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A CONCRETE ROAD WITH A CROSS SECTION BALANCED FOR LOAD STRESSES

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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

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# THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS 

## BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by L. W. TELLER, Senior Engineer of Tests; and EARL C. SUTHERLAND, Associate Highway Engineer
PART 3.-A STUDY OF CONCRETE PAVEMENT CROSS SECTIONS

THE SHAPES of the cross sections used in concrete pavement construction in the United States have gone through an interesting period of development during the last 15 years.
The earliest concrete pavements were laid in slabs that were either thicker in the center than at the edges or else were of uniform thickness at all points. The thick-center-thin-edge design was probably the result of the influence of the distribution of material in those macadam pavements with which engineers were most familiar at the time the early concrete pavements were laid.

Some of the firstattempts at a mathematical analysis of the stresses created by wheel loads in the pavement slab treated the transverse section as a beam supported at the ends, and this type of analysis naturally indicated the need for a section which was thicker in the center than at the edges. The uncertainty of assumptions as to subgrade support tended to make engineers hesitant about accepting the suggested theories of design, with the result that a considerable amount of the concrete pavement laid was of a uniform thickness.

Upon the entry of the United States into the World War, the wheel loads on many of our main roads suddenly increased greatly, and instances of edge failure of thin-edge sections began to be reported. These edge failures frequently began with a corner break at a construction joint or transverse crack and often developed into a completely shattered area of considerable size. Many engineers began to suspect that the thick-center-thin-edge pavement was not properly designed.

DEVELOPMENT OF THICKENED-EDGE PAVEMENT DESIGN REVIEWED
On November 12, 1920, in Maricopa County, Ariz., construction was begun under a very extensive paving

[^0]DEVELOPMENTS in the design of cross sections for concrete pavements during the past 15 years have produced such marked improvements in the roads constructed that one is likely to feel that a high degree of perfection has been attained. Edge thickening and longitudinal joints have been generally adopted and the present slab lengths are much less than those formerly used.
The most complete study yet made of the stresses in typical pavement slabs resulting from both load and temperature effects confirms the belief that progress has been made in the right direction, but it is clear that better designs are possible with the more complete knowledge now available.

The results of this study are surprising as to the stresses that will exist in concrete pavements with certain combinations of load, temperature conditions, and slab thickness. However, the conclusions are thought to be sufficiently well established for application in current design.

If loads alone are considered, the maximum of economy in the use of material is obtained with a thickenededge cross section.

While increased edge thickness results in a reduction of the edge stresses from applied load, it also causes an increase in the edge stresses under certain conditions of restrained warping.

Since a balanced cross section should in all cases be designed on the basis of combined load and warping stresses, it is obvious that economy demands that the stresses resulting from warping be limited to low values. The most practical way of doing this is by constructing short pavement slabs.

In short slabs the cross section may be designed on the basis of load alone.

A balanced cross section for load stresses is obtained with a design such as is shown on the cover page.

Edge thickening strengthens slab corners regardless of the length of the slab.

At the present time application of the principles set forth above to the design of pavement slabs involves considerations other than those discussed in this report but necessary to the forming of a correct judgment as to whether or not a completely balanced design should be used or how closely it should be approached.
program, including some 130 miles of concrete pavement. The point of particular interest is that the cross section adopted for the entire project was 3 inches thicker at the edges than at the center, the edge thickening being gradually reduced to zero at a distance of 24 inches from the edge of the slab. The report on this work ${ }^{1}$ states that the cross section was " a modified inverted-curb section designed to streng then the edge and at the same time permit simple construction of the subgrade. The thickened edges add structural strength as the area of load distribution and subgrade resistance decreases, thereby securing a paving slab with a more uniform resisting strength." This is clearly a recognition of the principle of design of thickened-edge slabs as we know it today.

The development of the thickened-edge design created widespread interest among highway engineers and, as the probable worth of the new design began to be appreciated, it was adopted for trial in a number of places.

The test road at Pittsburg, Calif., built during the summer of 1921, contained one section of the new design, and at the conclusion of the test this section was given the highest rating of all of those included in the track. ${ }^{2}$
The sections of the Bates test road in Illinois laid in 1920 and 1921 contained no thickened-edge designs but, fortunately, in the fall of 1922 sections of the new design were added and subjected to heavy traffic during $1923 .{ }^{3}$ The result was another early demonstration of the superiority of the new design over sections of uniform thickness when subjected to concentrations of heavy wheel loads. In this test the structural weakness of the edges of slabs of uniform thickness was definitely shown.

[^1]The Bureau of Public Roads at about this time developed a method for determining the stresses in concrete pavement slabs caused by wheel loads and, during 1923 and 1924, made a number of studies of stress. ${ }^{4}$ The data obtained from these tests indicated clearly the soundness of the thickened-edge design from a load-carrying standpoint.

The fact that concrete possesses definite elastic properties has led to several attempts to develop some mathematical analysis that would make possible the prediction of the stresses caused by load in a given pavement slab design. The most serious obstacle encountered in these efforts was the difficulty in treating rationally the conditions of subgrade support. It was not until Westergaard presented his analysis of the stresses in a concrete pavement slab in 1925 that there was available an even approximately tenable theory of design. ${ }^{5}$ In this analysis it is assumed that the load is applied over a definite area to an elastic slab that rests upon an elastic support. By means of the formulas presented it is possible to calculate the critical stresses resulting from a given load applied at the corner, at the free edge, or in the interior of the slab.

The analysis indicated that for the assumed conditions a thickened-edge design is necessary if the section is to offer uniform resistance to load at all points, thus confirming the evidence obtained from the field tests and stress measurements.

The analysis showed for the first time how important a factor the area of load application is in determining the magnitude of the critical stress. It was not possible however, to determine the correct shape of the slab cross section, nor was the means provided for determining the value of the elastic constant that should be used for a given subgrade in the practical application of the analysis to specific cases of design. In spite of these deficiencies the Westergaard analysis represents one of the most important steps in the development of concrete pavement slab design.

By 1924 several progressive States that were building considerable mileages of concrete pavement had adopted as a standard some form of thickened-edge design, but there was a wide variety of opinion as to what the shape of the cross section should be. Today, a decade later, 41 States are using exclusively some type of thickened-edge design and there still appears to be a considerable divergence of opinion as to the amount and distribution of edge thickening needed, the reason being that neither theory nor experiment has so far supplied the information to enable engineers to design with precision a cross section with equal resistance to applied load at all points.

## FACTORS AFFECTING CROSS-SECTION DESIGN INVESTIGATED

The investigation of the structural action of various concrete pavement slab designs which was begun in 1930 by the Bureau at the Arlington Experiment Farm included, as one of its major parts, a study of the behavior of a number of designs of pavement cross section. ${ }^{6}$

While it is usual to think of the design of a cross section as being determined by the variation in the

[^2]bending moments resulting from applied load, in parements, as in other structures, other factors must also be considered if a satisfactory design is to result.

Most of the data relating to the effects of temperature variations have already been presented, ${ }^{7}$ and in the attending discussion the primary importance of these effects has been brought out. In this study of pavement cross sections particular attention has been given to the following:

1. The effect of the condition of warping on the stresses caused by applied loads.
2. The effect of the changes in the supporting power of the subgrade caused by freezing and thawing or by other causes.
3. The stresses caused by variations in the temperature conditions within the pavement.

While the discussion naturally centers around the load-stress relations developed for the various cross sections, consideration is given to the effects of each of the factors just mentioned as observed in the tests at Arlington.

The sections selected for these tests have already been described in part 1 of this series of papers, but for the convenience of the reader the details of the various cross sections are again shown in figure 1. It will be observed that the sections shown cover fairly well the range of designs in use in this country today. There are four sections of uniform thickness $(6,7,8$, and 9 inches thick). This type of slab is still in use in seven States. There are three thickened-edge slabs of the type in which the edge thickening is reduced at a uniform rate to zero at a short distance from the edge. This general type is used in 28 States at the present time. The parabolic cross section, in which the thickening extends to the center of the slab, is used in 9 States, while the design suggested by the American Association of State Highway Officials (sec. 3 in fig. 1) is used in 1 State.

