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UNITED STATES DEPARTMENT OF AGRICULTURE BUREAU OF PUBLC ROADS


LABORATORY APPARATUS FOR MEASURING MOTOR TRUCK IMPACT

# UNITED STATES DEPARTMENT OF AGRICULTURE BUREAU OF PUBLIC ROADS 

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#### Abstract

The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to the described conditions


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# CALIBRATIONS OF ACCELEROMETERS FOR USE IN MOTOR TRUCK IMPACT TESTS 

REPORT ON COOPERATIVE INVESTIGATION BY U. S. BUREAU OF PUBLIC ROADS AND U. S. BUREAU OF STANDARDS

Reported by J. A. BUCHANAN, Associate Engineer of Tests, and G. P. ST. CLAIR, Associate Engineer of Tests, U. S. Bureau of Public Roads

IN 1923 the Bureau of Public Roads, in cooperation with the Rubber Association of America and the Society of Automotive Engineers, commenced a series of tests to determine the magnitude of the impact reactions between a truck wheel and the pavement and to study the major factors influencing the magnitude of such reactions. The impact force was determined indirectly by computation, using the acceleration and mass of the unsprung weight on the truck wheel and the truck spring pressure or sprung weight at the instant of impact. To measure such accelerations an instrument known as the coil spring accelerometer was developed and used by the bureau.
This coil spring accelerometer was calibrated by comparing readings obtained fromit with the magnitude of accelerations determined by the analysis of displace-ment-time records of the primary motion. A special impact testing machine was used to drop a truck wheel (to which the accelerometer was attached) on a pavement slab. It was found that, for use in tests involving any particular type of tire, a certain range in the characteristics of the sensitive element, i. e., spring-weight combination, of the accelerometer produced satisfactory results. A system of calibration factors was set up for each sensitive element for use with the tire equipments for which it was adapted.

[^0]This report deals with an investigation of three methods of measuring accelerations due to motor truck impact. They are, (1) by analysis of records from the displacement-time apparatus, (2) by means of the cantilever-spring, contact accelerometer, and (3) by means of the coil spring accelerometer. These instruments are described and the theory involved in the use of each is given. The calibrations included static tests, dynamic tests on a machine producing simple harmonic motion having periods and amplitudes analogous to those of motor truck impact, and dynamic tests with a motor truck on a reaction wheel closely simulating road-operating conditions. Data are given showing the agreement between basic measurements of acceleration, the relation between theory and practice in the case of the coil spring accelerometer, the dispersion to be expected in coil spring accelerometer data, and the estimation of impact periods. Comparisons are made which show the effect that new calibration data would have on impact forces computed by original calibrations. . The report closes with a statement of conclusions reached as a result of the investigation, which may be briefly stated as follows:

1. The contact type of accelerometer may be used to obtain highly accurate measurements of acceleration of the order of magnitude and period encountered in motor truck impact work.
2. The coil spring accelerometer may be used to obtain reasonably accurate measurements of acceleration due to motor truck impact, when due consideration is given to the relation between the period of the impact reaction and the period of the accelerometer element.
3. A recomputation of published data using calibration factors obtained in this investigation indicates that the impact reaction values which are based on the original calibration factors are from 10 to 15 per cent too low, because of systematic errors in calibration. The comparisons made and conclusions drawn in reports based on such data, being general in nature and depending upon many tests rather than upon individual measurements, show with sufficient accuracy the relations and factors which influence the magnitude of the impact reactions.
4. The displacement-time apparatus may be used under laboratory conditions to determine the magnitude of the acceleration, the period of impact, and other characteristics of motortruck impact reactions.

REASONS FOR ACCELEROMETER INVESTIGATION outlined
Prior to the publication of the first report on these impact tests, an investigation was made to determine the accuracy with which the accelerations of motortruck impact were measured by the coil spring accelerometer. This investigation, which was also cooperatively conducted, resulted $n$ the finding that the instrument was sufficiently accurate for use in procuring data from which only generalized comparisons were made. The first impact report showing the influences of various factors on the magnitude of motor-truck impact reactions was then released and appeared in Public Roads, June, 1926.

Two additional research projects in motortruck impact were planned to supply definite information concerning immediate problems. One was to determine the influence of the thickness of tread rubber (in solid and cushion types of tire) on the magnitude of impact reactions produced. The other was to establish the correlation between road roughness, tire equipment, wheel load, and vehicle speed in influencing the magnitude of impact reactions under actual operating conditions. While working on these projects, the Bureau of Public Roads, in the summer of 1926, prepared for an extended investigation of the coil spring accelerometer under well-controlled laboratory conditions which closely simulated the reaction between the truck wheel and the road.

The two motor-truck impact projects were completed in the fall of 1927, prior to the completion of the extensive instrument investigation. The reports on the impact tests were approved for publication, but before this was accomplished certain objections were made concerning the accuracy of the data. The bureau then decided to withhold all information from publication pending a thorough investigation of the instrumentation by some agency satisfactory to all parties for the purpose of developing complete knowledge concerning the accuracy of the instruments.


Figure 1.-General View of Truck and Instrument Arrangement for Reaction Wheel Tests

An agreement to conduct a cooperative accelerometer investigation was made between the United States Bureau of Standards and the United States Bureau of Public Roads in April, 1928. This agreement provided for carrying on a program of calibration of accelerometers which had been used or proposed for use in the motor-truck impact research. This investigation has been completed and a detailed report written. The calibration data developed by the Bureau of Public Roads have been incorporated in the report and constitute the section dealing with the reaction wheel tests. Space is not available for the publication of the entire detailed report. The following summary report has been prepared to acquaint the reader with the general nature of tests and analyses made, which form the basis for the conclusions reached.

