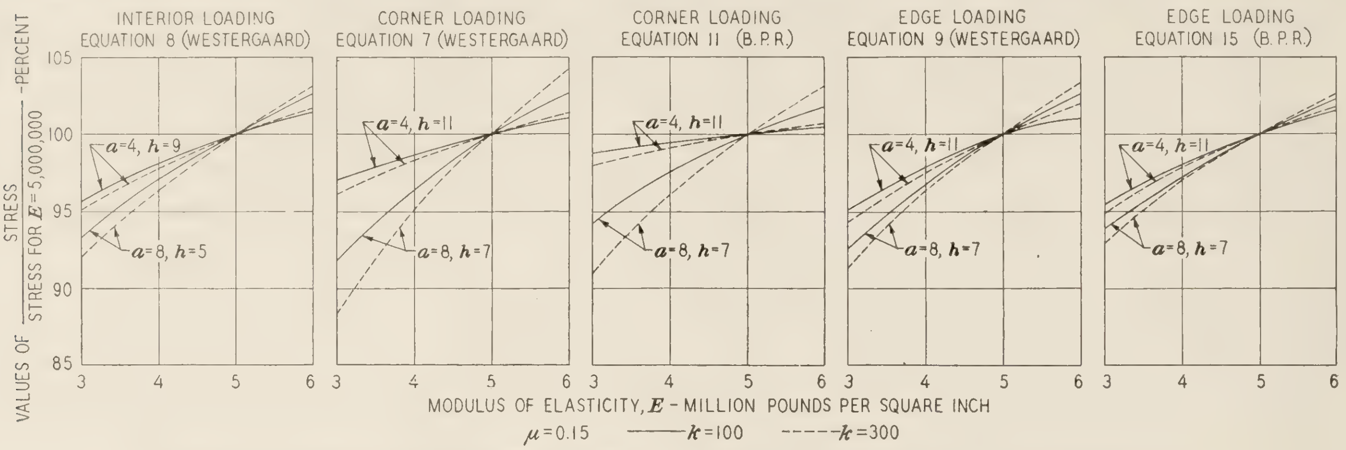

FIGURE 5.—EFFECT ON COMPUTED STRESSES OF VARIATIONS IN MODULUS OF ELASTICITY, E.

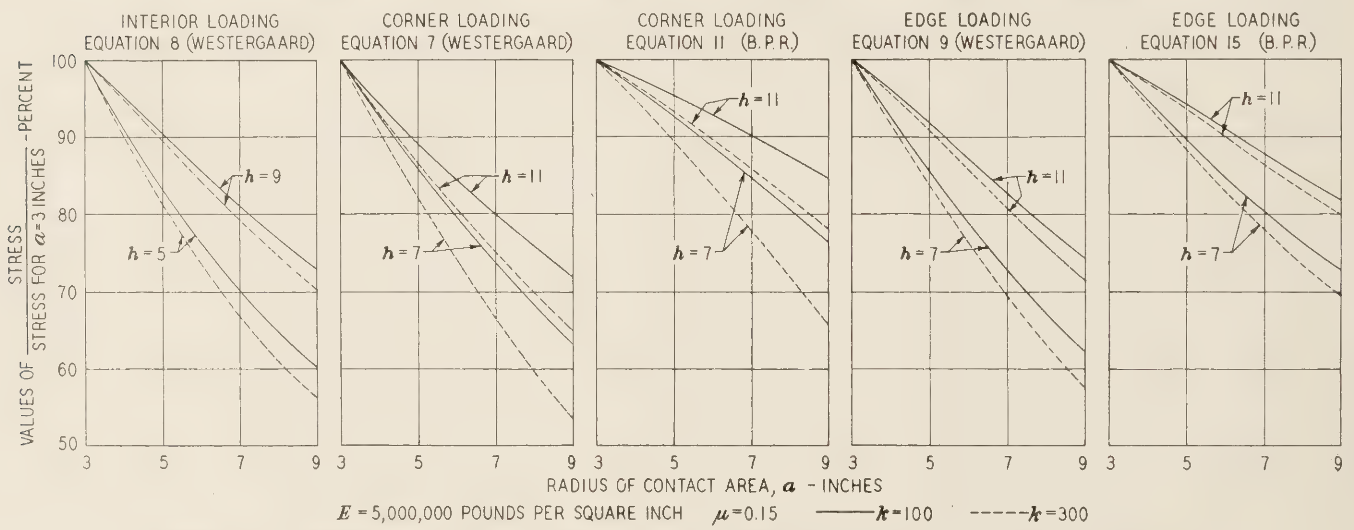

FIGURE 6.—EFFECT ON COMPUTED STRESSES OF VARIATIONS IN RADIUS OF CONTACT AREA, a.

greatly influenced by variations in the subgrade modulus.

Since it is on the side of safety to use relatively high values of the modulus of elasticity and since it is believed that it is representative of what may be expected in practice, the value of $E=5{,}000{,}000$ pounds per square inch is suggested for general use.

Variations in radius of contact area.—The radius of contact area, a, appears directly in the equations for corner loading and, through the radius, b, indirectly in the equations for interior and edge loading. Its marked effect on computed stresses is not readily apparent except by some such means as the charts of figure 6.

This figure shows the effect of variations in the radius of contact area between 3 and 9 inches on stresses computed for interior, corner and edge loadings for the same range in values of h as in figures 4 and 5 and for values of $k=100$ and $k=300$. It will be observed that an increase in the radius, a, from 3 to 9 inches may reduce the computed stress by more than 40 percent. It will also be observed that variations in the value of a have less effect on corner stresses and edge stresses computed by equations 11 and 15 than on those computed by equations 7 and 9.

Values of the radius of contact area.—Figure 7 shows the relation between static load and contact area for single and dual high-pressure and balloon tires. The curves are based on data developed by the Bureau of Public Roads in tests of single high-pressure and balloon tires, each in a range of sizes, subjected to static loads ranging from rated tire capacity to more than twice the rated capacity. The curves for single tires shown in figure 7 are closely representative of individual test results throughout the entire range of loadings, indicating that the relation between load and contact area is not appreciably affected by loads in excess of the rated tire capacity.

The curves of figure 7 for dual tires were developed from the data for single tires by assuming the tires to be spaced in accordance with the recommendations of the Tire and Rim Association, and adding to twice the contact area of one tire the area between the two tire impressions.

Figure 8 shows the relation between the wheel load and the radius of tire contact area. These curves were developed from those of figure 7 by assuming the tire contact area to be circular. The further assumption is made in connection with these data that they apply to both static and impact wheel loads.

All the assumptions that have been mentioned, and the additional one that the load is uniformly distributed over the contact area, require discussion.

ASSUMPTIONS REGARDING CONTACT AREAS OF TIRES DISCUSSED

It is known that the distribution of load under a pneumatic tire is not uniform (*30*) and that the shape of the tire impression tends to be elliptical rather than

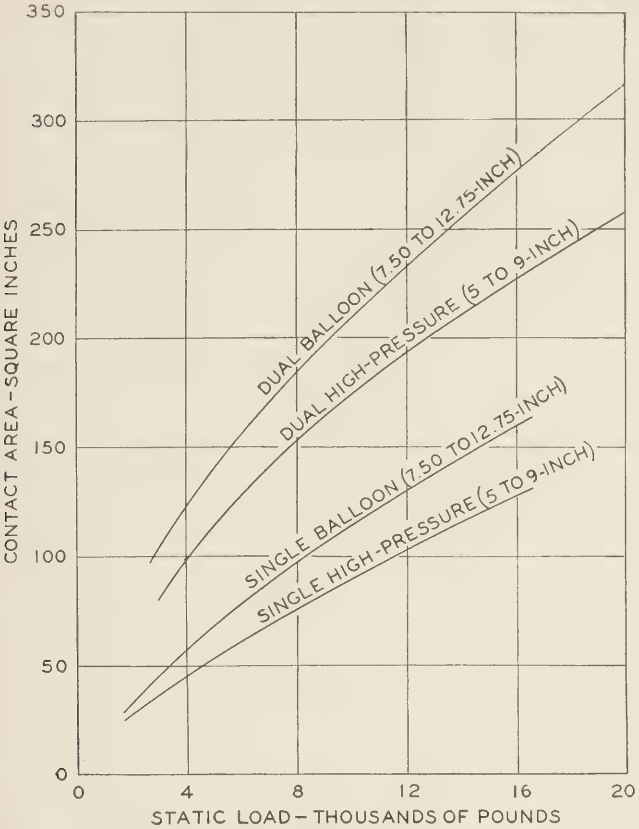

Figure 7.—Relation Between Static Load and Area of Contact for Pneumatic Tires; Average Results of Tests with Static Loads Ranging From Rated Tire Capacity to More Than Twice the Rated Capacity. (Areas of Contact for Dual Tires Computed From Tests with Single Tires and Include the Areas Between the Two Tire Impressions With the Tires Spaced in Accordance With the Recommendations of the Tire and Rim Association.)

circular. Nevertheless, it is believed that the assumption of uniform loading over a circular area equivalent to the measured contact area will lead to no serious error.

In computing the contact area for dual tires from the data for single tires, the area between the tire contacts is included. Since the area between the tire contacts actually receives no load, this procedure has been questioned. No tests have been made to determine the correctness of the assumption but very limited analysis of certain data developed in the Arlington tests indicate that it is not wholly unreasonable.

Unreported tests by the Bureau of Public Roads indicate that contact areas under impact and equivalent static loads are not greatly different for pneumatic tires of the high-pressure and balloon types. There are also data *(31)* indicating that the vertical deflections of solid and cushion tires are practically the same for the two types of load. While not conclusive, this information appears to justify the assumption that the curves of figure 8 are applicable to impact loads as well as to static loads.

Much additional research work is necessary to prove or disprove the validity of the assumptions that have been discussed. In the absence of such investigations it is necessary to make some assumptions and it is believed that those suggested are reasonable. Also, in the absence of more information than is now available, it is believed that further refinement in the use of existing data is unwarranted.