Although the actual dimensions of the cross sections used by some of the States will be found to differ somewhat from those of the sections shown in figure 1, the differences are not large and it is believed that complete tests on these sections provide adequate data upon which to base a judgment as to the efficiency or balance of almost any of the cross sections in use in the United States at the present time.

## PROGRAM OF LOAD TESTING DESCRIBED

The schedule of load testing followed in developing the data on the relative efficiency of the several cross sections can be most readily explained by referring to figure 2 , in which the four quarters or quadrants of one of the 20 -by 40 -foot test sections are shown. In this figure the small circles indicate the points where a load was applied, the load being placed successively on each point beginning at the free edge of the panel; the short lines within the circles in quadrants 1 and 3 show the position and direction of the strain gages; and the broken line $A^{\prime}-B^{\prime}$ in quadrant 4 is the line along which the curvature of the slab was measured. The loading schedule shown in quadrant 1 was followed on at least 1 quadrant of each of the test sections and on more than 1 quadrant if thought desirable. The schedule shown in quadrant 3 was followed on all four quadrants of all of the test sections. Deflection measurements

[^3]

Figure 1.-Designs of Pavement Cross Sections Studied.


Figure 2.-Points Wmere Loads Were Applied and Deflections and Strains Were Measured.
were made on one quadrant of each test section. On a number of the sections load-stress data were obtained during the different seasons of the year, particularly to determine the effect of subgrade condition.

In all of these loadings a bearing block 8 inches in diameter and of the grooved type was used in order that both deflection and strain measurements might be made within the area of load application. Because of the height of the clinometer it was necessary to use a split bearing block approximately 4 inches in height when making deflection measurements. The method of applying the load for deflection and strain measurements is shown in figures 3 and 4, respectively.

For the loading schedule shown in quadrant 1, recording strain gages of the type described in part 1 of this series of papers were installed at each of the 10 positions, being placed either all transversely or all longitudinally in any one test. The selected load was then applied successively at each of the 10 points, the strains in both the longitudinal and transverse directions being recorded at each gage position for each loading.

The tests scheduled in quadrants 1 and 4 were actually performed in the same quadrant of the test section in order that the closest possible relation would exist between the strain and deflection data.


Figure 3.-Applying a Test Load at the Edge of a Slab Panel. The Deflections Were Referred to a Bench Mark Located in the Shoulder.

All of the deflection data were obtained with a 10 inch clinometer using the procedure described in part 1 of this series of papers. The measurements were started at a bench mark set close to the edge of the test section (see fig. 3) and were continued entirely across the slab. The elevation of each clinometer point, with respect to the bench mark, including that directly underneath the load, was thus determined and the true shape of the transverse center line of the slab established. This determination was made first with no load upon the slab and again after the given load had been applied, the difference between the two curves at any point being the vertical deflection of that point caused by the applied load. In this manner the deflection of the slab between points $A^{\prime}$ and $B^{\prime}$ (see fig. 2) was obtained for a load acting at each of the several loading points.

Only a short period of time was permitted to elapse between the strain and the deflection measurements in order that the condition of the slab might not change.

## LARGE DEFLECTIONS AND STRAINS OBTAINED WITHOUT

 OVERSTRESSING CONCRETEIt was desirable to use loads of sufficient magnitude to cause large deflections and large strains in the concrete since this would reduce the error of measurement. On the other hand, it was necessary to put a limit on the strains produced by loading since it was very important that no injury be done to any of the test


Figure 4.-Strain Measurement With Load Placed Near Edge of Test Section. A Recording Strain Gage is Installed Directly Under the Load in a Direction Perpendicular to the Edge of the Slab.
sections. It was decided to limit the loads applied to the extent that the maximum stresses produced in the concrete would not, in general, exceed one-half of the average modulus of rupture as determined by the strength tests made at the beginning of the investigation. This criterion has proved to be satisfactory. With but one exception, all of the forty 10 - by 20 -foot test slab panels are apparently intact after 4 years of intensive load testing. The strains and deflections produced by loads of this magnitude were of sufficient size that they could be measured with satisfactory accuracy with the instruments available.
In the preceding paper it was shown that the condition of warping of a pavement slab had a definite effect upon the magnitude of the maximum stress a given load might be expected to produce, particularly if the load were applied at the corner or along the edges of the slab. It has also been established that, for the concrete used in this pavement at least, the moisture condition in the concrete had an important effect upon its elastic properties. These two facts made it necessary to take special precautions to maintain uniform conditions of moisture and of temperature within the concrete of the test sections during any period of load testing.

As previously described, the method adopted for protecting the slabs during these tests consisted of a covering layer of approximately 4 inches of dry straw laid directly on the concrete, with protection from rain, snow, or direct sunlight afforded by a large canvas shelter supported by a framework on the loading tank.

Temperature measurements made within the concrete of the slabs thus protected showed that the differentials in temperature between the upper and lower surfaces of the pavement were always very small, frequently so small as to be unmeasurable. At the time of testing there could have been but little if any temperature warping present in the test sections. It was
not possible to determine the variation in moisture throughout the depth of the concrete in the slabs and therefore no positive statement regarding the condition of moisture warping can be made. It seems reasonable, however, that the dead-air spaces in the dry straw layer which provided the thermal insulation would also greatly decrease the rate of moisture evaporation from the surface of the concrete and would reduce correspondingly the tendency for a differential in moisture content to develop. Whatever differential may have existed was held constant during the period of test by this method of protecting the test section.

## FORMULAS FOR CALCULATING LOAD STRESSES GIVEN

The strength and elastic properties of the concrete were determined by tests made upon drilled cores and sawed beams obtained from short sections of pavement constructed especially for this purpose at the same time that the test sections were constructed. One of the matters investigated in these collateral tests was the effect of moisture on the stiffness of the concrete, and it was found that the value of the modulus of elasticity varied to some extent with the moisture content, being highest for moist concrete. The details of these tests will be covered in a subsequent paper. It was decided as a result of the tests that the proper value of the modulus of elasticity of the concrete, in bending, and containing as nearly as could be determined the same percentage of moisture as the concrete in the test sections, was $5,500,000$ pounds per square inch. This value is used throughout this series of papers in computing stress values from the measured strains.

If a load is applied at a certain point on a pavement slab, the stresses developed at the point of load application can be determined from measured strains by means of the following formulas as described in part 2 of this series of papers:

$$
\begin{align*}
\sigma_{x} & =\frac{E}{1-\mu^{2}}\left(e_{x}+\mu e_{y}\right)  \tag{1}\\
\sigma_{y} & =\frac{E}{1-\mu^{2}}\left(e_{y}+\mu e_{x}\right) \tag{2}
\end{align*}
$$

in which $\sigma_{x}=$ the stress in the direction of the $x$-axis.
$\sigma_{\nu}=$ the stress in the direction of the $y$-axis.
$e_{x}=$ the unit deformation caused by stress in the direction of the $x$-axis.
$e_{y}=$ the unit deformation caused by stress in the direction of the $y$-axis.
$E=$ the modulus of elasticity of the concrete. $\mu=$ Poisson's ratio for the concrete.
The value of Poisson's ratio was not determined for these tests but was assumed as being 0.15 , which seemed to be a fair average value considering such test data as are available. ${ }^{8}$

In carrying out the test schedules described in the preceding paragraphs, a large number of observations were made. In all cases tests were repeated on the same or a different quadrant of the test section until the data obtained were considered to be well established. On some sections more tests were necessary than on others and in certain cases it was deemed desirable to repeat the tests under both summer and winter conditions. It is neither practicable nor desirable to present all of the data which were obtained, but an effort has

[^4]been made to include all significant data and to show representative data in all cases.

It has been found that the stresses caused by load in the vicinity of the longitudinal joint are affected to a considerable degree by the structural action of the joints. In designing a cross section, therefore, the shape of that portion immediately adjoining the longitudinal joint will be controlled by the design of the joint. Since the characteristics and design of joints are to be discussed in a subsequent paper, it is thought advisable to eliminate from the present discussion detailed consideration of the effect of the joint design upon the cross section. For this reason the data pertaining to that portion of the pavement within 3 feet of the longitudinal joints have been omitted (except in the case of the influence lines). There is every indication that at 3 feet from the longitudinal joint the effect of the joint action is so small as to be unimportant in all cases.