## THE MACHINE FOR PRODUCING SIMPLE HARMONIC MOTION DESCRIBED

It was decided by the committee that the instruments to be investigated should be calibrated on a device which could be made to produce accelerations of known magnitude. For this purpose the Bureau of Standards obtained the loan of a machine for reproducing simple harmonic motion which had been designed and built by the Firestone Tire \& Rubber Co. The rotating parts of this machine consist of two flywheels on a 3-inch shaft supported by large bearings in a cast-iron frame.

The combined weight of flywheels and shaft is about 1,000 pounds. A driving pulley is attached to one wheel and an adjustable mounting for an eccentric pin is provided on the other wheel. The eccentric pin floats in the yoke of a cross-head frame in such a manner that it is capable of transmitting only the vertical component of its motion. Practically all of the reciprocating parts are of aluminum alloy, their total weight, exclusive of instruments, being about 30 pounds. The machine is belt driven by an electric motor and an independent high-pressure oil system is provided to lubricate all bearings. All bearings are adjustable, so that minimum clearances may be maintained. The rated capacity of the machine is 1,800 revolutions per minute and 4 -inch stroke. Tests were run at speeds ranging from 300 to 2,100 revolutions per minute and with strokes ranging from 0.1 to 3 inches.

The speed of the machine was measured by means of a Veeder liquid tachometer, connected by a chain and sprokets directly to the main shaft between the two flywheels. It was calibrated by the Bureau of Standards prior to these tests, and the speed could be determined to the nearest revolution per minute without difficulty. For convenience in making minor changes in speed the machine was equipped with a small handbrake bearing on the periphery of one of the flywheels.

The reciprocating yoke is guided in a housing which covers the flywheel and cross-head assembly, An aluminum platen about 10 inches square is screwed on the upper end of a tubular extension of the yoke protruding above the housing. Instruments to be tested were mounted on this platen. The stroke of the machine was measured by means of a micrometer depth gage. Figures 4, 8, and 17 show the various instruments mounted for test.

## REACTION WHEEL USED TO PRODUCE IMPACT

In the summer of 1926 the Bureau of Public Roads purchased a steam engine flywheel having a diameter of 6 feet $6 \frac{1}{2}$ inches and a 19 -inch face. This wheel was mounted on a 6 -inch shaft carried by a massive reinforced concrete emplacement and furnished an excellent means for conducting motor-truck impact tests under well-controlled laboratory conditions. The pit in which the reaction wheel is situated is covered with a heavy timber floor so that a test truck can be driven into position with one rear wheel resting on the surface of the reaction wheel directly above the center of its shaft. The front axle of the truck is restrained from any motion whatever and the left rear wheel is prevented from moving in the fore-and-aft or lateral directions by means of chocks. The truck wheel is driven in the ordinary manner with a resulting rotation of the reaction wheel. Various obstructions were bolted to the reaction wheel to create impact conditions. To avoid interference between successive impacts, only one obstruction was in use during any given test. Figure 1 shows a test truck mounted on the reaction wheel and Figure 2 shows a closer view of the instrument set-up.

The vertical motion of the truck wheel is transferred to the instrument mounting in the following manner. A hub extension is securely bolted to the wheel and revolves in a self-aligning bearing which is encased in a rectangular bronze block. This block moves with practically no clearance within a steel yoke, which in turn moves freely in vertical guides. The bronze block


Figure 2.-Installation of Instruments for Reaction Wheel Tests
may move in and out and forward and back, transmitting only the vertical component of the motion of the wheel to the yoke. As may be seen on the cover page, the yoke is a part of the rigid mounting which carries the three instruments used in the tests-the displacement-time recorder, the contact accelerometer, and the coil spring accelerometer. The displacementtime instrument is connected to the base of the yoke, while the contact instrument is mounted directly above it. A rod extending above the truck body carries the coil spring accelerometer, which is restrained from lateral motion by vertical guides. Various devices which are not described here were installed to obtain the semiautomatic control of the instruments which was necessitated by the rapidly repeated impacts.
The two trucks used in tests on this reaction wheel were those which had been used in conducting the major portion of the motor-truck impact tests and the method of loading was the same as in the road tests. The tire equipments used on the trucks were selected from and were representative of those used in the earlier tests. Static load-deflection curves for these tires are given in Figure 3, and Table 1 contains data concerning principal dimensions of tires and weights.

Table 1.-Tire and weight data
No.
${ }^{1}$ Includes demountable rim.