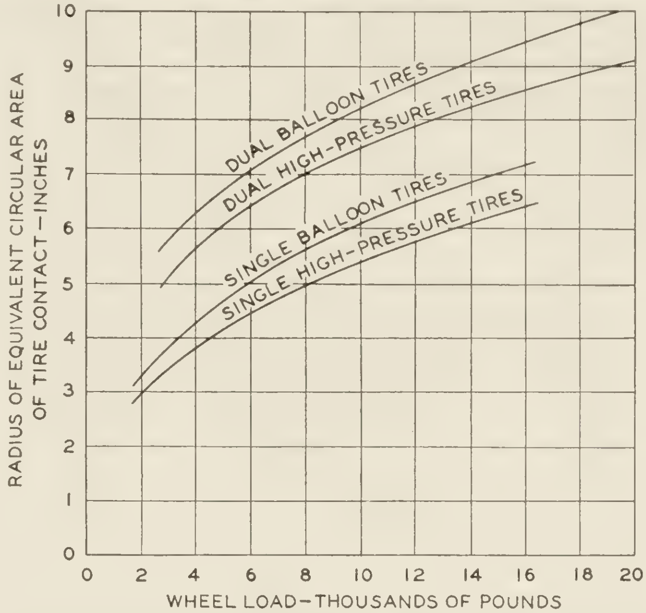

Figure 8.—Relation Between Wheel Load (Static or Impact) and Radius of Equivalent Circular Area of Tire Contact. Radii Correspond to Contact Areas Shown in Figure 7.

Radius of contact area for edge loading.—The Westergaard analysis assumes that interior and corner loads are applied on circular bearing areas and that edge loads are applied on semicircular bearing areas. Therefore it is necessary to decide: (1) If the semicircle used for edge loading is to have the same area as the circle used for interior and corner loading, or (2) if the semicircle is to have the same radius as the circle. The first procedure involves the assumption of equal unit pressure on the circular and semicircular areas and the second involves the assumption that the unit pressure on the semicircular area is twice as great as on the circular area.

When a wheel equipped with a single pneumatic tire moves along the edge of a pavement slab with depressed shoulders in such manner that only a part of the tire tread is in contact with the slab, the shape of the area of tire contact is undoubtedly changed but the effect on its area is unknown. For this case either assumption as to radius of contact area might be justified.

However, the situation is somewhat different with respect to the dual tires that are common equipment for the heavier wheel loads. It is not uncommon to see wheels with dual tires operated so close to the edge of the pavement that the entire wheel load is carried by the inside tire. In this case the tire load is doubled without a corresponding increase in contact area. For example, assuming an 8,000 pound static wheel load on dual high-pressure tires, table 1 shows that 11,800 pounds is the total impact reaction for this wheel load, and figure 7 shows a corresponding contact area of approximately 194 square inches. Also from figure 7 it is found that for this same load on a single tire the contact area is approximately 102 square inches. The corresponding unit pressures are about 61 and 116 pounds per square inch respectively. In the same manner it may be shown that the same wheel load on dual balloon tires may be expected to develop unit pressures of approximately 49 pounds per square inch over the full area of contact and 88 pounds per square inch when the load is concentrated on one tire.

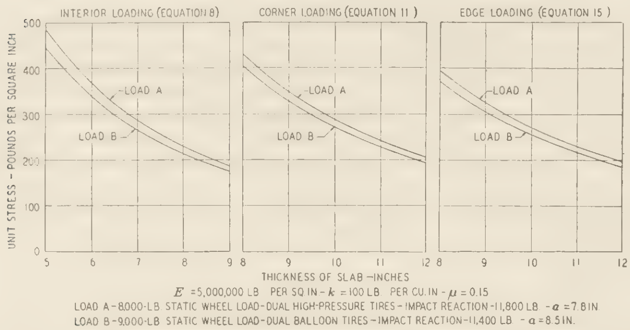

FIGURE 9.—COMPARISON OF STRESSES DUE TO 8,000-POUND WHEEL LOAD ON HIGH-PRESSURE TIRES AND 9,000-POUND WHEEL LOAD ON BALLOON TIRES.

In view of these facts it is recommended that, when the design is based on dual-tire equipment, the radius of area of contact for edge loadings be the same as for interior and corner loadings. Also, in view of the uncertainty regarding single tires, it is suggested that when the design is based on single-tire equipment, the area of contact for edge loadings be the same as for interior and corner loadings. If r is the radius of a circle then the radius of a semicircle of equivalent area equals $r\sqrt{2}$.

Variations in thickness of slab, h.—The fact that the thickness of the slab, h, exerts a major influence on computed stresses is evident from the stress equations. Since an exponential value of h appears twice in each stress equation and, in the equations for interior and edge loading an exponential value of h is also involved in the derivation of the radius, b, the relation between slab thickness and computed stress is not a simple one.

The relation between slab thickness and load stresses is shown graphically in figure 9 for two loads; one a static load of 8,000 pounds on a wheel equipped with dual high-pressure pneumatic tires, and the other a static load of 9,000 pounds on a wheel equipped with dual balloon tires. The impact reactions corresponding to these wheel loads are taken from table 1 and the corresponding radii of contact areas from figure 8. For the slab thicknesses ordinarily encountered in practice, the heavier wheel load on balloon tires gives stresses lower than those for the lighter wheel load on high-pressure tires by about 20 pounds per square inch. Here is justification for the requirement of the Uniform Vehicle Code (*32*) that the maximum wheel load on high-pressure tires be limited to 8,000 pounds and that on balloon tires to 9,000 pounds. It may also be noted that, for slabs of equal thickness, the stress due to corner loading is only slightly in excess of that due to edge loading.

EQUATIONS FOR COMPUTING TEMPERATURE WARPING STRESSES PRESENTED

Warping stresses due to temperature differential.—Changes in the temperature of concrete produce corresponding changes in its volume. A rise in temperature causes expansion of the concrete and a drop in temperature causes it to contract.

The temperature of a concrete pavement is constantly changing owing to variations in air temperature and during these changes in air temperature, which take place at a relatively rapid rate, the temperature in the slab does not remain constant throughout its depth. During the heat of the day in summer the top of the slab is warmer than the bottom while at night the reverse may be true. This differential in temperature between the two surfaces of the slab causes it to warp or curl and, since free warping is prevented by the weight of the slab, bending stresses are developed.

As early as 1926 Westergaard (*33*) presented a theoretical analysis of warping stresses due to temperature but their importance has not been generally recognized, possibly owing to the fact that in his stress computations he assumed a rather low value for the temperature differential. It remained for the Arlington tests (*16*) to demonstrate that these warping stresses may be as great as those produced by heavy wheel loads.

Westergaard's analysis covers slabs of infinite length and width, those of finite width and infinite length, and suggests a procedure to be followed in slabs having finite dimensions in both directions. On the basis of this analysis Bradbury (*9*) has developed general equations for the computation of temperature-warping stresses in the edge and interior of pavement slabs of the usual dimensions.

The following equations are not in exactly the same form as Bradbury's but they give identical results:

Edge Stresses

$$\sigma_{xe}=\frac{C_xEet}{2} \qquad (17)$$

Interior Stresses

$$\sigma_x=\frac{Eet}{2}\left(\frac{C_x+\mu C_y}{1-\mu^2}\right) \qquad (18)$$

$$\sigma_y=\frac{Eet}{2}\left(\frac{C_y+\mu C_x}{1-\mu^2}\right) \qquad (19)$$

in which

σ_{xe} = maximum stress, in pounds per square inch, in the extreme fiber at the edge of the slab, in the direction of slab length. At the extreme edge the stress at right angles to the edge is zero;

σ_x = maximum stress, in pounds per square inch, in the extreme fiber at the interior of the slab, in the direction of slab length;

σ_y = maximum stress, in pounds per square inch, in the extreme fiber at the interior of the slab, in the direction of slab width;

E = modulus of elasticity of concrete, in pounds per square inch;

e = thermal coefficient of expansion and contraction of concrete per degree Fahrenheit;

t = difference in temperature between top and bottom of slab, in degrees Fahrenheit;

C_x and C_y are coefficients determined from the curve in figure 10.

In figure 10:

L_x = length of slab in inches;

L_y = width of slab in inches;

l = radius of relative stiffness in inches (equation 6);

C_x corresponds to the value of $\frac{L_x}{l}$

C_y corresponds to the value of $\frac{L_y}{l}$

The data in figure 10 are also given in table 9.

The direction of slab warping is determined by the relation between the temperature in the top of the slab and that in the bottom and this in turn determines whether the resulting stress is a tensile stress in the top

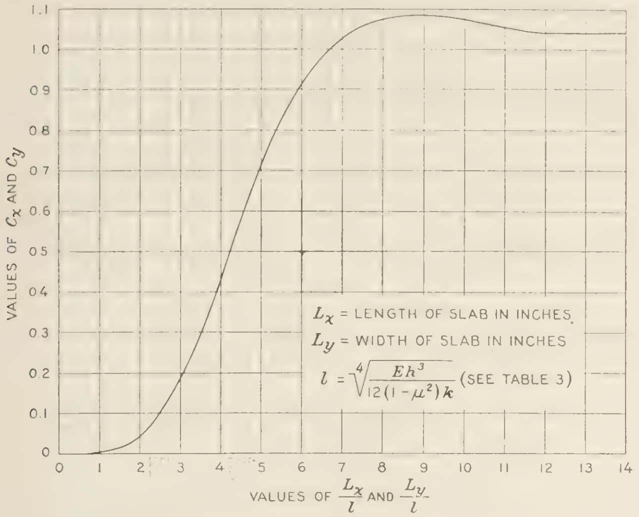

FIGURE 10.—COEFFICIENTS FOR WARPING STRESSES DUE TO TEMPERATURE.

of the slab or a tensile stress in the bottom of the slab. Of course, in either case an equal compressive stress is created in the opposite surface. For convenience the temperature differential will be considered positive when the top of the slab is at a higher temperature than the bottom and negative when the top of the slab is at a lower temperature than the bottom. A positive differential creates tensile stress in the bottom of the slab and a negative differential creates tensile stress in the top of the slab.