DEFLECTION AND STRESS VARIATION DATA OBTAINED FOR THE various cross-section designs

Figure 5 shows deflection curves along a transverse section for a slab of uniform thickness and for one of a conventional thickened-edge design. The magnitudes and positions of the load that produced the deflections are indicated in each case, and attention is called to the difference in the magnitude of the loads used in the two designs. These data show that the slabs are tipped slightly when the load is applied at the free edge, the opposite edge or center of the pavement being actually raised slightly. When the load is applied in the center of the 10 -foot section the slab is deflected over practically its entire width. Both of these effects are probably due to the comparative narrowness of the slab. The width, however, is typical of modern pavement construction. It is possible that the narrowness of the panel affects somewhat the elastic curvature and hence the stresses caused by a given load. This point has a bearing on any comparison of measured stresses with stresses calculated by an analysis that assumes an infinitely large panel.

The deflections of the thickened-edge slab are much smaller than those of the slab whose thickness is constant, for loads applied near the free edge. As the load is moved away from the edge, however, this difference decreases, becoming very small at a point 7 feet from the free edge.

Figure 6 shows longitudinal and transverse stresses produced by the same loads on the same two test sections for which deflection data were presented in figure 5. Each curve shows the variation of stress across the transverse section for the magnitude and particular position of load indicated. Longitudinal stress is measured parallel to the long axis of the pavement slab and transverse stress is measured perpendicular to it. These values are based on strains measured in the upper surface of the pavement and hence apply only to that surface. The corresponding stresses in the lower surface of the pavement would be of opposite sense and of very nearly the same magnitude.

## area of load influence found to be relatively small

It will be observed, from an examination of the stress variation diagrams of figure 6 , that a load applied at the free edge of a pavement slab produces two quite different stress conditions in the vicinity of the load. In the transverse direction the section acts somewhat as a cantilever and a fairly high transverse tensile


Figure 5.-Elastic Curves for Two Typical Cross Sections for Various Positions of Load.
stress is produced in the upper surface of the pavement, this stress reaching its maximum value at a distance of approximately 2 to 3 feet from the free edge. In the longitudinal direction, the maximum stress produced by the edge loading is a tension in the bottom of the pavement directly underneath the loaded area, with a corresponding compression in the upper surface. For the condition of these tests, in the case of the 6-inch uniform-thickness section, the magnitude of maximum longitudinal stress caused by edge loading is about $31 / 2$ times the magnitude of the corresponding maximum stress in the transverse direction and, in the case of the 9-6-9 section, about 6 times that in the transverse direction.

As the position of the load is moved gradually along a transverse section, the stress conditions created change from those just described to those of an interior loading. A load applied at the interior of a panel of infinite extent will cause deflection and stress conditions which do not vary with direction. Even on a panel of the limited dimensions used in these tests, it will be noted that when the loaded area is near the center of the panel the magnitudes of the maximum longitudinal and transverse stresses are practically equal. The variation curves reflect the effect of slab dimension, however, in that for loads applied near the center of the panel the slab is stressed for a somewhat greater distance in the direction of the long dimension than it is in the direction of the short dimension. These curves also show quite definitely that for the interior loading the critical stresses are highly concentrated and that the magnitude
of the stress decreases rapidly as the distance from the center of load application increases.

The area of influence of the load, i. e., the area within which appreciable stresses or deflections are produced, will naturally vary with the thickness and design of the cross section and perhaps with other factors. For the designs studied in these tests the data indicate that for the edge loading the area of influence for stresses is roughly a semicircle with a radius of approximately 3 feet, while for the interior loading the area of influence is roughly a circle with a radius of approximately 3 feet. These are only approximate values but the point of interest is the contrast with the areas over which appreciable deflections occur.

The data show that increasing the thickness of the slab near the free edge is an effective method for reducing the maximum stress that can be caused by an applied wheel load. The greatest reduction is thus found in longitudinal tension on the bottom of the slab with the load applied at the free edge of the pavement. There is also a slight reduction in the longitudinal stresses in the interior of the slab which is probably caused by a stiffening effect from the thickened edge.

## MAXIMUM DEFLECTION AND MAXIMUM STRESS DIAGRAMS CONSTRUCTED

If a given load is applied successively at a series of positions across a transverse section of a pavement slab, for each position of the load a certain maximum deflection and a certain maximum stress will be pro-


O-Stress in transverse direction
Flitre: 6. -trbs Variations in Two Typleal Sections for Various Positions of Load. Values Above thle Axis Indicate Tension and Values Below the Axis Indicate Compression in the Upper Surface of the Slab.
duced and the values of these can be determined. If these maximum values are ploted as ordinates at the points on the transverse section at which they were observed, a curve may be drawn through them which is in reality the envelop of all the deflection (or stress) variation curves that might be developed along the section by the load in question. Since this envelop curve shows the maximum deflections (or stresses) that could be developed by the given load regardless of its position, these graphs have been termed maximum deflection (or stress) diagrams in this paper.

Figure 7 shows maximum deflection diagrams for each of the 10 test sections. It should be noted that the diagrams apply only to the case of a single load and that the magnitude of this load varies with the design of the cross section under test.

Sections 10, 9, 7, and 6 are the sections of uniform thickness. The data from these sections show that the maximum deflection caused by a load applied at the free edge of a constant-thickness slab is slightly more than 3 times the maximum deflection caused by a load of the same magnitude applied at an interior point.



As the area of load application is moved from the free edge toward the interior the magnitude of the maximum deflection decreases rapidly until, with the center of load application approximately 3 feet from the edge, the maximum deflection is practically the same as it is for the load at the mid-point of the panel.

Sections 5,2 , and 1 are thickened-edge sections of the type in which the edge thickening is decreased at a uniform rate to zero at a short distance from the edge. The shape of the maximum deflection diagram for this type of section appears to be very similar to that of a section having a constant thickness, although the maximum deflection with the load applied at the edge of the slab is in this case on the average only about 2.35 times the maximum deflection mealsured for the same load applied at the interior of the panel. It is apparent that the edge thickening of these sections has a relatively small effect on the maximum deflections.

Section 4 is one in which the cross section is bounded by two parabolic curves, the center thickness being seven-tenths of the edge thickness. The edge deflection of this section is also approximately three times the deflection at the interior.

Section 3 has a heavy-edge cross section of a design once suggested by the American Association of State Highway Officials. The deflection of a slab of this design under an edge loading is approximately one and one-half times the deflection under an interior loading.

Section 8 is the section having the lip curb on the upper surface. As previously explained, two panels of this section (called sec. 8A) had the lip curb added to a slab of otherwise constant thickness while the remaining two panels were of the thickened edge type before the addition of the lip curb (sec. 8B in fig. 1). The deflection of the edge of section 8 A is a little more than 3 times the deflection of the interior, while in section 8 B the deflection under an edge loading is approximately 2 times the deflection under the same load at the interior of the slab.

These data show that in all cases a given load caused greater deflections when applied at the free edge of the pavement than when applied at the interior points. If the magnitude of the deflections were to be used as a criterion for balancing a design, one might conclude that none of the sections has been strengthened sufficiently along the free edge. It will be of interest to keep these curves in mind while examining the indications of the stress data which follow.

The maximum stress diagrams shown in figure 8 were prepared in a manner similar to that used in developing the deflection data just discussed. These curves show the variations in the maximum longitudinal and maximum transverse stresses as the given load is placed successively at the points shown in figure 2, quadrant 1 . The stresses are those directly under the load and generally are the maximum in each direction, the one exception being the transverse stress developed by the load applied at the edge of the slab. In this case the maximum transverse stress is developed 2 or more feet from the edge of the pavement. Because of the fact that a greater transverse stress is developed at the same point when the load is placed directly over it, the exception noted above does not affect the accuracy of the maximum stress diagrams of figure 8 .