Table 2.-Schedule of reaction wheel tests completed

| - Tire | Truck | Wheel load | Shape and size of obstruction in inches | Speed <br> range, miles per hour |
| :---: | :---: | :---: | :---: | :---: |
| Dual pneumatic (underinflated). | B | Pounds $3,500$ | 1 by 3, rectangular.-...... | 8 to 20 |
| Dual pneumatic (standard inflation). | B | $3,500$ | $1 / 2$ by 3 , rectangular <br> $3 / 4$ by 3 , rectangular <br> 1 by 3 , rectangular. <br> $11 / 2$ by 3 , rectangular. <br> $11 / 2$ by 3 , rounded. | $\begin{aligned} & 8 \text { to } 31 \\ & 8 \text { to } 25 \\ & 8 \text { to } 25 \\ & 8 \text { to } 20 \\ & 8 \text { to } 19 \end{aligned}$ |
|  |  | 5,000 | $1 / 2$ by 3 , rectangular. 1 by 3, rectangular $\qquad$ $11 / 2$ by 3 , rounded. .------- | $\begin{array}{r} 8 \text { to } 25 \\ 8 \text { to } 20 \\ 10 \text { to } 16 \end{array}$ |
| Dual cushion_(new) ...........- | A | 13,300 | $3 / 4$ by 3 , rectangular | 8 to 28 |
|  |  | 7,500 | 1/2 by 3 , rectangular----.-. <br> $3 / 4$ by 3 , rectangular. <br> 1 by 3 , rectangular. <br> $11 / 2$ by 3 , rectangular. <br> $11 / 2$ by 3 , rounded | $\begin{aligned} & 7 \text { to } 25 \\ & 6 \text { to } 25 \\ & 7 \text { to } 27 \\ & 7 \text { to } 15 \\ & 8 \text { to } 22 \end{aligned}$ |
|  |  | 10,000 | 1/2 by 3 , rectangular......... $3 / 4$ by 3 , rectangular. <br> 1 by 3, rectangular. <br> $11 / 2$ by 3 , rectangular <br> $11 / 2$ by 3 , rounded. | 7 to 25 <br> 4 to 25 <br> 5 to 25 <br> 6 to 15 <br> 5 to 15 |
| Dual_solid (new) .-.............- | A | 7,500 | $1 / 2$ by 3 , rectangular $3 / 4$ by 3 , rectangular <br> 1 by 3 , rectangular <br> $11 / 2$ by 3 , rounded | $\begin{aligned} & 8 \text { to } 20 \\ & 8 \text { to } 20 \\ & 7 \text { to } 25 \\ & 8 \text { to } 14 \end{aligned}$ |
|  |  | 10,000 | 1/2 by 3 , rectangular $3 / 4$ by 3 , rectangular. <br> 1 by 3 , rectangular <br> $11 / 2$ by 3 , rounded | 8 to 20 <br> 8 to 20 <br> 8 to 16 <br> 8 to 12 |
| Dual worn solid (1 inch visible rubber). | B | 3, 500 | 1/2 by 3 , rectangular........ $3 / 4$ by 3 , rectangular | $\begin{aligned} & 9 \text { to } 17 \\ & 9 \text { to } 14 \end{aligned}$ |
| Dual worn solid ( 0.6 inch visible rubber). | B | 3,500 | $1 / 2$ by 3 , rectangular. | 8 to 14 |

## TEST PROGRAM AND SCHEDULE OF COMPUTATIONS DISCUSSED

Table 2 is a schedule of the reaction wheel test program as completed. Tests were made with each of the six tire types, ranging from underinflated pneumatics to badly worn solids. The heavy duty truck, A, was used for the tests with cushion and new solid tires; the lighter truck, B, was used for the pneumatic and worn solid tire tests. Because of the enormous amount of computation involved, it was necessary to select representative tests for analysis, so as to cover as wide a range of conditions as possible. Table 3 is a schedule of tests for which the data were analyzed in full. This analysis included computations of acceleration from the records of the contact accelerometer and of the displacement-time instrument, computation of impact period from the displacement-time record,

Table 3.-Schedule of reaction wheel tests analyzed in full

| Tire | Truck | Wheel load | Shape and size of obstruction in inches | Type of impact | $\begin{gathered} \text { Num- } \\ \text { ber } \\ \text { of } \\ \text { tests } \end{gathered}$ | Speed range, miles per hour |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dual pneumatic (underinflated). | B | $\begin{array}{r} \text { Pounds } \\ 3,500 \end{array}$ | 1 by 3, rectangular .-.- | Shock <br> Drop. | 6 6 | $\begin{aligned} & 8 \text { to } 19 \\ & 8 \text { to } 20 \end{aligned}$ |
| Dual pneumatic (standard inflation). | B | 3, 500 | 1/2 by 3 , rectangular... <br> 1 by 3 , rectangular.... | Shock.-- <br> Drop. $\qquad$ <br> Shock <br> Drop. $\qquad$ | 1 1 7 7 | $\begin{array}{r} 20 \\ 19 \\ 8 \text { to } 25 \\ 8 \text { to } 25 \end{array}$ |
|  |  | 5,000 | 1 by 3 , rectangular.. | Shock.-. <br> Drop.... | 6 6 | $\begin{aligned} & 8 \text { to } 21 \\ & 8 \text { to } 20 \end{aligned}$ |
| Dual cushion (new). | A | 7, 500 | 1 by 3, rectangular.. | Shock... <br> Drop..-- | 5 | $\begin{array}{r} 10 \text { to } 27 \\ 9 \text { to } 26 \end{array}$ |
|  |  | 10,000 | 1/2 by 3 , rectangular... <br> $3 / 4$ by 3 , rectangular $\qquad$ <br> 1 by 3 , rectangular $\qquad$ <br> 11/2 by 3, rectangular .- <br> $11 / 2$ by 3 , rounded..... |  | $\begin{aligned} & 5 \\ & 3 \\ & 9 \\ & 5 \\ & 8 \\ & 7 \\ & 3 \\ & 4 \\ & 4 \\ & 3 \end{aligned}$ | 12 to 27 7 to 20 7 to 24 11 to 26 5 to 24 4 to 25 4 to 14 8 to 14 4 to 14 |
| $\overline{\text { Dual solid (new) -- }}$ | A | 7,500 | $1 / 2$ by 3 , rectangular... <br> 1 by 3 , rectangular. $\qquad$ | Shock.. <br> Drop <br> Shock <br> Drop. | $\begin{array}{r} 1 \\ 1 \\ 13 \\ 10 \\ \hline \end{array}$ | $\begin{array}{r} 20 \\ 19 \\ 8 \text { to } 23 \\ 7 \text { to } 26 \\ \hline \end{array}$ |
|  |  | 10,000 | 1 by 3, rectangular...- | Shock.-. <br> Drop.... | 4 | $\begin{aligned} & 7 \text { to } 14 \\ & 8 \text { to } 13 \end{aligned}$ |
| Dual worn solid (1.0 inch visible rubber). | B | 3,500 | $1 / 2$ by 3 , rectangular... <br> $3 / 4$ by 3 , rectangular | Shock.-. Drop. Shock <br> Drop.-- | 4 <br> 5 <br> 4 <br> 4 | 10 to 18 10 to 18 9 to 15 10 to 14 |
| Dual worn solid (0.6 inch visible rubber). | B | 3, 500 | $1 / 2$ by 3 , rectangular... | Shock.-- <br> Drop...- | 3 | $\begin{aligned} & 8 \text { to } 12 \\ & 9 \text { to } 14 \end{aligned}$ |