TABLE 9.—*Coordinates of curve of figure 10*

$\frac{L_x}{l}$ or $\frac{L_y}{l}$	C_x or C_y	$\frac{L_x}{l}$ or $\frac{L_y}{l}$	C_x or C_y	$\frac{L_x}{l}$ or $\frac{L_y}{l}$	C_x or C_y
1.41	0.010	4.95	.701	7.78	1.069
2.12	.051	5.66	.856	8.49	1.084
2.83	.148	6.37	.964	9.90	1.078
3.54	.309	6.69	1.000	11.31 [1]	1.052
4.24	.508	7.07	1.032		

[1] For values of $\frac{L_x}{l}$ or $\frac{L_y}{l}$ greater than 11.31, the values of C_x and C_y are determined by a composite curve constructed as follows:

Extend the curve plotted from the data in the above table from $\left(\frac{L_x}{l}=11.31,\ C_x=1.052\right)$ toward $\left(\frac{L_x}{l}=14.14,\ C_x=1.009\right)$ until it intersects a horizontal line drawn through $C_x=1.043$. C_x or C_y for all values of $\frac{L_x}{l}$ or $\frac{L_y}{l}$ to the right of this intersection is equal to 1.043.

Value of temperature differential.—The data developed in the Arlington tests (*16*) showed that the maximum temperature differential varies with the depth of the slab, being greater in thick slabs than in thin ones. The maximum positive differential occurs in the daytime and is greater in summer than in winter. The maximum negative differential occurs at night and is much the same in both winter and summer. The published data are summerized in tables 10 and 11.

From these data Bradbury (*9*) concluded that, for purposes of design computations, the maximum positive temperature differential might be assumed as 3.0° *F.* per inch of slab thickness and the maximum negative differential as 1.0° F. per inch of slab thickness. These appear to be reasonable figures for general use but it should be recognized that they are merely average figures and will result in computed stresses that may be appreciably lower than the stresses that will occur at times in the pavement.

TABLE 10.—*Summary of values of maximum positive temperature differentials observed in Arlington tests on 27 days between April 3 and June 4, 1934* [1]

	At edge of slab of uniform thickness		Thickened-edge section 9–6–9 inch		
	6-inch slab	9-inch slab	Edge	18 inches from edge	36 inches from edge
	° *F.*	° *F.*	° *F.*	° *F.*	° *F.*
Maximum	+24	+33	+33	+31	+28
Minimum	+14	+20	+18	+17	+15
Average	+19	+27	+27	+25	+22

[1] Data from table 2, PUBLIC ROADS, November 1935.

TABLE 11.—*Summary of values of maximum temperature differentials observed in Arlington tests on 17 days during 1931, 1932 and 1933* [1]

	6-inch slab				9-inch slab	
	April to August, inclusive		September to February, inclusive		April to August, inclusive	
	Day	Night	Day	Night	Day	Night
	° *F.*	° *F.*	° *F.*	° *F.*	° *F.*	° *F.*
Maximum	+24.3	−6.5	+15.6	−6.7	+31.0	−9.2
Minimum	+18.7	−4.5	+8.2	−1.3	+22.3	−5.7
Average	+21.2	−5.8	+11.8	−4.1	+26.9	−7.5

[1] Data from table 1, PUBLIC ROADS, November 1935.

FOR TEMPERATURE WARPING, INTERIOR STRESSES EXCEED EDGE STRESSES

Value of the thermal coefficient of expansion.—The thermal coefficient of expansion and contraction of concrete depends on a number of factors, among which the character of the aggregate appears to be the most important. Data from a number of investigations indicate that in general the highest thermal coefficient will be found in concrete containing siliceous aggregates and that considerably lower values may be expected in concrete made with granite, limestone, or diabase aggregates. A summary of data given by various authorities (*34*) shows values of the thermal coefficient ranging from about 0.000004 to about 0.000007 per degree Fahrenheit for concrete having a cement content comparable to that used in pavement construction.

The concrete used in the Arlington tests, with a limestone coarse aggregate and a siliceous fine aggregate, had a coefficient of approximately 0.000005 per degree Fahrenheit and this value appears to be a satisfactory one for general use. However, when the circumstances are such as to make this possible, it will be well to select a value appropriate for the character of concrete that is under consideration.

Computed warping stresses.—The Arlington tests were all made on slabs that varied in dimensions only in depth. Within these limitations the observed warping stresses due to temperature differential were in reasonably good agreement with computed stresses.

Stresses computed by the Bradbury equations are shown graphically in figure 11 for the interior, and in figure 12 for the edge, of slabs 10 feet wide and of various lengths, depths of 6 and 9 inches, and values of the subgrade modulus of 100 and 300 pounds per cubic inch.

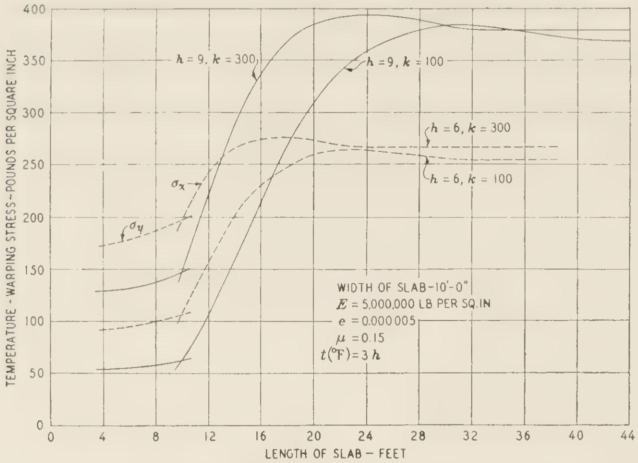

FIGURE 11.—TEMPERATURE-WARPING STRESSES, INTERIOR OF SLAB.

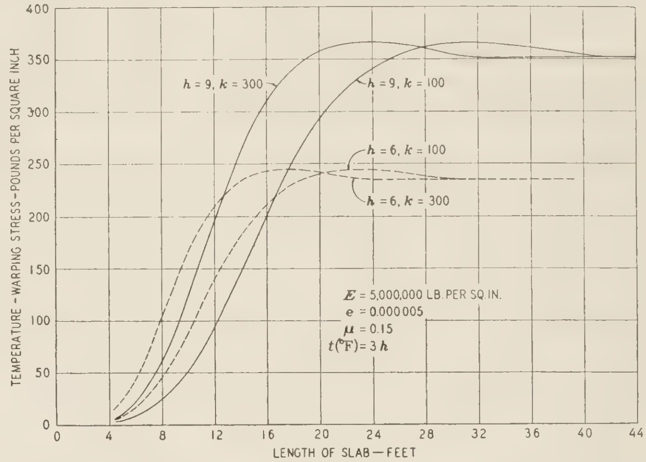

FIGURE 12.—TEMPERATURE-WARPING STRESSES, EDGE OF SLAB.

The most striking fact shown by these curves is the magnitude of the maximum temperature-warping stresses, which are of the order of 275 and 375 pounds per square inch, respectively, for the 6-inch and 9-inch slabs. Other interesting observations that may be made are enumerated as follows:

1. A comparison of figures 11 and 12 shows that maximum edge stresses are always lower than maximum interior stresses but the difference is not great except in slabs having a length less than the width. (In this discussion the length of the slab is considered as the dimension in the direction of the longitudinal axis of the pavement even though it may be less than the width of the slab.)

2. Increases in the length of the slab beyond about 18 feet for the 6-inch slab, and about 24 feet for the 9-inch slab, have no great influence on maximum edge or interior stresses. Below these limits, decreases in slab length result in rapid reduction in stress.

3. In the interior of the slab, $\sigma_x=\sigma_y$ when the slab is square. When the length exceeds the width, σ_x is greater than σ_y and when the length is less than the width the reverse is true. Between the upper limits of slab length that have been mentioned and the point at which the length equals the width, reduction in slab length results in rapid reduction in maximum interior stresses. When the length is less than the width the critical warping stress is influenced primarily by the width and variations in length have little effect on its magnitude. In contrast to this, edge stresses decrease continuously with decreasing slab length.

4. For the longer slabs the maximum stresses in the 9-inch slab exceed those in the 6-inch slab by 40 to 50 percent. However, for slab lengths less than about 17 feet for $k=100$ and 13 feet for $k=300$, the stresses in the 6-inch slab exceed those in the 9-inch slab by as much as 50 pounds per square inch.

5. Variations in the value of the subgrade modulus have no significant influence on the stresses in long slabs. However, for short slabs increases in the value of the subgrade modulus result in considerable increases in the computed stresses. Figures 11 and 12 show that the stresses in the 9-inch slab for $k=300$ may exceed those for $k=100$ by more than 100 pounds per square inch. The difference is somewhat less in the case of the 6-inch slab.

This effect of subgrade modulus on temperature stresses is the reverse of its effect on stresses due to wheel loads where low values of the modulus give higher stresses than do high values. In the case of combined stresses due to load and temperature warping this reversal of influence tends to compensate somewhat for possible errors in computed stresses owing to the assumption of a subgrade modulus different from that which may actually exist.

For example, assuming an 8,000-pound static wheel load on high-pressure dual tires, table 1 shows the total impact reaction to be 11,800 pounds and figure 8 gives a value of a equal to 7.8 inches. For $\mu=0.15$ and $E=5{,}000{,}000$, equation 8 gives interior stresses in a 6-inch slab of approximately 365 pounds per square inch for $k=100$ and 315 pounds per square inch for $k=300$. From figure 11 the corresponding warping stresses in a slab 14 feet long are 200 and 265 pounds per square inch. The combined stresses due to load and temperature are then 565 pounds per square inch for $k=100$ and 580 pounds per square inch for $k=300$.

Thus it appears that, for short slabs, variations in the subgrade modulus may be expected to have a minor influence on combined stresses. However, for slabs of the length commonly used in pavements, the effect of subgrade modulus on warping stresses is slight, with the result that it will have a noticeable effect on combined stresses. Therefore, the value of $k=100$ pounds per cubic inch appears to be a desirable figure for general use in the computation of combined stresses as well as for stresses due to wheel loads only.