The values plotted in these diagrams are generally the average of two sets of determinations (the data such as are shown in figure 6 being considered as one set) from tests made on one or (wo quadrants of the
test section. In order to provide a general check and to determine more closely the relation between the maximum stresses resulting from edge and interior loadings, the test schedule shown in quadrant is of figure 2 was carried out on all 4 quadrants of all the sections, except section \&, the lip-curb) design, where only 2 quadrants of each type were a wailable

## more uniform load-stress distribution fol: id on

 thickened-edge desiensIn general, the average maximum stresses for the edere and interior loadings as determined by these supplementary tests agree closely with corresponding values determined during the measurements mentioned above, i. e., the schedule shown in quadrant 1, figure 2. The measurements that detemined the stress rariation curves give a good indication of the variation in strengeth in the various parts of the different cross-section designs. The supplementary tests, although they in olve measurements at only the two positions on the cross section (the free edge and the interior), were made at a considerable number of points. For this reason, the average values for each test section (shown as triangular points in figure 8 and some of the other figures that follow) are believed to give the most reliable indication of the relative strengths of the edge and interior of any particular design. Some discrepancies may be noted in this figure, if a comparison is made between the stresses found in different test sections having comparable edge. or interior thicknesses. These discrepancies are believed to result in large part from the fact that different sections were tested at different times of the year.
There were variations in the condition of the concrete and of the subgrade that influence these data to some extent so far as the relation between slab thickness and stress caused by a given load is concerned. Other data obtained within a relatively short period of time were used in the comparison of slab thicknesses to be discussed in a subsequent report. Since all of the tests on any one section were made during a short period of time, it is believed that the effect of the seasonal variations in slab and subgrade condition do not enter any of the comparisons that are made in this discussion of pavement cross sections.

If a pavement cross section is to carry loads with a maximum of efficiency, the design, insofar as applied loads are concerned, should theoretically be such that a given load will produce a certain maximum stress regardless of the point of application of the load on the slab. The maximum stress diagram for such a design would have the same ordinates at the free edge, the interior, and all intermediate points. Any variation in the magnitude of the maximum stress at different points across the section is an indication of too much or too little strength at that particular point. The maximum stress diagrams can be used, therefore, as a hasis for judging the efficiency or "balance" of the various cross sections that were tested.
Sections 10, 9, 7, and 6 have a uniform thickness at all points. Referring to the stress diagrams for these sections in figure 8, it will be observed that the shape of the curves showing the variation in maximum stress across the slab are very similar for these four sections. An increase in the magnitude of the maximum stress in the longitudinal direction as the position of the load approaches the free edge of the slab is apparent in all of the sections, showing clearly the lower load-ramying capacity of this portion of a slab of uniform thickiness



Fifiure 9.-Maximum Stress Diagrams for UniformThickness Sections Deyeloped From Data Obtained for Sections Nos. $10,9,7$, AND 6.
when load stresses alone are considered. In the transverse direction the stresses are practically zero at the extreme edge of the sections but increase to their maximum value in a distance of approximately 4 feet from the edge. In the case of the longitudinal stresses the maximum value is at the free edge and this value decreases over a distance of approximately 3 feet to a minimum value. The maximum value of the transverse stress is equal to the minimum value of the longitudinal stress throughout the interior region of these sections.

The characteristics of a section of constant thickness are possibly shown better by the average diagram of figure 9. This diagram was constructed by assuming that stress varies directly with load and then averaging the maximum stresses that would be caused in each of the four pavement sections by an average load of 10,250 pounds.
These data indicate that there is a relatively large part of the cross section of a slab of uniform thickness over which a given load will produce practically the same maximum stress and that there is only a relatively small part of the cross section adjacent to a free edge that requires modification to "balance" the cross section. It is indicated that this adjustment of cross section need not extend more than $21 / 2$ feet from the free edge of the slab.

The average maximum stress at the extreme edge of these sections of constant thickness is approximately 60 percent greater than the average maximum stress in the interior portions under the given load.
Sections 5, 2, and 1 have thickened-edge cross sections of a similar type, the interior area of each being of a constant thickness. The maximum stress diagrams for these sections (fig. 8) give direct evidence on the reduction in the maximum edge stress resulting from various degrees of edge thickening. Although a definite reduction is apparent for all three of these sections it is indicated that in none of them is the relation between edge and interior thicknesses proper for a constant maximum stress value. It will be noted that the 9-6-9 section is the most nearly balanced, yet even with an edge thickness that is 50 percent greater than the interior thickness, the load placed 6 inches from the extreme edge of this section produced a maximum stress somewhat greater than that caused by the same load applied in the interior of the slab.

## EDGE THICKENING FOR BALANCED DESIGN DETERMINED

The relation between edge and interior stresses in the slabs of constant thickness and in the three thickenedeige sections just mentioned provides a means for estimating the extent to which the edge thickness should


Figure 10.-Effect of Edge Thickening on the Reduction of Edge Stress.
be increased if a perfect balance of the cross section is to be attained. In figure 10 the increase of edge stress over interior stress (expressed as a percentage of the latter) has been related to the increase of edge thickness over the interior thickness, similarly expressed, for each of these sections. In the case of the four constantthickness sections the average value was used. By drawing a straight line through the plotted points and prolonging this line a short distance to the horizontal axis of the graph, the intercept on this axis indicates approximately the percentage of increase in edge thickness that would be required to reduce the edge stress to the same value as the interior stress.

It will be observed that this method of analysis indicates that, for the type of cross section in which the interior of the slab is of constant thickness and the edge thickening is developed at a uniform rate in the outer ? or 3 feet of the pavement, the thickness of the extreme edge must be about one and two-thirds times that of the interior of the slab, if the section is to be completely balanced for stresses caused by load.

This relation would not necessarily hold for other cross sections of radically different shape and might be affected somewhat by the conditions of subgrade support.

Section 4 has a cross section bounded by two parabolic curves with a thickness of 6.3 inches at the longitudinal joint and 9 inches at the free edge. The maximum stress diagram for this section shows a slightly greater stress at the edge of the slab than at the interior but there is a section at a short distance from the edge in which the stresses are definitely lower than those in the interior of the panel (see fig. 8). The diagram indicates, therefore, that at the extreme edge the section is not of sufficient thickness to balance completely the edge and interior stress values, and that at a short distance from the edge the section is thicker than is necessary. It must be concluded that so far as stress from applied load is concerned this section is not well balanced.

Section 3 has the same center and edge thicknesses as section 4 but the shape of the cross section is quite different. In section 3 the 9 -inch edge thickness is maintained in the outer 2 feet and then reduced to the center thickness of 6.3 inches in the next 2 feet of the cross section. The remainder of the slab width is uniformly 6.3 inches thick. The maximum stress diagram for this section shows that the maximum


Figitre 11.- Application of a Test Load on the Extreme Edge of the Lip-Curb Section
stress at the free edge is approximately the same as that in the interior of the slab. That part of the cross section at a short distance from the edge is shown to be much stronger than either the edge or the interior and it is this part of the cross section that throws the design out of balance. The diagram also indicates that the heavy edge of this section exerts an influence on the magnitude of the stresses for a short distance heyond the point where the edge thickening is discontinned. The contrast between the deflection diagram (fig. 7) and the stress diagram (fig. 8) for this section is worthy of note. From the deflections it might be concluded that this section was the most nearly balanced of all of those tested, while the stress diagram leads to the conclusion that this section is not well balanced and therefore the most economical use is not made of the material in it.

The stress diagram for section 3 was compared with those for section 4 and the three sections in which the edge thickening was decreased at a uniform rate from the edge of parement inward (secs. 5, 2, and 1) and it Was found that the shape of the cross section has some influence on the relation between the edge and interior thicknesses necessary to balance the maximum stresses for a given load. For example, it has been noted that with the type of cross section represented by sections 5,2 , and 1 , an increase in edge thickness of approximately 66 pereent over the interior thickness is necessary to balance these stresses, whereas in section 3 an essential batance of edge and interior stresses is obtained with an edge thickness that is only 43 percent greater than that of the interior of the slab. This is undoubtedly due to the greater stiffness of those desions in which the edge thickness is not decreased rapidly from the edge of the pavement, designs which the stress diagrams show to be poorly balanced elsewhere.

SPECIAL METHODS OF LOADING LIP-CURB SECTIONS DESCRIBED
Section 8 was provided with a lip curb on either edge. As mentioned earlier, the two halves of this section, divided longitudinally, are of different design. One half, marked section 8 A on figure 1 , is of uniform thickness except where the lip curb was added, while on the other half, marked section 8 B on figure 1 , the lip curb was added to a 9 -inch edge, 7 -inch interiorthickness cross section. Thus section $8 \Lambda$ may be considered as being thickened on the top and section $8 B$ on both bottom and top.