measurement of records from the coil spring accelerometer, and application of a method of correlation between the coil spring accelerometer and the other two instruments. Table 4 is a schedule of tests for which computations of impact periods were made by analysis of displacement-time records. The technique of the analyses and the data obtained will be described in subsequent paragraphs.

## METHOD OF ANALYSIS DESCRIBED

The reaction wheel tests were conducted primarily to calibrate the coil spring accelerometer under the conditions of motor-truck impact. The displacement-time recorder and the contact accelerometer were used as control instruments, and the analysis of the data obtained from the coil spring accelerometer is based upon the data obtained from the other two instruments.

This report is concerned chiefly with the question of the accuracy of the three instruments used for measuring accelerations. In considering the error and dispersion of a given instrument or the agreement between two instruments, it has been found necessary to employ certain terms and methods analogous to those used in statistical work without adopting the rigorous procedure of that science. An error or a deviation from a mean value is generally expressed as a percentage of the correct or mean value. Thus if $A_{M}$ is a mean or assumed correct value of acceleration and $A_{e}$ is a corresponding value deviating from it, the percentage of error is defined by the equation $e=100\left(\frac{A_{e}}{A_{M}}-1\right)$. The value of $e$ is generally given absolute without regard to sign.

When data showing the relation between two variables exhibit considerable dispersion, a frequency diagram

Table 4.-Schedule of reaction wheel tests analyzed for period only

| Tire | Truck | Wheel load | Shape and size of obstruction in inches | Type of impact | $\begin{gathered} \text { Num- } \\ \text { ber } \\ \text { of } \\ \text { tests } \end{gathered}$ | Speed <br> range, <br> miles <br> per <br> hour |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dual pneumatic (standard inflation). | B | $\begin{gathered} \text { Pounds } \\ 3,500 \end{gathered}$ | $1 / 2$ by 3 , rectangular... <br> $3 / 4$ by 3 , rectangular $\qquad$ <br> $11 / 2$ by 3 , rectangular .- <br> $11 / 2$ by 3 , rounded. $\qquad$ | Shock.-- <br> Drop <br> Shock <br> Drop <br> Shock... <br> Drop <br> Shock <br> Drop. | $\begin{aligned} & 3 \\ & 2 \\ & \\ & 3 \\ & 3 \\ & 3 \\ & 3 \\ & 3 \\ & 3 \\ & 3 \end{aligned}$ | 8 to 22 . <br> 10 and 31. <br> 8 to 26. <br> 8 to 26 . <br> 8 to 21. <br> 8 to 20. <br> 8 to 18. <br> 8 to 19. |
|  |  | 5,000 | $1 / 2$ by 3 , rectangular... <br> $11 / 2$ by 3 , rounded | Shock <br> Drop. <br> Shock <br> Drop. | 3 3 3 3 | $\begin{aligned} & 8 \text { to } 20 . \\ & 8 \text { to } 24 . \\ & 10 \text { to } 16 . \\ & 10 \text { to } 15 . \end{aligned}$ |
| Dual cushion (new). | A | ${ }^{1} 3,330$ | 3/4 by 3, rectangular ..- | Shock .-. Drop.... | $\begin{array}{r} 11 \\ 7 \end{array}$ | $\begin{aligned} & 8 \text { to } 27 . \\ & 8 \text { to } 27 \text {. } \end{aligned}$ |
|  |  | 7,500 | $1 / 2$ by 3 , rectangular $\qquad$ <br> $3 / 4$ by 3 , rectangular $\qquad$ <br> 1 by 3 , rectangular $\qquad$ $11 / 2$ by 3 , rectangular. - <br> $11 / 2$ by 3 , rounded $\qquad$ | Shock <br> Drop $\qquad$ <br> Shock. $\qquad$ <br> Drop $\qquad$ <br> Shock. <br> Shock. $\qquad$ <br> Drop. $\qquad$ $\qquad$ <br> Drop. $\qquad$ | $\begin{array}{r} 6 \\ 5 \\ 7 \\ 11 \\ 1 \\ 4 \\ 4 \\ 5 \\ 4 \end{array}$ | 8 to 18 . <br> 7 to 25. <br> 8 to 23. <br> 8 to 22. <br> 9. <br> 7 to 14 . <br> 8 to 15 . <br> 8 to 21 . <br> 8 to 20 . |
|  |  | 10,000 | $1 / 2$ by 3 , rectangular ... <br> $3 / 4$ by 3 , rectangular <br> 1 by 3 , rectangular <br> $11 / 2$ by 3 , rectangular .- <br> $11 / 2$ by 3 , rounded | Shock. <br> Drop <br> Shock. <br> Drop.... <br> Shock.-- <br> Drop $\qquad$ <br> Shock.-. <br> Drop. $\qquad$ <br> Shock_-- <br> Drop. $\qquad$ | $\begin{array}{r} 4 \\ 6 \\ 6 \\ 10 \\ 2 \\ 6 \\ 4 \\ 4 \\ 5 \\ 2 \end{array}$ | 7 to 20. <br> 7 to 27 . <br> 5 to 26 . <br> 5 to 24. <br> 15 and 20. <br> 5 to 24. <br> 8 to 14 . <br> 4 to 14. <br> 4 to 15. <br> 11 and 15. |
| Dual solid (new)- | A | 7,500 | $1 / 2$ by 3 , rectangular... <br> $3 / 4$ by 3 , rectangular <br> $11 / 2$ by 3 , rounded | Shock $\qquad$ <br> Drop $\qquad$ <br> Shock $\qquad$ <br> Drop $\qquad$ <br> Shock $\qquad$ <br> Drop. $\qquad$ | 2 2 3 3 3 3 | 8 and 14. <br> 8 and 14. <br> 8 to 21. <br> 8 to 21. <br> 8 to 14. <br> 8 to 14 . |
|  |  | 10,000 | $1 / 2$ by 3 , rectangular ... <br> $3 / 4$ by 3 , rectangular <br> $11 / 2$ by 3 , rounded. | Shock $\qquad$ <br> Drop $\qquad$ <br> Shock $\qquad$ <br> Drop $\qquad$ <br> Shock $\qquad$ <br> Drop. $\qquad$ | 3 3 3 3 3 3 | $\begin{aligned} & 8 \text { to } 19 . \\ & 8 \text { to } 20 . \\ & 8 \text { to } 19 . \\ & 8 \text { to } 20 . \\ & 8 \text { to } 12 . \\ & 8 \text { to } 11 . \end{aligned}$ |