TEMPERATURE WARPING STRESSES CAUSE MUCH CRACKING OF CONCRETE PAVEMENTS

Table 12 is presented to show the effect of width of pavement on transverse warping stresses. The figures indicate that the warping stresses in a slab 20 feet wide may exceed 300 pounds per square inch and may be more than twice as great as the stresses in a slab 10 feet wide. Figures such as these show the reason for the use of longitudinal joints in concrete pavements, the necessity for which has been thoroughly demonstrated by practical experience.

It is evident from equations 17, 18, and 19 that the computed warping stress due to temperature differential varies directly with values of the modulus of elasticity, E, the thermal coefficient, e, and the temperature differential, t. The stress values shown in figures 11 and 12 are based on assumed values of E, e and t that may be considered as average rather than maximum.

The value of E may exceed 5,000,000 pounds per square inch, the value of e may exceed 0.000005 per degree Fahrenheit and, at times, the value of t is very likely to exceed 3° F. per inch of slab thickness. In the Arlington tests (tables 10 and 11) values of the temperature differential as high as 4° F. per inch of slab thickness were observed occasionally. Therefore the warping stresses that may exist at certain times in concrete pavements having a high modulus of elasticity and a high thermal coefficient may be more than twice as great as the stresses shown in figures 11 and 12.

TABLE 12.—*Transverse temperature-warping stresses in slabs 30 feet long*

$\mu = 0.15$.
$E = 5{,}000{,}000$ pounds per square inch.
$e = 0.000005$.
$t(°F.) = 3h$ (inches).

Subgrade modulus k	Width of slab	Depth of slab		
		6 inches	7 inches	8 inches
Lb. per cu. in.	*Feet*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*
100	10	130	120	115
	20	280	320	340
300	10	210	200	190
	20	285	335	380

It should be noted also that the assumption of a 10-foot width of slab for the computation of the longitudinal interior warping stresses shown in figure 12 involves also the assumption that the longitudinal joint offers no restraint to warping. Actually the types of longitudinal joints in common use may be expected to develop some restraint to warping and such restraint as may exist serves to increase the computed interior warping stresses, both in the longitudinal and transverse directions.

It seems reasonable to conclude that the magnitude of the stress that may be induced by temperature warping explains much of the cracking that takes place in concrete pavements which, in the past, has frequently been attributed to other causes. The possible magnitude of these stresses indicates the importance of the use of curing methods that will protect the concrete from extreme changes of temperature during its early life when its strength is low.

Corner warping stresses.—An exact mathematical analysis of stresses produced by temperature warping near the corner of a slab is not available and an approximate solution must be used for stress computation. Both theory and experiment (*16*) indicate that the warping stress increases as the distance from the corner along the diagonal bisector increases. The warping stress that is important is that which occurs at the point of maximum load stress. Bradbury (*9*) has developed an approximate equation for this stress, which is

$$\sigma_{cw} = \frac{Eet}{3(1-\mu)}\sqrt{\frac{a}{l}} \qquad (20)$$

Combinations of simultaneous stresses due to load and temperature:

Corner.—When the temperature differential is positive it produces compressive stress in the top of the slab, whereas corner loading produces tensile stress. Therefore, since the combined stress due to warping and load is less than stress due to load alone, this condition requires no further consideration. At night, when the slab is warped upward, the two stresses are of the same sign and therefore the warping stress tends to increase the combined stress. However, the effect is not great since at night the temperature differential, and the resultant warping stress, are small.

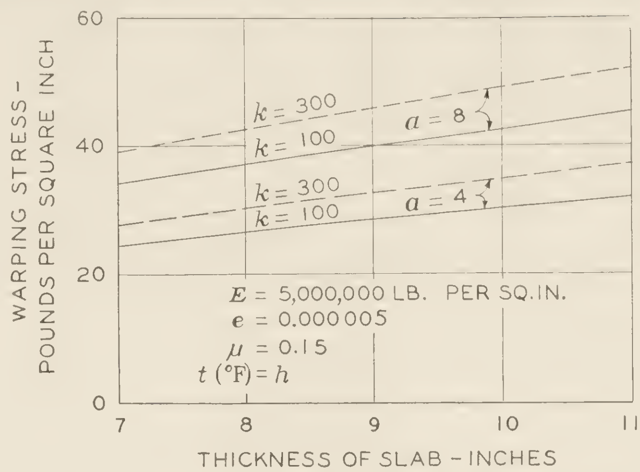

FIGURE 13.—TEMPERATURE-WARPING STRESSES, CORNER OF SLAB.

Corner-warping stresses computed by equation 20 are shown in figure 13 for an assumed temperature differential of 1° F. per inch of slab thickness. The curves show no great effect of any of the variables considered and the assumption of a flat value for the warping stress of about 40 pounds per square inch would probably be sufficiently accurate for all practical purposes. This value is in good agreement with observed values ((*18*), table 14).

Edge.—When temperature-warping stresses in the edge of the slab are combined with load stresses, two combinations require consideration. In the daytime, when the edge of the slab is warped down so that it is in contact with the subgrade, the load stresses are computed by Westergaard's formula (equation 9) and these should be combined with warping stresses computed for the daytime temperature differential of 3° F. per inch of slab depth. In this case both load and temperature create tensile stress in the bottom of the slab.

The second combination is that of maximum load stresses, which occur at night when the edge of the slab is warped upward, with the warping stresses computed for the nighttime temperature differential of 1° F. per inch of slab thickness. For these assumed temperature differentials the warping stress at night is one-third as large as that which occurs during the day and it is of opposite sign from stress due to load. Therefore, the combined stress at night is less than the stress due to load alone.

MOISTURE WARPING STRESSES CAN BE SAFELY IGNORED IN DESIGN

Interior.—In the Arlington tests (*16*) it was found that the condition of slab warping had a negligible effect on the magnitude of the maximum stress produced by a load applied at the interior of the slab. The maximum load stress at the interior is about the same at night when the edges of the slab are warped upward as in the daytime when the edges are warped down. Therefore, in the determination of the maximum combined stress due to load and temperature warping, the maximum load stress should be combined with the warping stress produced by the temperature differential that occurs in the daytime.

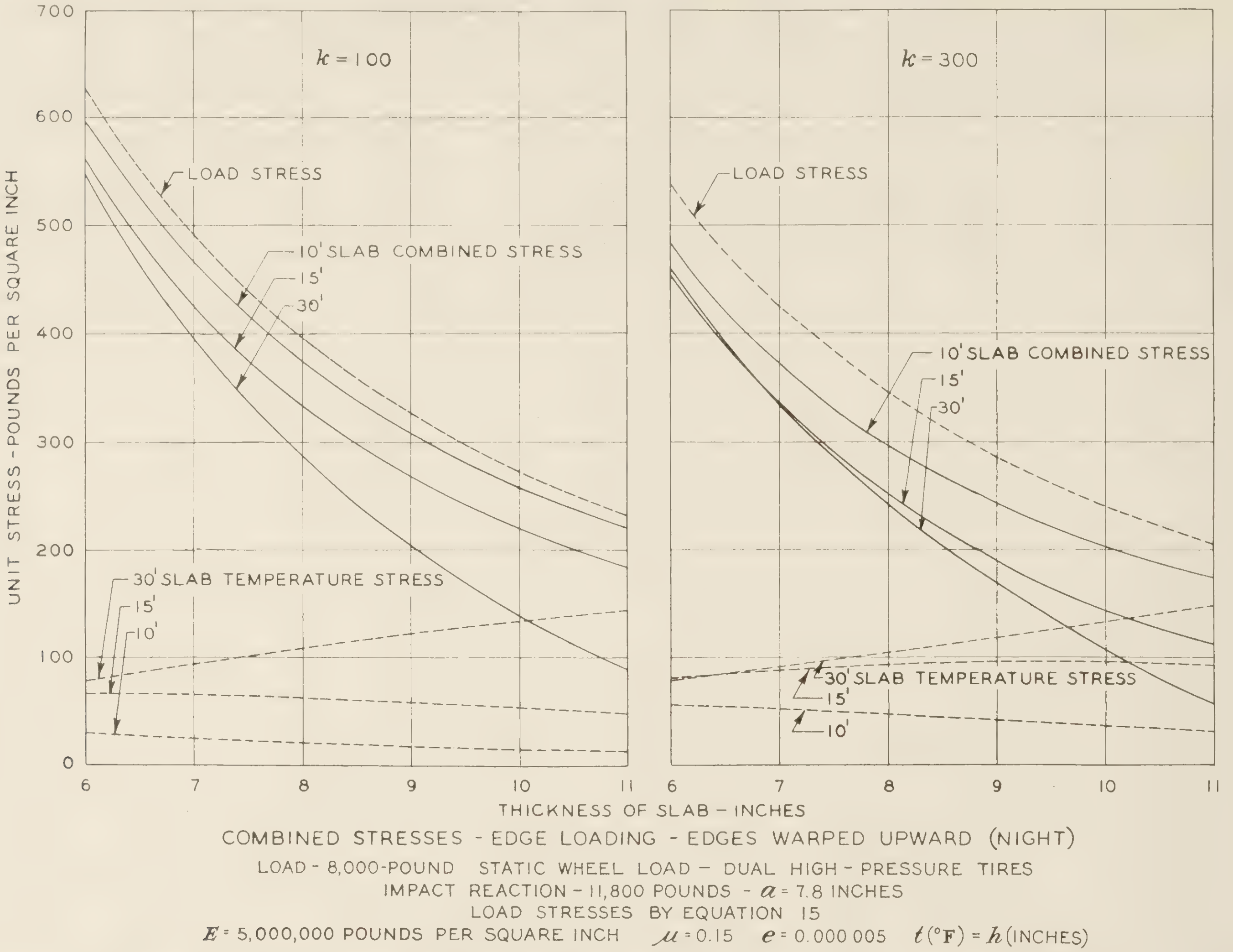

FIGURE 14.—EFFECT OF SLAB THICKNESS, SUBGRADE MODULUS, AND SLAB LENGTH ON COMBINED STRESSES DUE TO LOAD AND TEMPERATURE WARPING IN THE EDGE OF A SLAB 10 FEET WIDE.