In making tests at the extreme edge of these sections it was necessary to place loads and to measure strains in a manner slightly different from that used on the other sections, because of the presence of the lip curb. The bearing block used was solid, i. e., there was no groove in the bottom, and it was placed on top of the curb, the center of the block being in the center of the level section of the curb, as shown in figure 11. Since this level section is only 3 inches wide, the full area of the bearing block was not in contact with the pavement, and in this position the center of the loaded area was somewhat closer to the edge of the slab than was the case with the other sections. It was considered that this method of loading was justified as it represents the condition of a wheel load "riding" the curb.

The longitudinal strains were measured by installing the two strain gages on the vertical face of the edge of the pavement, one as near the upper and the other as near the lower surface of the pavement as possible.

For reasons that are obvious, it was not practicable to measure the strains in the transverse direction directly under the load when the load was placed at the edge of the lip-curb section. The transverse stress computed from the transverse strain was found to be practically zero at the edges of all of the other sections, so it was assumed to be zero at the edge of the lip-curb section. Consequently, in order to complete the transverse-stress curve for this section, the value plotted represents the transverse effect of the longitudinal stress. All of the other values shown in the stress diagrams are the direct result of strain measurements.

The maximum stress diagram for section 8 A shows a slightly higher stress at the edge than at the interior and a slightly lower stress about 18 inches from the edge. Across the remainder of the section the stress is practically constant. It appears that, except for loads applied at the top of the lip curb, this design can be considered as balanced and, since with a lip curb in normal service the application of loads on the top of the curb would probably occur but rarely, it appears that where a lip curb of this design is used no other edge strengthening is necessary. The maximum stress diagram for section 8 B confirms this conclusion. It will be seen in figure 8 that, in that part of the cross section which includes the thickening on the bottom of the slab, the stresses are consistently less than in the interior.

It might possibly be argued that, in spite of the fact that the very presence of a lip curb tends to keep the wheel loads away from the edge of the pavement, the section should provide fully for this occasional loading. If such a design is desired, the data indicate that the cross section, exclusive of the lip curb, should be made slightly thicker at the edge than at the interior although not to the same extent as in section 8 B . It is probable that an additional inch at the edge, decreased to zero at from 12 to 18 inches from the edge, would accomplish the desired result.

As previously remarked in the discussion of section 3, there is a marked difference between the deductions concerning the relative balance of designs that might be made from the maximum deflection diagrams and those that might be made from the maximum stress diagrams. The deflection data might be taken to indicate that all of the sections are very poorly balanced, while the stress data show that a number of the sections are fairly well balanced. The reason for the difference is that for a given loading the maximum stress is found where the change in the rate of curvature of the elastic curve is greatest and is a highly localized condition, whereas the maximum deflection noted may be the result of deflection over a large area and thus may not be associated with a marked change in the shape of the elastic curve.

This is one of a number of instances in these investigations in which the relations indicated by the deflection data are not in agreement with those shown by the stress data, and in all of the analyses which have been made much more weight has been given to the stress data since the latter are a direct measure of the loadcarrying ability of the slab. It is believed that the deflection measurements are, in all cases, as accurate as it is possible to make them by direct methods and that the data show very closely the relative movements which occurred. However, the shapes of the elastic curves of the deflected slabs are not determined by these methods with sufficient precision to warrant any deductions from the deflection data regarding the stress conditions associated with the deflection.

## EFFECTS OF MULTIPLE-WHEEL LOADINGS DISCUSSED

The data and discussion thus far presented have related solely to the effects produced by a single load. It has been shown by these and by earlier studies ${ }^{9}$ that the area of a pavement slab that is stressed by a single load is relatively small, and the data indicate that if several loads are applied simultaneously, as would occur in the case of a vehicle on the pavement, the critical stress under each load will not be increased by the presence of the other loads, providing the distance between the loads is approximately 3 to 4 feet or more. Thus it is to be expected that 4- and 6-wheel vehicles of the usual types will not have wheel loads so closely spaced that the effect of adjacent wheel loads on stress need be considered. On rare occasions, however, pavements are subjected to heavy loads on closely spaced wheels as, for example, where such cargoes as power shovels, road rollers, electrical transformers, or other concentrated weights are moved over the highway on what are generally known as heavy-duty trailers. These trailers are usually equipped with four wheels at the rear articulated in order to distribute the load to the pavement uniformly.

Figure 12 shows typical wheel spacings (along a plane through the rear axle) for two trailers of the type referred to. It will be noted that the close spacing of the wheels may possibly affect the critical stress in the pavement. Thus, this type of loading might have an influence on the design of the cross section of pavement slabs for certain locations.

No direct tests to determine stresses in the test sections under multiple loadings have been made up to the present time. However, data developed in the

[^5]

Figure 12.-Typical Wheel Arrangements for Heavy-Duty Trailers.
tests which have been described in this report furnish a means for investigating the probable effect of loadings such as are shown in figure 12 . It was thought that an analysis of these two cases, from the standpoint of a possible influence on the design of the slab cross section, would be of value in the present discussion.

In the description of the testing procedure it was stated that a load was placed successively at each of the points shown in quadrant 4, figure 2, for deflection measurements, or in quadrant 1 for strain measurements, and that the deflection or strain was measured at each of these points for every position of the load. From the data obtained it is possible to construct diagrams that show the variation in deflection or stress at any one of the points of measurement as the point of loading was moved transversely across the slab. These diagrams are in reality influence lines for deflection or for stress for the several points along the cross section.

Taking again the 6-inch uniform-thickness slab (sec. 10) and the 9-6-9 thickened-edge slab (sec. 5) as representative sections, influence lines for deflection for each of the 7 load positions in quadrant 4 are shown in figure 13, and corresponding influence lines for stress for each of the 10 load positions in quadrant 1 are shown in figure 14. These values are for a 5,000 -pound load applied on a circular area 8 inches in diameter.

It was mentioned earlier in this paper that the structural action of the longitudinal joint exerted an influence on the magnitude of the stresses produced by a given load and that the effect might extend as far as 3 feet from the joint. For this reason the data taken within this distance of the joint have been omitted generally from the figures. An exception was made in the case of the influence lines because it is believed that they will be more useful if shown in this way. The values given for points within approximately 3 feet of the longitudinal joint apply strictly only to those designs that include longitudinal joints having the same structural


Figtire 13.-Influence Lines for Deflection on Two Typical Sections.
action as those used in the tests, but should apply with reasonable accuracy for other designs of a similar type and degree of joint efficiency

The longitudinal joint of section 10 is a corrugated steel plate, with $1 / 2$-inch diameter deformed steel bars in bond and placed transversely at 60 -inch intervals. The longitudinal joint for section 5 is a steel plate with a single triangular tongue with steel bars placed in the same manner as in section 10. In tests of the efficiency of the longitudinal joints of the various sections it was found that both of the joints described were of approximately the same efficiency and that both were fairly efficient. It is believed that the data shown will apply with a reasonable degree of accuracy to longiludinal joints of the tongue type where longitudinal continuity is provided: If the longitudinal joint is of a different type then only the values for the outer 7 feet of the cross section apply.

The magnitude of the stresses developed in a concrete pavement slab by a load is affected by the type and character of the subgrade as well as by the elastic properties of the pavement. The absolute values of deflection and stress shown in these diagrams apply only to the particular conditions of the tests. However, as has been said before, the subgrade conditions for the lests at Arlington might well be considered as average and it is believed that the influence lines as developed
in these tests should give a good general indication of the effects of different types of loading on pavement slabs.

## MAXIMUM STRESS DIAGRAMS FOR MULTIPLE-WHEEL LOADINGS DEVELOPED

From the influence lines for stress, stress-variation curves, similar to those shown in figure 6 but for the particular load positions desired, were constructed for each of the four wheel loads of the two types of trailer. These are shown as light solid lines in figure 15 , and since they are all longitudinal stresses they may be combined as in the heavy solid lines to show the stress variation along the pavement cross section under the influence of the four loads applied simultaneously. The light dotted lines indicate the general shape of the enveloping curves, or maximum stress diagrams, that might be expected should this combination of loads be shifted transversely across the section. It should be noted that the curves in this figure are for the 6 -inch uniform-thickness section.