${ }^{1}$ Empty truck.
is substituted for the customary plot of coordinates. The frequency diagram is divided into squares, the length of the sides of each square representing convenient intervals in the two variables involved. Within each square is written a number equal to the number of points which would appear within that square if the coordinates were plotted. In this manner the distribution of the points about a mean position is brought out clearly. Figures $12,18,20,22$, and 25 are examples of such frequency diagrams.
In a number of cases it was desirable to develop functions expressing the probable error of the data. This was done by drawing curves in such a manner that half the data shown in the given frequency diagram lies below and half above the curve. A similar curve is drawn bounding a zone which includes 90 per cent of the data. These curves are denoted by the symbols $e_{50}$ and $e_{90}$ as in Figures 13, 21, and 26.

## THE DISPLACEMENT-TIME RECORDER DESCRIBED

It is possible, with the reaction wheel set-up, to obtain an autographic record of the vertical motion of a truck wheel under impact. If such a record is taken on a drum rotating at a known constant speed, a curve is obtained from which the acceleration-time


Figure 3.-Static Load-Deflection Curves for Tires used in Reaction Wheel Tests. Curves for Dual Tires Obtained by Adding Loads for Individual Trres at Given Deflections
relation may be derived by differentiation. The Bureau of Public Roads has used this method of measuring acceleration for a number of years. The early calibration of the coil spring accelerometer was based on displacement-time tests conducted with a stationary drop-impact machine. A new instrument was constructed for use in the reaction-wheel tests. Figure 4 shows this instrument mounted to record the vertical movement of the yoke on the simple harmonic motion machine at the Bureau of Standards. The cover page shows its installation for the reaction-wheel tests.

The essential elements of the displacement-time recorder are, (1) a vertically moving rod which carries a recording stylus, (2) a drum which revolves at a constant speed about a vertical axis, and (3) a directrecording tuning fork. The rod moves in specially constructed adjustable bronze bearings which are lapped in so as to allow free movement with minimum clearance. The rod is connected to the body whose motion it is desired to record by a short stud having a necked section one-eighth inch in diameter and onehalf inch in length. This flexible link was designed to eliminate binding of the rod in its bearings due to slight lateral displacements of the moving body, and, in case of accident, to serve as a weak link protecting the instrument from violent distortion. The recording stylus is mounted within a solenoid and is moved into contact with the paper by the operation of a key switch.

The drum is of aluminum and is approximately $7^{1 / 4}$ inches in diameter and $91 / 2$ inches in height. It turns in conical bearings, and is driven by a small motor


Figure 4.-Displacement-Time Apparatus on the Simple Harmonic Motion Machine
through a 100 to 1 worm-gear reduction. The motor speed is about 10,000 revolutions per minute, producing a tangential velocity of approximately 38 inches per second at the surface of the drum. The record paper is attached to the face of the drum with rubber cement and is coated with a light film of smoke. Upon removal from the drum the record is fixed by being dipped in a thin solution of pure white shellac in grain alcohol.