Moisture warping.—Since concrete expands and contracts with changes in moisture content, it follows that a difference in the moisture content between the top and bottom of a concrete pavement slab causes the slab to warp or curl in much the same manner as does a differential in temperature. When the top of the slab is dryer than the bottom the edges of the slab curl upward and when the moisture differential is in the opposite direction the edges of the slab curl downward.

As a result of the extensive observations made in the Arlington tests (*16*) it was concluded that, for the climatic conditions that prevailed, the moisture content of a pavement slab is at a maximum, and the moisture gradient that causes warping is at a minimum, during the period from January to March. As compared with the conditions that prevailed during this period, it was found that the edges of the slab were curled upward during the summer months, when the top of the slab was dryer than the bottom, and began to curl downward again during the fall.

Thus the warping of the slab caused by moisture differential is a seasonal change which takes place slowly over a considerable period of time during which there is opportunity for plastic yield of the concrete to take place. Also it was observed in the Arlington tests that as the seasonal warping takes place the slab settles into the subgrade, thus reducing the restraint to warping due to the weight of the slab. Because of the time element and its effect on the adjustment between slab and subgrade and on the plastic flow of the concrete, it seems very probable that stresses due to moisture warping are not as great as the deformations in the concrete would indicate.

For these reasons the strains due to moisture warping that have been measured in connection with the Arlington tests cannot be translated into stress with any certainty. However, the observations made indicate that the curvature caused by moisture is principally an upward warping of the edges caused by moisture loss from the top of the slab during the warm season of the year, and that the downward warping that takes place when the moisture in the top of the slab exceeds that in the bottom may be expected to be considerably smaller. Thus, during hot summer days when moisture and temperature differentials are both a maximum, the curvature caused by one is in the opposite direction to that caused by the other and such stress as may be caused by moisture serves to reduce rather than to increase the stress due to temperature warping. Since the stresses due to moisture warping cannot be evaluated, it is fortunate that the evidence indicates that they may be disregarded with safety in computing the stresses in pavement slabs. To ignore them appears

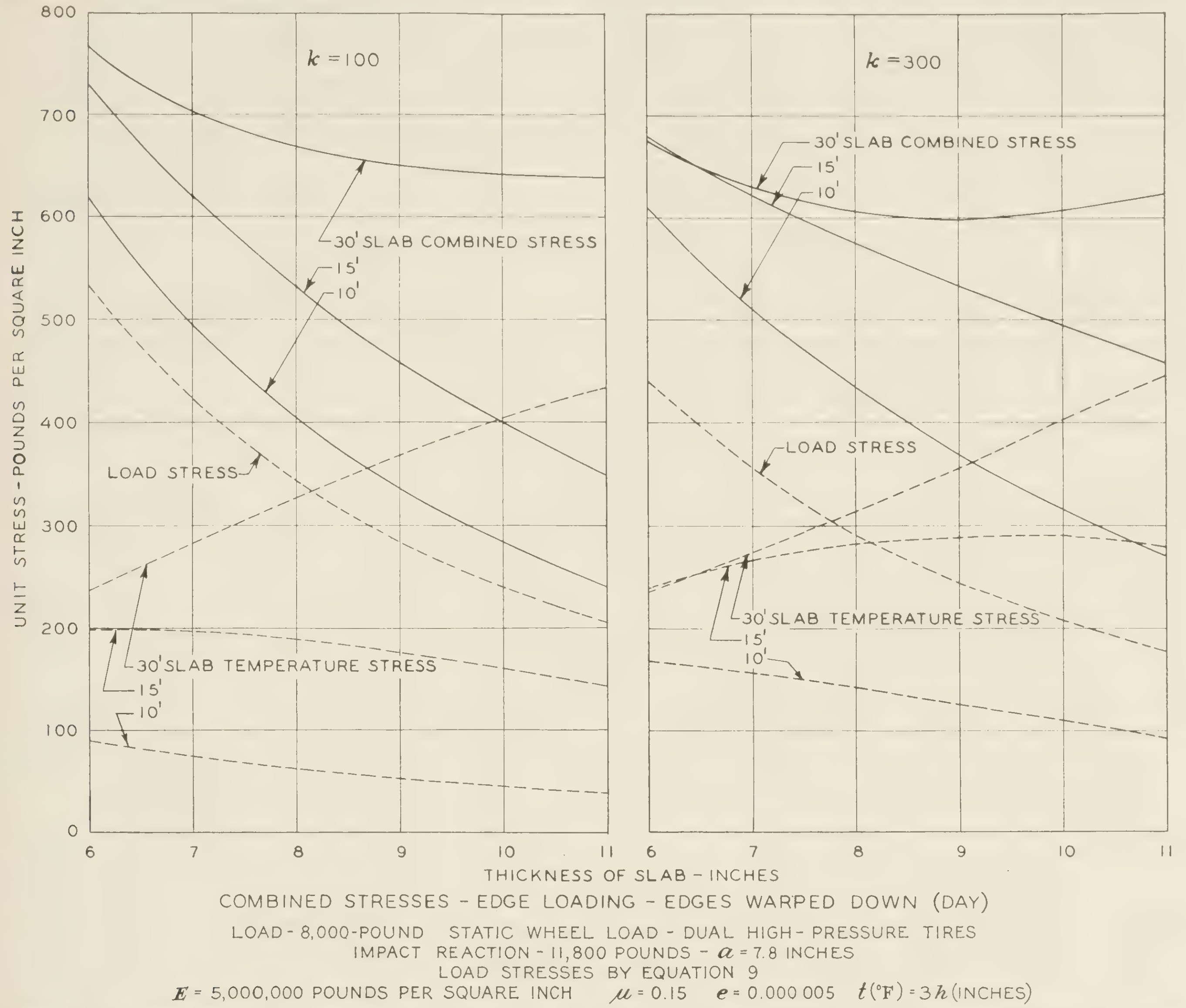

Figure 15.—Effect of Slab Thickness, Subgrade Modulus, and Slab Length on Combined Stresses due to Load and Temperature Warping in the Edge of a Slab 10 Feet Wide.

to add some factor of safety of unknown magnitude and importance.

Combined stresses.—Total combined stresses due to load and temperature warping are shown in figures 14, 15, and 16 for the edge and interior of slabs of different depths, a width of 10 feet and lengths of 10, 15, and 30 feet. Combined corner stresses, which are not influenced by the dimensions of the slab other than depth, are shown in the left part of figure 17. The assumed load is an 8,000-pound wheel load on dual high-pressure tires. The edge-load stresses of figure 14 are computed by equation 15 for the nighttime condition of upward warping and therefore the assumed temperature differential for the warping stresses is taken as 1° F. per inch of slab thickness. Since the warping stresses and load stresses are of opposite sign, the combined edge stresses of figure 14 are less than the load stresses. For the reasons that have been given, the assumed temperature differential for the corner warping stresses of figure 17 is also taken as 1° F. per inch of slab thickness. The edge-load stresses of figure 15 are computed by equation 9 for daytime conditions and therefore the assumed temperature differential for the warping stresses is taken as 3° F. per inch of slab thickness. The same differential is also used for computing interior warping stresses to be combined with interior load stresses in figure 16.

As would be expected from the previous discussion, the computed corner warping stresses are small, ranging from about 30 to 50 pounds per square inch for the range of variables assumed, and their effect on combined corner stresses is practically negligible.

REDUCING SLAB LENGTH TO 10 FEET GREATLY REDUCES COMBINED STRESSES

It may be observed that in all cases, for a given thickness of slab and the same value of the subgrade modulus, the combined edge stresses of figure 15 are larger than those of figure 14. The somewhat larger load stresses that may occur at night (equation 15), when reduced by the warping stresses, are less than the lower load stresses of equation 9 in combination with the high warping stresses that occur during the day. Except in slabs 10 feet long the differences are of considerable

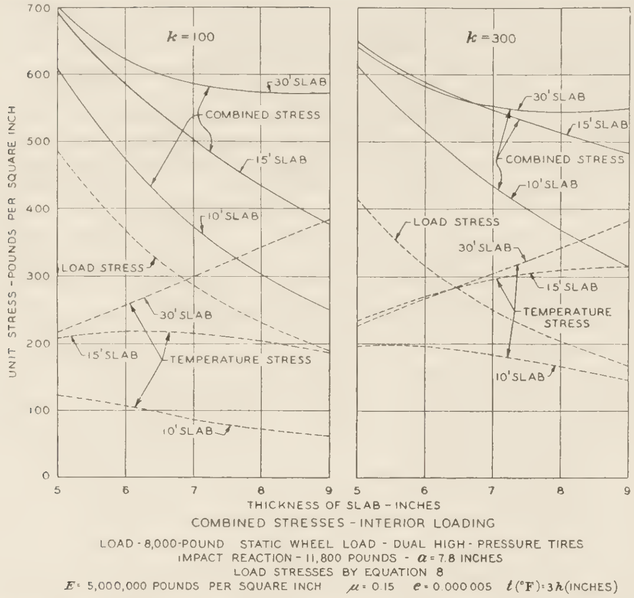

FIGURE 16.—EFFECT OF SLAB THICKNESS, SUBGRADE MODULUS, AND SLAB LENGTH ON COMBINED STRESSES DUE TO LOAD AND TEMPERATURE WARPING IN THE INTERIOR OF A SLAB 10 FEET WIDE.

magnitude. In view of this, the combined stresses of figure 14 will be disregarded in the subsequent discussion although it should be recognized that other assumptions than those which determine the curves of figures 14 and 15 might lead to different relative values.

Bearing in mind that the temperature warping stresses shown in figures 15 and 16 may be regarded as average rather than probable maximum values, the following interesting observations may be made with respect to the combined edge stresses of figure 15 and the combined interior stresses of figure 16, both being for a slab 10 feet wide.

1. In slabs 30 feet long an increase in the depth of slab does not effect any marked decrease in the total combined stress. In fact, for $k=300$, there is a slight increase in interior stress as the slab thickness is increased beyond 8 inches and in the edge stress as the thickness is increased beyond 9 inches.

2. In slabs 30 feet long a high value of the subgrade modulus results in a lower combined stress than a low value of the modulus, but for values between $k=100$ and $k=300$ the difference is not great enough to be significant.