For the type of loading designated as " $\Lambda$ " (fig. 12) the diagram shows that the interior stress has a constant value to within approximately 3 feet of the free edge of the pavement, and that the maximum edge stress is about 50 percent greater than the interior stress. This differs but very little from the conditions


Fighe 14.- Infliente Lines for stress on Two Typical Sections. Valdes Above the Axis Indicite Tension, and Values Below the Axis Indicate Compression in the (ipper Stirface of the Sbab.


Figure 15.-Construction of Maximum Stress Diagram for Traller Loadings-Outer Wheel of Trailer Near Free Edge of Stab.
developed under the single loading, from which it seems reasonable to conclude that a section designed for balanced stresses under single or widely separated loads would prove to be reasonably well balanced under the trailer loading " $\Lambda$."

For a loading of the type designated as "B" the stress conditions are slightly different due to the greater separation and more uniform distribution of the loads. For the same total load the critical stresses are slightly lower than they were for loading "A" and the stress decreases across the full width of the loaded area from the edge toward the center. Again the maximum or cdge stress is about 50 percent higher than that developed in the interior of the panel. If a section were to be designed especially for this type of loading, the analysis indicates that it would be economical to extend the alge thickening toward the center of the pavement more than is necessary when designing for conventional types of loading. However, it is indicated again that ia pavement balanced for single loadings will also be reusonably well balanced for trailer loads of this type.

In the above analysis the outer load was centered 6 inches from the edge of the pavement. This is an extreme condition and not likely to be encountered in the moving of very heavy loads. It is of interest to make a similar study for the more probable condition of the center of the load group being coincident with the center of the 10 -foot slab width. In making this analysis the 9-6-9 thickened-edge section (sec. 5) was used, since this is a fairly well-balanced section and one which is widely used.

The stress variation curves for both trailer loadings o11 this section are shown in figure 16. These curves were developed in exactly the same manner as those in the preceding figure.

It appears from this diagram that on this cross section the maximum stresses under the four loads are of practically equal magnitude. It is also indicated that at these four critical points the stress developed by the muitiple loading is about 30 pereent greater than under the single load.


Figure 16.-Construction of Maximom Stress Diagram for Trailer Loadings-Center of Trailer Over Center Line of Slab Panel.

This means that 5,000-pound wheel loads distributed in this manner would cause critical stresses some 30 percent higher than would single or more widely separated wheel loads of the same magnitude.

As stated at the beginning of this discussion of multiple-wheel effects no direct tests of such loadings have yet been made nor have the relations between stresses developed under an 8 -inch diameter circular bearing block and those under various types and sizes of motor-vehicle tires been established. For these reasons the data should not be extended too far and the analysis should be considered indicative rather than conclusive.
There is another factor involved in an evaluation of multiple-wheel loading concerning which no data are available. When a single load is placed on a pavement slab and a certain maximum stress is developed, this maximum stress extends over a distance approximately equal to the diameter of the bearing area and then diminishes rapidly. For trailer loadings of the type described the maximum stress value obtains over a much greater percentage of the pavement width, and whether or not for a given stress value its effect on the slab is more severe has not been established.

In the preceding paper the importance of the stresses caused by restrained warping was pointed out. If heavy loads are to be moved on trailers having closely spaced wheels, consideration should be given to the possibility of over-stressing the pavement through a combination of warping stresses and load stresses. It may be desirable to confine the movement of certain types of heavy vehicles to those parts of the day when warping stresses are lowest, i. e., at night or in the early morning.

## CRITICAL STRESSES RESULTING FROM LOAD NOT GREATLY AFFECTED BY SLAB WARPING

In the first part of this paper attention was called to certain efleets of temperature that may influence the load-stress relation in concrete pavements, and it was stated that in this investigation consideration was given to the following:

1. The effect of the condition of warping on the stresses caused by applied loads.
2. The effect of the changes in the supporting power of the subgrade caused by freezing and thawing or by other causes.
3. The stresses caused by variations in the temperature of the pavement.

In the following paragraphs the significance of the data obtained with respect to the design of the cross section will be discussed.

One of the important effects of temperature changes on a concrete pavement is that temperature differentials are developed which cause marked changes in the shape of the slab. Since this results in a modification of the condition of support, it affects the magnitude of the stresses caused by a given applied load, as was brought out in part 2 of this series of papers.

In studying the design of a parement cross section some consideration should be given to the changes in the load-stress relation caused by slab warping in order to determine what correction, if any, should be applied to basic data obtained from pavement sections tested in a flat or unwarped condition.

A study was made of the effect of both upward and downward warping on the load-stress relation and these data were presented and analyzed in part 2. It was shown that the effect of slab warping on the magnitude of the stresses caused by load was negligible in the interior portion of the slab panel. In figure 35 of part 2 it was shown that:

1. Maximum downward warping redused the critical stresses caused by load at the edge of the pavement by about 6 percent in the 7 -inch slab (sec. 9) and about 2 percent in the 9 -inch slab (sec. 6).
2. Maximum upward warping increased the critical stresses caused by load at the edge of the pavement by about 8 percent in the 7 -inch slab and about 20 percent in the 9 -inch-slab.

From these data it would be concluded that it is not necessary to make any correction to the load-stress relation at the interior of the section, but that small arljustments of the critical stresses at the outer end of the cross section may be desirable to compensate for the effects of slab warping.

## FREEZING AND THAWING OF THE SUBGRADE PRODUCES CONSIDERABLE EFFECT ON STRESSES

In part 2 there was included a description of certain load tests conducted during a period of freezing and thawing of the subgrade and a rather full discussion of the effect of the condition of the subgrade on the loadstress relation was given. This is a matter having possible bearing on the design of pavement cross sections and it is pertinent to reexamine the data with this in mind.

It will be recalled that at the time of these particular tests the subgrade was at first frozen solidly to a depth of $2 \frac{1}{2}$ inches under the pavement and to a depth of 7 inches below the surface of the earth shoulder, and that later it thawed rapidly until near the surface it became very wet and completely plastic. During the time that the carth was frozen, the pavement sections were displaced upward or heaved about one-half inch.
The conditions of pavement support are probably more uncertain during the period immediately followinge a thawing of the subgrade than at any other time. For this reason stress data obtained after a thaw are of more importance than those obtained with the subgrade in the frozen state.

Figure 17 shows the maximum stress diagram for section 5 ( $9-6-9$ cross section) under a 7,000 -pound load with the subgrade in a softened condition immediately after a hlaiw. The values shown are the averages of two sets of strain measurements.


Figure 17.-Maximum Stress Diagiram for One of the Test Sections With the Subgrade in a Softened Condition Immediately After a Thaw.

The maximum stresses at the various points in the interior of the slab are much less uniform than they were found to be with the subgrade in its normal condition. (See fig. 8.) The lack of uniformity in the stresses is probably the reflection of a similar lack of uniformity in the physical condition of the subgrade. Also the stresses for a 7,000 -pound load (fig. 17) are of approximately the same magnitude as those shown for the 8,000-pound load (fig. 8).

It will be noted that the curve showing the variation of maximum stress across the section has been drawn through the maximum stress values rather than as an average of all the values. This was done deliberately so that the relation between edge and interior stresses as shown by the diagram for the condition of a thawing subgrade would represent the most critical value developed by these tests. As the curve is drawn the maximum edge stress is approximately 25 percent above the value of the maximum stress at the interior. This is practically the same relation as was found in other tests during the winter months when the subgrade is wet, so it appears that, for this section at least, it might safely be assumed to be representative of winter conditions. For the remainder of the year the relation between edge and interior stress is as shown in figure 8 .

## COMBINED LOAD AND TEMPERATURE WARPING STRESSES DISCUSSED

When the temperature of the pavement changes two effects are produced. First, there is a change in the average temperature of the concrete; and second, there is almost certain to be a change in the relation between the temperatures of the upper and lower surfaces of the slab.