Figure 5.-Photograph of Displacement-Time Record

The tuning fork is driven by an electromagnet operating on a make-and-break circuit. The record is made by a small stylus of very thin brass projecting from the end of one prong and resting very lightly on the record paper. The tuning fork has a frequency of approximately 58 cycles per second.
The accuracy with which accelerations can be determined depends largely upon the precision with which the coordinates of displacement and of time can be measured. It is therefore necessary that the amplitude and period of the motion to be analyzed be large with respect to the errors in the measurement of these quantities. The accuracy of the determination depends further upon the nature of the motion involved and it may be stated as a general rule that the curve to be analyzed should be the record of a fundamental motion of great regularity or "smoothness," disturbed only slightly if at all by parasitic vibrations or irregularities.

Tire deformations under impact are generally of considerable amplitude, varying from a few tenths of an inch in the case of worn solid tires to an inch or more in the case of pneumatic tires. The periods of impact reaction vary from less than 0.01 second for badly worn solids to as much as 0.10 second for pneumatics. In almost all cases the displacement-time curves are of a regular character, exhibiting a gradual change of curvature with the passage of time. Small parasitic oscillations, however, are always present. These oscillations may be due to engine vibrations, to small local structural deformations, or to disturbances within the recording mechanism itself. They cause minute irregularities in the curve, and are eliminated in the analysis by a process of smoothing. It may be stated that, in general, displacement-time records of impact reactions may be accurately analyzed

However, because of the wide range of amplitudes and periods and the varying effect of parasitic irregularities, the accuracy is necessarily variable. The most uniform results were obtained from tests with cushion and new solid tires. Tests with worn solid tires were rather unsatisfactory because of the extremely violent reactions occurring with such tires.

## SOURCES OF ERROR IN RECORDING ANALYZED

In the complete report of these investigations there is given a detailed discussion of the errors to which the displacement-time recording mechanism is subject. The
following sources of error are considered, and estimates are made of their probable effect, based on such data as are available:
(1) Sources of error in displacement only
a. Elastic deformations of the vertically moving rod and its flexible connection to the body whose motion is being measured.
b. Movement of the supports on which the instrument is mounted.
(2) Sources of error in time only.
a. Error in calibration of the tuning fork.
$b$. Irregularities in the functioning of the tuning fork.
c. Fluctuation in speed of the drum.
(3) Sources of error in both displacement and time.
$a$. Errors in alignment and verticality of the rod and the drum.
b. Eccentricity of the drum.
c. End play in the drum.
d. Irregularities in the record paper and in the manner of its mounting on the drum.
$e$. Malfunctioning of the recording stylus, due to-

1. Eccentricity of the point.
2. Play in the barrel.
3. Bending of the projecting portion of stylus.
$f$. Shrinkage or expansion of the paper after fixing with shellac solution.
It was found that for moderate test conditions, such as those encountered with cushion and solid tires, the probable error in acceleration due to imperfections in the recording device lies between 1 and 2 per cent. For severe conditions, as in the case of tests with worn solid tires, the error is undoubtedly much greater. For highly favorable conditions it may be much less.

## METHOD OF DIFFERENCES USED IN ANALYZING RECORDS

For use in analyzing displacement-time curves a special comparator was built to order. By means of this comparator one may read rectangular coordinates, over a 4 -inch range for each coordinate axis, directly in ten-thousandths of an inch. It was calibrated by the Bureau of Standards and found to be highly accurate. Errors inherent in the comparator are entirely negligible in comparison with the errors of the recording device, discussed in the preceding paragraph.

Figure 5 is a photograph of a displacement-time curve obtained in the reaction-wheel tests. The significant portions, or ares, of the curve are marked "shock" and "drop." The former occurs as the tire encounters and mounts an obstruction, the latter as it again reacts upon the surface of the reaction wheel. The object of the analysis is to obtain for either or both of these ares a curve or tabulation of values expressing the variation of acceleration corresponding to the displacement-time variation given by the are in question.

For the desired portion of the curve comparator readings of displacement are made at regular intervals of the time coordinate and tabulated. The method used in analysis is numerical rather than graphical and is based on the calculus of differences. ${ }^{2}$ This method is described in full with examples in the detailed report. The first step in the analysis is the elimination, by the process known as "smoothing," of irregularities caused by parasitic occillations of high frequency or by irregularities in the width of the record line. This is accomplished by replacing the series of displacement coordinates given by the comparator readings with a smooth series differing from it only by very small amounts. It has been found in practice that in almost all cases the smoothing can be restricted to changes of less than 0.002 inch in magnitude and that the great majority of changes can be held below 0.001 inch. The fact that the process of smoothing involves changes in the fourth decimal place with occasional changes of 1 or 2 in the third place indicates that the probable errors due to the use of that process are well within the limits of accuracy imposed by the imperfections of the recording apparatus and the varying qualities of the displacement-time curves themselves.

The first differences of a tabulated series are obtained by subtracting each tabulated value from the value next following it. The second differences are obtained by applying the same process to the first differences, and so on. If the series is tabulated for regular intervals of the independent variable or argument (time in this case) values of the derivatives of the tabulated function with respect to the argument may be obtained by the use of formulas involving the tabular differences as defined above. Accelerations were computed by the application of such formulas to the smoothed series of displacement coordinates, tabulated at regular intervals of time.

Given a smooth series, no error is involved in this method of analysis except those "last-place" errors which are characteristic of all computations where values are given in a fixed number of decimal places. These inaccuracies may be eliminated by adding one or more fictitious decimal places and carrying out the process of smoothing until the series is "smooth" in the last added place. Such a procedure does not, of course, affect the true accuracy of the acceleration measurement, which depends on other factors.