3. In slabs 30 feet long the combined edge stresses are somewhat higher than those in the interior of the slab. For an 8-inch slab the difference is about 100 pounds per square inch for $k=100$ and 60 pounds per square inch for $k=300$.

4. Reducing the slab length from 30 to 15 feet results in some reduction in interior stress when $k=100$ but has very little effect when $k=300$. In general, this reduction in slab length has a greater effect on combined edge stresses than on combined interior stresses and the reduction in stress is considerably greater when $k=100$ than when $k=300$.

5. In slabs 15 feet long in contrast to those 30 feet long, a high value of the subgrade modulus generally results in a higher combined stress than does a low value of the modulus. In an 8-inch slab, interior and edge stresses for $k=300$ exceed those for $k=100$ by about 80 pounds per square inch and 40 pounds per square inch, respectively.

6. Reducing the slab length from 30 to 10 feet results in an appreciable reduction in combined interior and edge stresses. The combined stresses in an 8-inch slab, as shown in figures 15 and 16, are given in table 13.

The combined stresses which may occur in the daytime in the free edge of a transverse joint in a slab 10 feet wide are shown in the second chart of figure 17. The curves show that the depth of slab has a marked influence on combined stresses but that the effect of variations in the subgrade modulus between $k=100$ and $k=300$ is negligible.

From the above discussion it may be concluded, for the stress-producing conditions assumed, that:

1. In slabs as long as 30 feet, the depth of slab has very little influence on the magnitude of combined interior and edge stresses.

2. In slabs as long as 30 feet, combined edge stresses and combined interior stresses of the order of 600 pounds per square inch are to be expected under what may be considered average conditions. When the concrete has a higher thermal coefficient and a higher modulus of elasticity than the values used in these computations and when the temperature differential is higher than that assumed, these combined stresses may be greatly increased.

TABLE 13.—*Combined edge and interior stresses in a slab 10 feet wide and 8 inches thick* [1]

Slab length	Combined edge stress		Combined interior stress	
	$k=100$	$k=300$	$k=100$	$k=300$
Feet	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*
30	670	610	570	550
15	530	570	430	510
10	400	430	300	370

[1] From figs. 15 and 16.

3. In order to effect any significant reduction in combined stresses in the edge and interior of the slab it is necessary to reduce the slab length to about 10 feet. In a slab 10 feet long and 8 inches thick the combined stresses will be of the order of 400 pounds per square inch as compared with 600 pounds per square inch in a slab 30 feet long.

4. In short slabs the depth of the slab has a very marked influence on combined stresses at the edge and interior. In slabs of any length the depth of slab has a marked influence on combined stresses at the corners and edges of free transverse joints.

5. The character of the subgrade, as measured by variations in the subgrade modulus between $k=100$ and $k=300$, does not have a great effect or a consistent effect on the magnitude of combined stresses. In long slabs the higher interior and edge stresses are associated with the lower values of the modulus while in short slabs the reverse is true.

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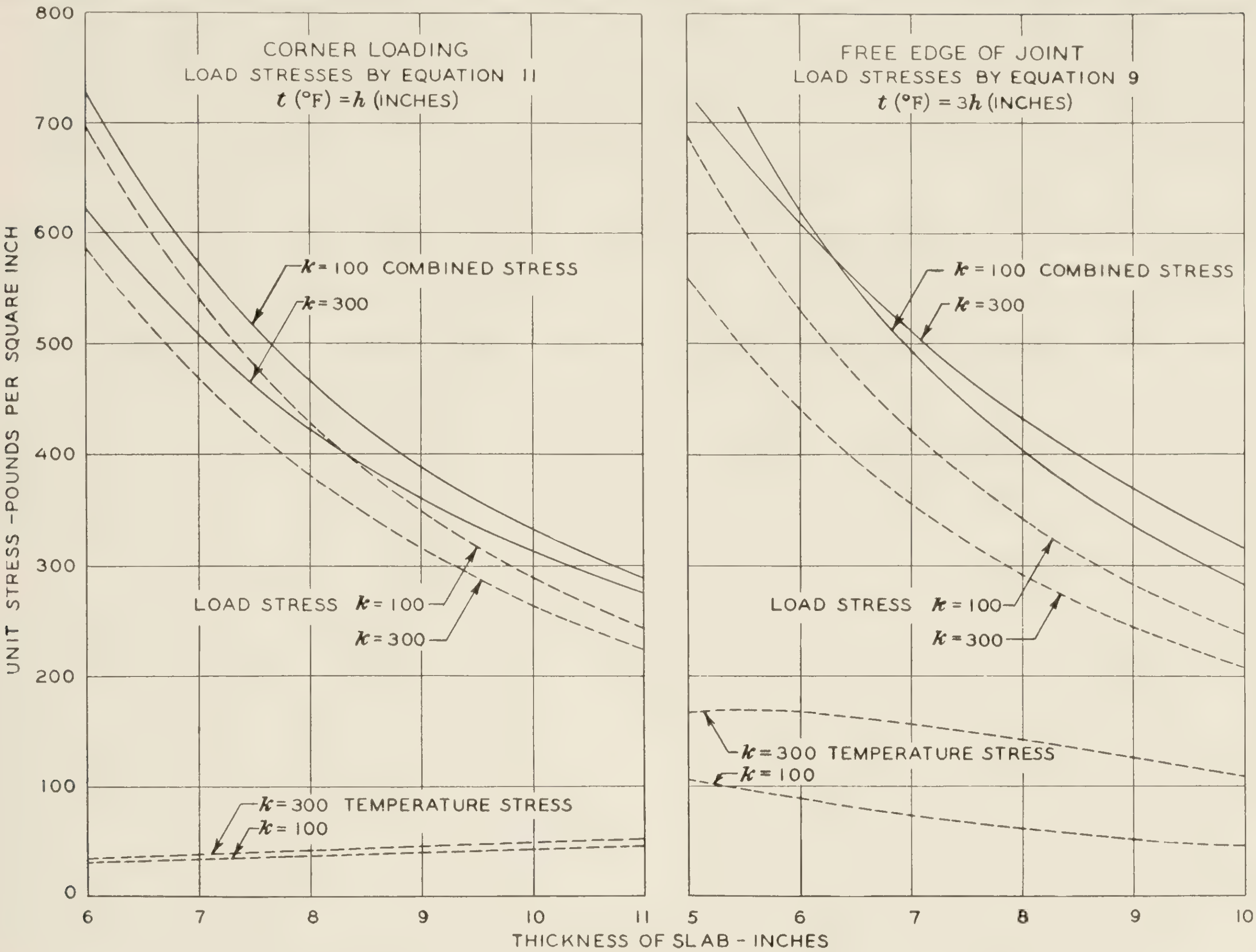

FIGURE 17.—EFFECT OF SLAB THICKNESS AND SUBGRADE MODULUS ON COMBINED STRESSES DUE TO LOAD AND TEMPERATURE WARPING IN THE CORNER AND EDGE OF A FREE TRANSVERSE JOINT IN A SLAB 10 FEET WIDE.

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STATUS OF FEDERAL-AID HIGHWAY PROJECTS