The first effect changes the dimensions of the slab and, because there is a resistance to horizontal movement developed in the earth of the subgrade, direet tensile or compressive stresses (depending upon the direction of movement) are created, which attain their maximum value in the mid-section of the slah. The unit stresses developed in this manner are normally of the same magnitude across the entire width of any given section and for this reason do not enter as a factor in the design of the cross section. Their principal influence is a general increase or decrease in the amount of stress resistance. that is available for carrying wheel loads.
The stresses developed as a result of restrained warping under the action of a temperature differential (the second effect mentioned above) are of more importance than the direct stresses described in the preceding paragraph, becanse at certain times they may be of considerably greater magnitude, but more particularly because they vary in magnitude from point to point


Figure 18. -Adrangement of Recording Strain Gages and Thermocouple Equipment for Studying the Stresses Caused by Restrained Temperature Warping.


Figure 19. - Maximum Stress Diagrams for Combined Load and Warping S'tresses for Two Typical Cross Sections. The Double-Hatched Area Shows the Small Reduction Appled to the Orserved Load Stress Values to Corfect fol the Effect of Wariping.
across a given section. It is desirable, therefore, to examine these stresses as they combine with the stresses caused by applied loads in order to determine their influence on the design of the cross section.

The stresses resulting from restrained warping were not determined for all of the test sections, but the data for one constant-thickness section and one of the thickened-edge sections will be given as typical examples. The two sections selected were the 6 -inch constant-thickness section (sec. 10) and the 9-6-9 thickened-edge section (sec. 5). The technique employed in these tests has been described in part 2 and a typical installation of the apparatus is shown in figure 18.

Figure 19 shows the maximum stress diagrams for these two sections and the stress values shown are the result of restrained warping and applied load combined.

The load stresses are those shown in figure 8, while the warping stresses are average maximum values as determined by measurement during the spring and summer months when the temperature warping is at a maximum, as shown in figure 35 of part 2 .

It was not possible to measure the warping stresses on the sections having thickened edges in the usual manner, due to the difficulty of measuring the change in length which should occur in the interior of the slab for the condition of free warping. It is probable, however, that the warping stresses for the $9-6-9$ section approximate in magnitude those of the 6 -inch uniform-thickness section except in the vicinity of the edges. It was shown in part 2 that the average warping stresses at the edge of the 9-6-9 section are approximately 30 percent above those at the edges of the 6 -inch section of constant thickness, and this relation has been used to establish the warping stress at the edge of the 9-6-9 section in figure 19.
Since the load stresses near the edge are reduced slightly by the same downward warping of the edges
that produces the critical warping stresses, a slight correction has been applied to the load stress values in this diagram. The correction was made in accordance with the experimental data on this point previously referred to.

The curves in figure 19 for the maximum combined stress across the two typical pavement sections show a very uniform stress condition throughout the interior area of both sections. Since the interior of both of these sections is of constant thickness, it is indicated that for a balanced cross section the interior of the slab should be of a constant thickness. This was shown to be the requirement when only the stresses developed by applied loads were considered. The shape of the interior portion of the cross section is the same, therefore, whether determined by load stresses alone or by combined stresses.

Referring to the diagram for the 6 -inch constantthickness section in figure 19 , it will be seen that the combined stress at the edge is about 90 pounds per square inch, or 15 percent above that in the interior. This compares with an increase of 165 pounds per square inch or 60 percent where load stresses only are considered (see fig. 8), and the comparison indicates that when the combination of the stresses from restrained warping and from applied loads is considered, the pavement of constant thickness is more nearly balanced than it is for the stresses from applied load alone. In order to balance the combined stresses for this section it would be necessary to reduce the edge stresses by slightly more than one half of the amount that would be necessary to balance the load stresses only

For the 9-6-9 section, the diagram in figure 19 shows that combined stress at the edge of the pavement is about 85 pounds per square inch or 13 percent above the interior stress. This is practically the same relation as was found to exist in the slab of constant thickness and indicates that so far as maximum combined stresses across a mid-slab section are concerned, the 9-6-9 section is no better balanced than the section of constant thickness. This somewhat startling result is explained by the fact that the increase in the warping stress at the edge of the thickened-edge slab, caused by the increased temperature differential which results from the greater depth, is approximately equal to the reduction in load stress at this point accomplished by the increased edge thickness.
SHORT SLAB LENGTHS NEEDED TO MINIMIZE STRESSES CAUSED BY TEMPERATURE WARPING

It has been shown, during the discussion of warping stresses in part 2 , that the magnitude of the critical warping stress that most directly affects the load capacity of the pavement is a function of slab length. The data show that in a slab length of 10 feet the maximum warping stresses are small but that they increase with length until the length is such that complete restraint to warping exists in the interior of the slab. The length necessary for complete restraint in slabs of different thicknesses has not been determined in this investigation. It is indicated that practically complete restraint is developed by a 20 -foot length in a slab of 6-inch constant thickness, and also that this length in a 9-6-9 thickened-edge section is not sufficient to develop full restraint at the mid-length of the panel.

Because of the effect of slab length, it is apparent that the analysis of combined stresses given above can be applied exactly only to slab lengths of 20 feet.

It is not known whether the stresses caused by load in the test sections are larger or smaller than those developed in actual pavements by traflie, so that the comparison between the combined stresses observed in the test sections and the combined stresses that occur in parement slabs in service is not certain. It is apparent, however, that in parement slabs as they are designed today the factor of safety against breaking must be very small at times when conditions are such as to produce high warping stresses. The relatively frequent transverse cracking in our more heavily traveled concrete pavements is also an indication that the combined stresses in them often exceed the flexural strength of the concrete.

At first thought one might expect immediate cracking when the combined stresses exceeded the strength of the concrete, and this would probably occur if the high stresses extended completely across the slab. When the critical stress is highly localized, as it appears to be under an isolated load, a single application of an excessive stress may produce no immediate effect. In cases where load stresses are responsible either wholly or in part for the cracking of a pavement, it is but natural to expect the transverse cracking to develop gradually and to continue over a period of years.

Consider, for example, the case of a heavy wheel load moving longitudinally on the parement at a time when high warping stresses are present in the slab. Being of the same sense in the mid-section of the slab, the stresses combine and may exceed the ultimate flexural strength of the concrete. These tests indicate that the high combined stress will be found to exist only for a very short distance directly under each wheel. The remainder of the cross section will not be overstressed. It is not probable that such a stress condition would cause a full-length transverse crack immediately, but it is reasonable to believe that a great many repetitions of the condition would cause such a crack.

If the length and width of a pavement slab were equal, the warping stresses would be the same in the two directions and it would be reasonable to expect the added stress caused by wheel loads to cause longitudinal cracking to precede transverse cracking, because the wheel load traverses the complete length of the slab, stressing every point in the longitudinal section, yet this wheel stresses to the same degree only a comparatively small part of any traverse section. This may explain the early appearance of longitudinal cracks in some of the older pavements that were built without a longitudinal joint.

In concrete slabs where the combination of the length, depth, and flexural strength are such that warping stresses alone may cause the slab to crack, it is to be expected that such cracks will develop suddenly, since the high stress condition extends over the entire cross section.

SHORT SLABS MAY BE DESIGNED ON THE BASIS OF LOAD STRESSES ALONE

Before proceeding with a discussion of the application of the results of the investigation to the design of a pavement cross section, it may be well to review brielly some of the points developed in parts 1 and 2 .

It has been demonstrated conclusively that variations in the temperature of the concrete in the parement slab frequently cause large stresses in certain parts of the slab. These stresses combine with those caused by applied loads, and the parement design, to be adequate, must be based upon combined stresses rather than upon load stresses alone

It will be recalled that loads were applied with the test sections in the flat condition and in the warped condition and that tests were made when the subgrade was in its normal condition of moisture and also at a time of excessive moisture immediately after a thaw. Data were presented to show the influence of concrete temperature on (1) the direct stresses resulting from the resistance offered by the subgrade to horizontal slab movement, and (2) the bending stresses caused by restraint to free warping which exist in practically every part of the slab.

Since the magnitude of the stresses caused by a given applied load is affected by the warping of the slab to a certain degree at the edge of the slab and to a different degree at the interior, it is evident that this is a factor that must be considered.

It was shown in figure 17 that the relation between calge and interior stresses is practically the same during the very wet conditions that prevail immediately after a thaw as during the balance of the winter. Many of the tests that supplied the data for the maximum stress diagrams were conducted during normal winter conditions. It is believed to be unnecessary, therefore, to make an allowance for subgrade condition in setting the ratio of edge to interior thickness.