DISPLACEMENT-TIME RECORDER TESTED ON SIMPLE HARMONIC MOTION MACHINE
A number of tests were conducted with the instrument mounted as shown in Figure 4 to record the reciprocating movement of the simple harmonic motion machine. Tests were made at the following strokes and speeds:

[^1]| Stroke, <br> inches <br> (depth <br> gage) | Revolutions <br> per minute <br> (liquid ta- <br> chometer) |
| :---: | :---: |
|  |  |
| 0.997 | $299,599.899$ |
| 2.000 | $299,599,899$ |
| 2.998 | $299,599,899$ |

The corresponding periods of cycle, computed by means of the formula, $T_{1}=\frac{60}{\text { r.p.m.' }}$ are $0.2007,0.1002$, and 0.0667 second, respectively.

In each case the downward half-cycle was chosen for analysis, because it is analogous to the "drop" in motor truck impact. Values of acceleration were obtained by three different methods, and given the symbols $A_{1}, A_{2}$, and $A_{3}$. They are defined as follows:

$$
A_{1}=\frac{4 \pi^{2}}{T_{1}^{2}} a
$$

where $a$ is one-half the stroke as measured by the depth gage and $T_{1}=\frac{60}{\text { r.p.m. }}$ as measured by the liquid tachometer.

$$
A_{2}=\frac{4 \pi^{2}}{T_{1}^{2}} a,
$$

where $a$ is one-half the stroke and $T_{1}$ is twice the period of the lower half-cycle, both quantities being obtained by lineal measurements on the displacement-time record.
$A_{3}=$ acceleration obtained from analysis of the dis-placement-time record by the method of differences.
The value of $A_{1}$ is independent of the displacementtime record and may be regarded as the basic value with which $A_{2}$ and $A_{3}$ are to be compared. The accuracy of $A_{2}$ depends upon the precision with which displacement and time are recorded and measured. In the case of $A_{3}$ the method of analysis, including the process of smoothing, is involved.
Table 5 gives the values of $A_{1}, A_{2}$, and $A_{3}$ for the 16 curves which were analyzed. In Figure 6 the following combinations are plotted: $A_{2}$ against $A_{1}, A_{3}$ against $A_{2}$, and $A_{3}$ against $A_{1}$. Lines are drawn having a 1 to 1 slope to denote complete agreement.

In order to indicate the extent to which the various values of acceleration deviate from complete agreement, values of the following quantities are given in Table 6:

$$
\begin{aligned}
& A_{2}-A_{1}, \\
& e_{2}=100\left(\frac{A_{2}}{A_{1}}-1\right), \text { (absolute value) }, \\
& A_{3}-A_{1}, \\
& e_{3}=100\left(\frac{A_{3}}{A_{1}}-1\right), \text { (absolute value) } .
\end{aligned}
$$

It is possible, without resort to statistical analysis, to obtain a fair idea of the probable errors of $A_{2}$ and $A_{3}$ on the assumption that $\Lambda_{1}$ is correct. By inspection of Table 6 we find that the median valines of $e_{2}$ and $e_{3}$ are as follows:

Median $e_{2}=0.98$ per cent.
Median $e_{3}=1.07$ Der cent.

Table 5.-Values of $A_{1}, A_{2}$, and $A_{3}$ for the 16 curves analyzed

| Test No. | Stroke | Speed | $A_{1}$ | $A_{2}$ | $A_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Revolutions per | Feet per second per | Feet per second per | Feet per second per |
|  | Inches | minute | second | second | second |
| 1 | 2. 000 | 299 | 81.7 | 82, 5 |  |
| 3 | 2. 000 | 599 | 327.9 | 329.3 | 327.7 |
| 4 | 2. 000 | 599 | 327.9 | 329.5 | 330.3 |
| 5 | 2. 000 | 899 | 738.5 | 735.4 | 770.6 |
| 6 | 2. 000 | 899 | 738.5 | 750.9 | 756.6 |
| 7 | 2. 998 | 299 | 122.4 | 124.9 | 125. 4 |
| 10. | 2. 998 | 599 | 491.5 | 491.6 | 495.4 |
| 11 | 2. 998 | 599 | 491.5 | 496.8 | 491. 1 |
| 15 | 2.998 | 899 | 1,107. 1 | 1,111. 1 | 1,118.9 |
| 16 | . 997 | 299 | 40.7 | 41.1 | 40.9 |
| 17 | . 997 | 299 | 40.7 | 41.7 | 42.4 |
| 18 | . 997 | 599 | 163.4 | 164.9 | 167.9 |
| 19. | . 997 | 599 | 163.4 | 165.0 | 163.4 |
| 20 | . 997 | 899 | 368.2 | 371.9 | 369.2 |
| 21. | . 997 | 899 | 368.2 | 372.4 | 375.1 |