AS OF JUNE 30, 1939

STATE	COMPLETED DURING CURRENT FISCAL YEAR			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	
Alabama	$ 7,005,608	$ 3,212,670	246.6	$ 8,323,267	$ 4,148,643	308.6	$ 727,350	$ 361,820	21.9	$ 3,101,328
Arizona	2,516,890	1,799,919	125.5	1,261,511	895,630	50.9	289,535	204,830	19.4	1,825,489
Arkansas	1,810,161	1,792,683	107.1	3,271,382	3,267,487	220.1	196,261	194,176	4.3	1,738,388
California	11,119,638	5,996,198	256.4	5,294,381	2,908,628	63.8	858,878	431,709	9.7	4,293,753
Colorado	3,615,134	1,936,720	139.0	3,857,872	2,145,923	88.4	454,385	254,526	8.6	2,202,372
Connecticut	1,191,520	582,221	11.6	1,389,368	690,879	15.7	694,532	343,528	5.6	1,331,528
Delaware	743,081	366,830	17.8	524,921	257,380	12.7	1,451,557	712,821	25.8	1,008,742
Florida	3,584,507	1,747,762	83.4	2,357,420	1,178,710	41.0	1,771,868	885,709	32.3	2,904,467
Georgia	5,625,422	2,700,961	281.2	5,545,490	2,772,745	285.1	3,630,798	1,815,399	220.9	5,652,962
Idaho	2,207,117	1,250,690	198.2	2,034,693	1,227,773	55.3				1,662,248
Illinois	13,131,577	6,485,708	330.2	8,199,507	4,096,859	193.6	3,570,527	1,794,623	66.9	3,545,138
Indiana	6,372,925	3,076,689	163.3	5,012,566	2,454,775	90.5	3,400,506	1,647,659	86.5	2,268,286
Iowa	8,459,615	4,016,131	290.0	4,297,566	1,874,633	158.9	1,598,718	747,200	81.4	1,533,270
Kansas	6,098,327	3,017,327	757.6	3,533,265	1,758,927	155.1	3,798,690	1,898,465	219.3	4,195,785
Kentucky	5,801,316	2,867,271	222.7	4,052,874	2,024,881	84.0	1,271,322	635,661	70.4	3,017,531
Louisiana	1,520,382	750,988	38.3	11,671,198	2,929,730	52.3	1,497,266	727,906	28.0	2,703,740
Maine	3,104,397	1,518,792	71.2	1,504,944	752,471	29.4	1,177,536	588,768	31.4	406,151
Maryland	1,223,576	609,522	19.9	2,799,245	1,388,291	45.7	1,376,636	679,505	21.0	1,824,539
Massachusetts	2,616,317	1,307,565	16.3	3,348,765	1,671,693	25.1	1,537,039	765,997	10.3	2,574,887
Michigan	8,416,730	3,949,240	174.1	4,962,839	2,478,972	145.8	1,498,700	651,700	32.8	3,145,182
Minnesota	5,172,346	2,480,193	317.9	6,165,319	3,066,676	305.3	2,305,132	1,149,598	175.5	3,815,595
Mississippi	6,571,088	2,879,023	284.0	7,579,632	2,710,048	315.6	1,303,300	480,134	54.9	2,824,715
Missouri	6,295,359	3,026,127	165.5	4,927,490	2,451,293	182.9	2,638,432	1,238,973	83.4	4,663,254
Montana	2,305,566	1,295,090	103.8	3,561,604	2,014,552	184.1	212,688	120,632	11.2	4,445,648
Nebraska	4,854,308	2,274,896	413.9	5,297,999	2,667,993	445.4	2,828,638	1,415,819	308.2	2,890,294
Nevada	2,255,396	1,914,681	202.5	963,736	833,163	42.0	46,009	39,572	3.3	1,603,560
New Hampshire	1,316,109	647,939	25.7	620,591	305,385	14.1	964,343	476,703	29.8	932,785
New Jersey	2,999,495	1,481,172	20.0	3,058,686	1,527,793	28.9	1,193,400	546,700	2.5	2,246,292
New Mexico	2,827,851	1,819,345	294.9	1,652,917	1,007,894	80.6	458,711	286,898	41.5	1,524,043
New York	16,218,410	7,724,629	273.1	10,708,900	5,231,287	190.3	3,033,410	1,286,710	41.3	4,110,959
North Carolina	7,815,839	3,713,373	336.7	6,506,843	3,247,822	391.1	1,512,260	694,460	64.2	2,112,429
North Dakota	3,622,515	3,334,141	292.3	256,090	137,089	26.1	3,282,864	1,759,544	323.0	3,430,254
Ohio	9,429,801	4,649,998	112.3	9,715,552	4,789,387	104.1	1,757,150	883,535	25.4	7,319,148
Oklahoma	6,819,168	3,562,656	272.7	1,977,680	1,046,474	29.7	2,693,791	1,428,185	93.3	3,841,245
Oregon	3,452,899	2,010,145	159.0	2,951,521	1,789,322	127.0	68,256	40,480	1.1	2,249,713
Pennsylvania	8,886,117	4,359,444	144.2	10,192,359	4,922,665	99.4	1,706,868	840,917	20.7	5,231,412
Rhode Island	1,382,093	681,850	17.9	508,476	254,051	6.1	629,776	314,400	8.0	1,073,748
South Carolina	5,396,142	2,404,078	267.1	2,862,034	1,277,487	86.3	117,200	53,000	23.7	2,408,179
South Dakota	2,363,488	1,319,786	300.5	4,323,949	2,391,170	404.8	1,557,870	898,070	145.3	3,427,465
Tennessee	6,950,643	3,442,913	207.5	3,852,210	1,926,105	106.3	512,280	256,140	14.8	4,604,324
Texas	18,600,528	9,149,390	1,156.4	12,501,058	6,176,003	548.1	1,264,449	570,515	133.8	7,136,819
Utah	1,854,605	1,258,418	133.1	2,333,900	1,564,310	87.4	190,825	129,487	28.8	1,076,602
Vermont	1,334,290	610,413	33.9	726,484	345,593	17.7	172,110	85,865	5.2	637,953
Virginia	7,545,084	3,764,699	255.8	2,385,237	1,190,554	67.0	2,178,394	1,031,904	48.1	1,066,943
Washington	4,882,817	2,538,963	114.3	2,752,950	1,440,116	30.0	1,649,892	741,242	16.2	1,080,604
West Virginia	2,220,412	1,460,013	71.3	1,465,862	757,011	38.1	1,805,740	899,168	43.2	2,263,331
Wisconsin	5,348,323	2,640,259	188.9	6,761,936	3,307,780	183.4	3,348,370	1,601,700	146.5	1,769,726
Wyoming	2,974,801	1,807,345	304.2	1,570,645	972,995	148.5	298,147	188,208	37.9	1,026,385
District of Columbia							297,500	148,750	2.0	338,750
Hawaii	1,287,547	637,043	23.3	808,350	395,200	12.7	892,407	427,968	14.3	1,058,510
Puerto Rico	703,366	347,080	14.2	1,632,798	811,330	33.4	171,477	80,925	2.3	482,540
TOTALS	249,850,646	128,220,989	10,057.3	203,865,882	101,483,578	6,458.4	71,893,793	35,458,204	2,945.9	133,629,011

STATUS OF FEDERAL-AID SECONDARY OR FEEDER ROAD PROJECTS

AS OF JUNE 30, 1939

STATE	COMPLETED DURING CURRENT FISCAL YEAR			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	Estimated Total Cost	Federal Aid	Miles	
Alabama	$ 284,756	$ 139,712	24.3	$ 778,250	$ 383,750	32.7	$ 281,200	$ 57,000	0.3	$ 782,784
Arizona	506,576	329,737	42.3	263,967	173,408	29.3	15,912	11,475		355,372
Arkansas	108,487	101,817	9.9	398,390	395,311	44.9	181,860	181,422	32.6	440,945
California	1,918,647	1,065,263	117.9	1,055,708	542,287	44.7	86,988	50,895	3.7	758,464
Colorado	1,160,418	606,547	64.8	540,850	274,564	21.5	161,270	76,292	4.2	235,353
Connecticut	106,870	53,215	1.6	172,794	72,417	2.9				286,249
Delaware	22,730	11,365	5.3	80,840	40,420	17.5	73,930	36,965	7.8	231,250
Florida	20,122	10,061		762,533	380,450	26.3	227,500	109,850	11.4	374,744
Georgia	525,621	252,520	70.1	442,406	221,203	55.2	155,980	77,990	20.3	1,083,865
Idaho	497,893	222,141	57.2	246,595	140,187	11.2				295,511
Illinois	2,011,457	977,930	167.4	1,501,632	696,816	83.8	380,000	190,000	22.8	770,576
Indiana	759,194	318,067	80.5	941,170	470,585	80.5	386,177	173,298	22.5	644,375
Iowa							47,751	22,015	35.4	1,657,792
Kansas	251,257	125,622	29.9	47,588	23,794	11.7	408,515	204,257	21.8	1,353,173
Kentucky	798,767	243,871	106.1	1,076,701	290,910	66.2	730,604	239,763	67.0	317,903
Louisiana	241,327	107,635	20.0	675,599	290,180	54.4	307,416	143,120	26.6	398,713
Maine	423,420	205,994	25.7	218,250	109,395	11.5	224,590	111,265	14.4	37,761
Maryland				188,974	94,487	15.1	142,000	52,355	10.2	388,839
Massachusetts	126,988	63,055	1.8	300,011	148,856	6.4	288,754	142,443	6.0	498,369
Michigan	512,333	251,636	37.4	1,165,104	580,052	68.8	404,300	188,450	42.4	967,350
Minnesota	273,069	126,700	42.2	702,410	349,161	61.1	213,446	106,723	16.8	1,202,621
Mississippi				399,262	199,631	31.8	305,500	152,650	29.5	798,585
Missouri	523,807	250,970	72.8	705,966	342,573	76.6	515,824	218,755	87.3	701,338
Montana	14,071	7,865		452,945	256,840	37.5	461,668	261,854	38.4	813,334
Nebraska	617,650	292,326	101.3	743,466	362,777	143.8	426,966	202,489	67.0	446,867
Nevada	427,436	345,390	68.8	120,169	104,184	15.5	51,737	44,685	9.5	192,987
New Hampshire	206,978	102,285	6.0	60,759	29,708	2.4				188,007
New Jersey	171,820	79,020	2.5	349,350	172,625	10.0	155,240	76,735	9.8	542,598
New Mexico	857,449	521,681	57.5	450,024	271,508	28.1				252,877
New York	2,400,190	1,161,481	167.9	1,807,100	903,550	99.1	464,000	172,900	13.0	851,452
North Carolina	769,492	383,616	89.3	1,253,454	622,055	111.9	162,570	75,110	17.5	349,602
North Dakota	108,510	56,615	26.8	115,030	61,606	8.3	42,770	22,907	8.2	875,949
Ohio	147,535	73,767	3.8	632,270	322,910	32.0	243,200	121,600	13.4	1,850,842
Oklahoma	394,585	205,059	42.1	82,986	44,156	.8	625,440	309,598	37.4	973,691
Oregon	471,113	274,000	63.2	509,332	306,332	55.0	261,327	153,035	20.6	269,990
Pennsylvania	1,903,132	900,337	133.5	2,121,525	1,042,981	116.3	185,900	92,950	7.6	714,676
Rhode Island	166,074	81,173	7.2	99,335	49,644	2.2				134,171
South Carolina	673,849	292,852	79.1	583,907	239,069	56.9	169,800	66,200	12.4	279,791
South Dakota	11,519	6,250					13,880	7,640		1,050,410
Tennessee	420,621	185,123	17.6	719,438	304,489	32.0				881,848
Texas	3,654,802	1,732,636	515.2	2,028,005	966,441	199.7	397,069	191,406	46.5	1,160,749
Utah	780,439	387,018	65.9	103,735	56,018	18.3	143,020	66,846	17.2	209,198
Vermont	232,410	106,201	13.8	90,306	45,153	4.0	109,100	53,400	3.1	77,967
Virginia	1,086,502	488,862	90.8	493,334	239,171	50.6	172,096	69,912	15.4	367,303
Washington	571,884	297,126	64.3	694,589	364,996	40.1	70,770	37,000	16.3	266,006
West Virginia	243,696	119,483	21.4	153,296	76,648	8.3				515,848
Wisconsin	682,683	326,289	28.9	846,640	419,575	29.5	312,992	135,550	7.0	693,622
Wyoming	416,758	254,565	59.0	356,182	220,069	20.2	333,324	209,724	31.6	88,192
District of Columbia										73,125
Hawaii				170,080	85,040	4.6	22,900	11,450	1.3	223,510
Puerto Rico	250,901	123,966	13.7	135,545	65,880	8.6	98,148	48,085	3.9	82,069
TOTALS	28,755,838	14,268,844	2,716.8	27,837,802	13,853,862	1,989.8	10,463,434	4,978,059	880.1	29,008,613

PUBLICATIONS of the PUBLIC ROADS ADMINISTRATION

(Formerly the *BUREAU OF PUBLIC ROADS*)

Any of the following publications may be purchased from the Superintendent of Documents, Government Printing Office, Washington, D. C. As his office is not connected with the Agency and as the Agency does not sell publications, please send no remittance to the Federal Works Agency.