The usual assumption concerning the direct stresses of tension or compression, caused by the resistance to horizontal movement offered by the subgrade, is that they are uniform over the full cross section of a slab of constant thickness. It is probable that the same assumption could be applied to the thickened-edge sections of this investigation with no great error. If this is assumed to be the case, the stresses referred to will not affect the shape of the cross section.

The effect of closely grouped wheel loads was investigated to a limited extent, and the maximum stress diagrams which were developed indicate that the shapes of the load-stress curves for the multiple loads are approximately the same as for single loads. This leads to the tentative conclusion that a cross section designed for single loadings will not need to be modified because of multiple loadings of the type described.

In order to design a cross section from the maximum stress diagrams shown in figure 19 it will be necessary to make allowance for the following:

1. The effect of the condition of warping on the stresses caused by loads applied along the cross section of the slab.
2. The stresses caused by the restrained warping resulting from temperature differentials within the slab.

The stresses caused by restrained warping are dependent in magnitude upon the length and the thickness of the pavement slab and this must be considered in a discussion of the design of the cross section. It W:as shown in table 5 of part 2 that if the length of slabs of the usual pavement thicknesses is reduced to approximately 10 feet, the maximum stresses caused by restrained warping are quite small. The difference between the warping stress at the edge and that at the interior points is reduced to such an extent as to be unimportant in its effect upon the design of the cross section. If a parement slab were to be deliberately designed with a length of 10 feet, it would be sufficient to) lesign the cross section on the basis of load stresses alone.

The maximum stress diagrams shown in figures $S$ and! will be used to design a balanced pavement cross section with the slab length limited to 10 feet. The principal decisions to be made are:

1. What should be the shape of the interior portion of the cross section?
2. What should be the relation between the depth of the slab at the interior where the conditions of support are most favorable and the depth at the free edge where they are the least favorable?
3. What should be the shape of the cross section near the free edge?

It is definitely indicated by all of the data and shown more clearly by the average curve for the four constantthickness sections that, except in the immediate vicinity of the slab edge, the pavement slab should be of uniform thickness (see fig. 9).

A load applied at an unsupported edge causes a maximum stress considerably higher than that produced by the same load placed at interior points of the slab. Designing for balanced load stresses therefore requires a strengthening of free slab edges. The degree of thickening required for this strengthening can be determined from figure 10, which indicates that the depth of the slab at the free edge should be one and two-thirds times the depth in the interior region. As mentioned before, this ratio applies only to cross sections in which the additional or edge thickening is reduced to zero approximately uniformly between the edge and a point a short distance from the edge.

The distance in which this reduction of edge thickness should be accomplished is determined by the variation in the stresses in the longitudinal direction since these are the critical load stresses in the vicinity of the edge. The average curve for stress for the four con-stant-thickness sections (see fig. 9) shows that the interior stress is practically uniform from the center out to a point approximately $2 \frac{1}{2}$ feet from the edge, and indicates that for this area no additional depth would be required. The data also indicate that the effect of the thickened edge on the maximum stress from load is felt even beyond the point at which the thickening ends. Considering the stress diagrams tor all of the sections, it seems reasonable to conclude that the edge thickening need not extend more than approximately 2 feet from the edge. Again this applies only to the type of cross section in which the edge thickness is reduced to the interior thickness at an essentially uniform rate, as in section 5 . While the data obtained with the test sections of this type indicate that this is probably the minimum distance over which the edge thickening should be reduced, as a practical matter in shaping the subgrade the tendency will be to increase rather than decrease this dimension. In balancing the cross section for load, it is probably permissible to adopt the minimum distance, or 2 feet, as this dimension.

## TEST RESULTS APPLIED TO THE DESIGN OF A BALANCED CROSS SECTION

It is not to be expected that the data from field tests, no matter how carefully the tests were conducted, would justify fine distinctions as to the shape of the cross section where the edge thickening is introduced. The data indicate quite definitely that the desired balance of stresses is accomplished to a satisfactory degree by a cross section in which the increase in slab thickness is developed at a uniform rate as the edge is approached, as in sections 1,2 , and 5 . None of these three sections had an erlge depth suflicient to balance fully the load stresses, yet the advantage of this type of edge thickening over that provided in sections is and 4 is readily apparent in the maximum stress diagrams.

The pavement cross section shown in figure 20 has been designed in accordance with the principles developed by this investigation, and it represents a section that is completely balanced so far as the stresses from applied loads are concerned. The interior of the cross section is of constant depth and the relation between this depth and that at the extreme edge is $3: 5$ and is constant for the usual range of pavement thicknesses. The difference between the depth at the edge and that at the interior is reduced to zero in a distance of 2 feet.

It is apparent that the design based on these principles is not radically different from that used by a number of States today. However, attention is called to the fact that this design does not take into account temperature stresses and for this reason can be effective only when conditions are such that large temperature stresses will not develop. Climatic conditions so constant as to accomplish this are rare indeed, but as mentioned previously, a very large reduction in the magnitude of the critical temperature stresses can be obtained by using short slabs.

It is indicated by the data presented in part 2 that even with a 10 -foot slab length the maximum stresses caused by restrained warping are greater in magnitude than the increase in load stress caused by upward warping of the edges of the pavement. Therefore, thie critical combined stresses to be considered in the design of the cross sections are those occuring during the daytime when the edges of the slab are warped downward.
It was assumed earlier that the small difference between the warping stresses at the edge and those at the interior of a 10 -foot slab would not cause the relation for combined stresses to be materially different from that which obtained for load stresses alone. In proceeding from the unwarped condition, for which the cross section shown in figure 20 was designed, to that of the combined stresses of the warped slab of the 10foot length, it is only necessary, therefore, to modify the load stresses slightly due to the effect of the condition of downward warping. It has already been shown that this decrease for a load applied at the edge of the 7 - and the 9 -inch sections was about 5 percent.

If the edge stresses shown in figure 9 are decreased by 5 percent, it is found that the edge thickness of a balanced slab should be approximately 1.6 times the interior thickness instead of 1.66 as was indicated for the unwarped slab condition. This is a small difference and is perhaps beyond the accuracy of this method for determining the relation between edge and interior thickness. The data do lead definitely to the conclusion that in slab lengths of approximately 10 feet a balanced design requires a maximum edge thickness that is slightly more than one and one-half times the interior thickness.


Increasing either slab length or parement thickness increases the warping stress up to the point at which the length for a given thickness is such that complete. restraint to warping exists in the center of the slab. For this reason, increasing the edge thickness of a long slab may not increase the load-carrying capacity of the slab edge as much as would be expected. As shown in the maximum stress diagrams for combined stress in the two 20 -foot slabs of figure 19 , the benefits derived from the thickened edge from the load-stress standpoint are practically offset in the 20 -font section by the increased warping stresses which unaroidably result. It is conceivable that there may be cases of slabs so long that complete restraint to warping is developed in which the thickening of the slab edge may actually lower the load-carrying ability of the edge.

It may seem from the above discussion that the thickening of slab edges should be eliminated in slabs with lengths of 20 feet or more. So far as strengthening the edge in the mid-length of the slab this is probably true. The high combined stresses in long slabs that carry heavy loads eventually lead to transverse cracking which shortens the slab length and thus reduces the warping stresses and consequently the combined stresses at the edges of the slab.

While the design of slab corners is not discussed in this paper, edge thickening is so intimately connected with corner design that some mention of the effect of edge thickening on the stress conditions in the region adjacent to the slab corner should be included. Thickening the edges of a pavement slab does not increase the combined stresses in the corner because the critical warping stresses are opposite in sense to those caused by load. Edge thickening is effective therefore in reducing the combined stresses at the corner.

This study leads inevitably to the conclusion that a condition of balanced stressesin a pavement cross section is possible only when the critical stresses arising from restraint to warping are definitely limited to low values. The most practical method of insuring this seems to be through the construction of short pavement slabs. Application of this conclusion to the design of pavement slabs involves considerations other than those discussed in this report but necessary to the forming of a correct judgment as to whether or not a completely balanced design should be used or how closely it should be approached.


CURRENT STATUS OF UNITED STATES WORKS PROGRAM GRADE CROSSING PROJECTS
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