ANNUAL REPORTS

Report of the Chief of the Bureau of Public Roads, 1931. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1933. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1934. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1935. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1936. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1937. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1938. 10 cents.

HOUSE DOCUMENT NO. 462

Part 1 . . . Nonuniformity of State Motor-Vehicle Traffic Laws. 15 cents.

Part 2 . . . Skilled Investigation at the Scene of the Accident Needed to Develop Causes. 10 cents.

Part 3 . . . Inadequacy of State Motor-Vehicle Accident Reporting. 10 cents.

Part 4 . . . Official Inspection of Vehicles. 10 cents.

Part 5 . . . Case Histories of Fatal Highway Accidents. 10 cents.

Part 6 . . . The Accident-Prone Driver. 10 cents.

MISCELLANEOUS PUBLICATIONS

No. 76MP . . The Results of Physical Tests of Road-Building Rock. 25 cents.

No. 191MP . . Roadside Improvement. 10 cents.

No. 272MP . . Construction of Private Driveways. 10 cents.

No. 279MP . . Bibliography on Highway Lighting. 5 cents.

Highway Accidents. 10 cents.

The Taxation of Motor Vehicles in 1932. 35 cents.

Guides to Traffic Safety. 10 cents.

Federal Legislation and Rules and Regulations Relating to Highway Construction. 15 cents.

An Economic and Statistical Analysis of Highway-Construction Expenditures. 15 cents.

Highway Bond Calculations. 10 cents.

Transition Curves for Highways. 60 cents.

DEPARTMENT BULLETINS

No. 1279D . . Rural Highway Mileage, Income, and Expenditures, 1921 and 1922. 15 cents.

No. 1486D . . Highway Bridge Location. 15 cents.

TECHNICAL BULLETINS

No. 55T . . . Highway Bridge Surveys. 20 cents.

No. 265T. . . Electrical Equipment on Movable Bridges. 35 cents.

Single copies of the following publications may be obtained from the Public Roads Administration upon request. They cannot be purchased from the Superintendent of Documents.

MISCELLANEOUS PUBLICATIONS

No. 296MP. . Bibliography on Highway Safety.

House Document No. 272 . . . Toll Roads and Free Roads.

SEPARATE REPRINT FROM THE YEARBOOK

No. 1036Y . . Road Work on Farm Outlets Needs Skill and Right Equipment.

TRANSPORTATION SURVEY REPORTS

Report of a Survey of Transportation on the State Highway System of Ohio (1927).

Report of a Survey of Transportation on the State Highways of Vermont (1927).

Report of a Survey of Transportation on the State Highways of New Hampshire (1927).

Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio (1928).

Report of a Survey of Transportation on the State Highways of Pennsylvania (1928).

Report of a Survey of Traffic on the Federal-Aid Highway Systems of Eleven Western States (1930).

UNIFORM VEHICLE CODE

Act I.—Uniform Motor Vehicle Administration, Registration, Certificate of Title, and Antitheft Act.

Act II.—Uniform Motor Vehicle Operators' and Chauffeurs' License Act.

Act III.—Uniform Motor Vehicle Civil Liability Act.

Act IV.—Uniform Motor Vehicle Safety Responsibility Act.

Act V.—Uniform Act Regulating Traffic on Highways.

Model Traffic Ordinances.

A complete list of the publications of the Public Roads Administration, (formerly the *Bureau of Public Roads*) classified according to subject and including the more important articles in PUBLIC ROADS, may be obtained upon request addressed to Public Roads Administration, Willard Bldg., Washington, D. C.

STATUS OF FEDERAL-AID GRADE CROSSING PROJECTS

AS OF JUNE 30, 1939

STATE	COMPLETED DURING CURRENT FISCAL YEAR					UNDER CONSTRUCTION					APPROVED FOR CONSTRUCTION					BALANCE OF FUNDS AVAILABLE FOR PROGRAMMED PROJECTS
			NUMBER					NUMBER					NUMBER			
	Estimated Total Cost	Federal Aid	Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise	Estimated Total Cost	Federal Aid	Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise	Estimated Total Cost	Federal Aid	Grade Crossings Eliminated by Separation or Relocation	Grade Crossing Structures Reconstructed	Grade Crossings Protected by Signals or Otherwise	
Alabama	$ 278,490	$ 278,291	7		6	$1,203,662	$ 1,201,724	14	1		$ 62,800	$ 55,800	2		1	$ 842,733
Arizona	30,741	30,741				469,516	443,841	5								281,092
Arkansas	618,179	615,817	14			189,891	189,891	3			71,693	71,693	1		4	1,225,099
California	1,362,358	1,361,783	5	3		1,691,373	1,690,278	10			80,272	80,272	1			1,296,732
Colorado	93,136	89,748	2		6	489,958	489,958	4		1	69,722	65,962			24	893,860
Connecticut						18,930	12,665				171,920	166,540		1		832,360
Delaware	66,368	65,889			23	9,150	9,150	2			2,320	2,320			1	513,891
Florida	17,416	17,416	1			428,094	428,094	2			79,700	79,700	1		1	1,158,058
Georgia	28,650	28,650	1			427,280	427,280	7			132,120	132,120	1	2	9	2,306,620
Idaho	256,974	249,144	5			312,066	280,535	4								454,970
Illinois	563,280	563,280	4		22	2,830,545	2,773,545	18	3	49	583,450	544,450	2		18	2,354,151
Indiana	736,388	626,218	4	4	21	848,183	848,183	3	1	69	484,475	484,475	2	1	9	969,772
Iowa	1,047,880	1,011,085	13	2	20	314,382	276,106	6			487,414	456,600	8		89	1,369,238
Kansas	596,658	596,533	6		5	934,612	934,612	11			473,030	473,030	5		8	1,075,292
Kentucky	249,688	249,688	1		31	586,597	586,597	9	4		277,438	226,991	2	1		1,107,615
Louisiana	98,714	91,980	1			441,463	440,678	4			328,603	328,570	11			1,026,699
Maine	54,882	54,247	2		1	473,676	473,676	4	3		90,800	90,800	1			207,671
Maryland						72,188	72,188	1			258,200	161,407	1		14	993,901
Massachusetts	74,505	74,505		2		520,631	519,367	4	2							1,727,702
Michigan	957,084	915,797	8	1	45	822,526	822,526	6	2		110,950	110,950			20	2,085,059
Minnesota	50,605	50,331		1	3	1,042,247	1,025,806	3	6	6	358,889	357,569	5		5	1,537,428
Mississippi	356,600	356,600	4			603,614	603,614	8								934,587
Missouri	319,890	318,351	4	1		1,059,640	1,059,640	5	1		567,910	564,120	4	2		1,679,326
Montana	365,654	360,772	4			860,225	860,225	9			37,469	37,469	1			327,257
Nebraska	175,709	172,676	8			931,327	931,327	24			457,389	457,389	4		35	550,707
Nevada	246,425	245,178	1	3	6	151,935	151,935		1		46,951	46,951	1		11	112,509
New Hampshire	70,205	69,765		1	2	100,927	100,459	7		1	105,035	104,987	2			316,039
New Jersey	188,941	183,715	2	1	1	493,541	493,541	1	2	1	255,740	255,740	1	1		1,426,875
New Mexico	264,915	264,649	7	1		75,081	75,081	2			2,861	2,861			1	675,857
New York	1,147,444	1,142,543	5	4	3	2,006,512	2,001,112	4	10		913,951	673,500	6	1		4,288,723
North Carolina	419,861	419,861	3	4	8	1,146,500	1,111,400	7	4		423,615	423,615	5	1	36	990,495
North Dakota	546,750	545,687	1	1		676,052	627,650	10	1	7	345,860	345,860	3			369,188
Ohio						844,902	808,140	10	1		1,049,680	1,000,490	6	1	2	3,254,391
Oklahoma	73,654	63,672			5	298,080	264,080	2		42	217,400	217,400	3		8	2,191,397
Oregon	595,514	594,865	3	2		39,002	39,002				266,457	261,372	3			314,891
Pennsylvania	213,129	197,923	2			1,967,294	1,755,395	3	3		570,653	362,200	3		2	4,545,633
Rhode Island						438,791	438,791	1	3							152,459
South Carolina	110,771	110,321	11	1	4	623,703	569,187	8	2		143,479	143,479		1	35	959,865
South Dakota	138,949	138,316	4		9	278,800	278,800	3	2	2	61,260	61,260			15	1,110,539
Tennessee	54,421	54,421		1	6	651,609	651,609	2	2	1	185,940	185,940	1	1	4	1,373,250
Texas	1,040,905	1,038,613	17	3		2,690,625	2,659,822	22	3		554,259	525,030	5		6	2,208,513
Utah	109,143	108,908	2		4	39,638	39,638	2			336,619	336,619	1		127	217,372
Vermont	245,681	230,614	6	2	8	14,256	14,256			2	16,180	15,990			5	317,471
Virginia	511,925	510,852	17	3	7	608,282	519,282	8	1	14	241,327	235,327	1	2	3	912,147
Washington	803,854	788,804	9	4	3	277,733	276,324	3		3	117,743	117,743		2	9	502,865
West Virginia	249,681	245,981	1		5	370,941	355,181	7			18,800	18,800	1		1	964,852
Wisconsin	202,131	200,987	3			1,432,157	1,387,305	15		1	208,478	208,462		2	3	1,162,829
Wyoming	155,409	154,992		2	4	202,010	122,590	1	1		17,010	17,010			7	514,272
District of Columbia	30,215	30,215		1							283,544	250,000	1			128,186
Hawaii	50,098	48,630	1			181,790	181,790	4	1							360,830
Puerto Rico	61,900	61,550	2			394,352	392,150	9								426,676
TOTALS	15,931,770	15,630,604	191	48	258	33,586,289	32,716,026	297	60	199	11,569,406	10,758,863	95	19	513	57,549,944

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