Table 6.-Values indicating the extent to which the various values of acceleration deviate from complete agreement

| Test No. | $A_{1}$ | $A_{2}-A_{1}$ | $\begin{aligned} & e_{2}=100 \times \\ & \left(\frac{A_{2}}{A_{1}}-1\right) \end{aligned}$ | $A_{3}-A_{1}$ | $\begin{aligned} & e_{3}=100 x \\ & \left(\frac{A_{3}}{A_{1}}-1\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feet per | Feet per |  | Feet per |  |
|  | second per second | second per |  | second per second |  |
| 1. | 81.7 | +0.8 | 0.98 | +1.3 | 1. 59 |
| 3 | 327.9 | +1.4 | . 43 | -. 2 | . 06 |
| 4 | 327.9 | +1.6 | . 49 | +2.4 | . 73 |
| 5 | 738.5 | -3.1 | . 42 | +32.1 | 4.35 |
| 6 | 738.5 | +12.4 | 1. 68 | +18.1 | 2.45 |
| 7 | 122.4 | $+2.5$ | 2.04 | +3.0 | 2.45 |
| 10. | 491.5 | +. 1 | . 02 | +3.9 | . 79 |
| 11. | 491.5 | +5.3 | 1.08 | -. 4 | . 08 |
| 15. | 1,107. 1 | +4.0 | . 36 | +11.8 | 1.07 |
| 16 | 40.7 | +. 4 | . 98 | +. 2 | . 49 |
| 17. | 40.7 | +1.0 | 2. 46 | $+1.7$ | 4.18 |
| 18 | 163.4 | $+1.5$ | . 92 | +4.5 | 2.75 |
| 19 | 163.4 | $+1.6$ | . 98 | 0 |  |
| 20 | 368.2 | +3.7 | 1. 00 | +1.0 | . 27 |
| 21. | 368.2 | +4.2 | 1. 14 | +6.9 | 1.87 |

The median is, by definition, a value such that the number of values exceeding it is equal to the number of values which it exceeds. The median values of $e_{2}$ and $e_{3}$ may be taken therefore as approximate probable errors of $A_{2}$ and $A_{3}$, respectively.

It may be observed that these values are of the same order of magnitude as that of the probable error obtained from an analysis of the sources of error inherent in the recording mechanism.

## DISPLACEMENT-TIME RECORD TAKEN ON ALL REACTION WHEEL TESTS

A displacement-time record was taken in each test conducted on the reaction wheel. As is shown in Figure 5, each curve so obtained contains both shock and drop segments. However, in any given test, the contact instrument was set to record either shock or drop as desired. In consequence, the analysis of the displacement-time record was confined to the segment corresponding to that for which the setting of the contact instrument was made. Both the acceleration $A_{D T}$, and the period of impact $T_{1}$ were obtained in the analysis. The term $T_{1}$, as used in motor-truck impact tests, is defined as twice the duration of the upward acceleration, in order to obtain a value analogous to the period of a complete cycle in simple harmonic motion. In addition, the amplitude of the impact displacement was measured for use in computing the acceleration $A_{C}$ as determined by the contact accelerometer.




Figure 6.-Tests of Displacement-Time Recorder on Simple Harmonic Motion Machine. Comparison of Accelerations Computed by Three Methods: $A_{1}$ : Stroke Measured by Means of Depth Gage, Period by Means of Liquid Tachometer; $A_{2}$ : Stroke and Period Obtained from Measurements on Displacement-Time Record; $A_{3}$ : Acceleration Obtained by Method of Differences

## THE CONTACT ACCELEROMETER DESCRIBED

The contact accelerometer ${ }^{3}$ is represented diagrammatically in Figure 7. The sensitive element is pivoted at $O$, to the left of which extends a flat cantilever spring $S$ of uniform section and to the right of which is an inertia element W. The spring is deflected and the deflection is measured by means of a micrometer M . The motion of the weight $W$ is limited by the opposed contact screws BC and MC . A suitable rigid frame carries the barrel of the micrometer $M$, the pivot $O$, and the contact screws BC and MC in fixed relation to one another.


Figure 7.-Diagram of Micrometer-Type Contact Accelerometer

When the spring S is deflected downward by the micrometer M , the weight W is pressed upward against the contact screw BC with a force determined by the load-deflection rate of the spring and the deflection imposed upon it. Now, if the weight be subjected to an acceleration acting downward with respect to it (or if the frame be subjected to an acceleration acting upward with respect to the weight), the weight will move from contact screw BC and bear against contact screw MC when the magnitude of the acceleration acting upon its mass creates a force sufficient to overcome the resistance of the deflected spring. By varying the spring deflection the critical acceleration at which this takes place is varied, and the spring deflection at the micrometer is a measure of such acceleration. Whether or not these contacts are broken or made is determined electrically by means of a telephone headset.

When the circuit is arranged to detect "break" at the upper contact screw BC, the instrument is so sensitive as to be practically useless in measuring accelerations due to motor-truck impact because of the highfrequency, low-amplitude vibrations which are nearly always parasitic to such impact reactions. However, when conditions are such that the break circuit can be used the micrometer deflection beyond the break zero setting can be used without resort to correction factors to determine the critical acceleration. With the break circuit no knowledge is required concerning the amplitude or period of the impressed acceleration, nor is the gap between the break and make contacts of importance.

When parasitic vibrations are present, particularly when such vibrations are unimportant with respect to the basic accelerations to be measured, their effect may be eliminated by arranging the telephone circuit to detect contact at the lower or "make" contact

[^2]371-30-2
screw MC, using a suitable "gap" clearance (as defined below). The sensitivity of the instrument, however, decreases with the magnitude of the gap opening (defined as the movement of the weight between the make and break contacts). It has been found that a gap of 0.002 inch at the center of gravity of the weight G is ordinarily sufficient to avoid interference by such parasitic vibrations. Since this movement corresponds to nearly 0.006 inch movement at the micrometer to cause the weight to move between the upper and lower contact screws, the uncorrected use of the "make" zero setting of the micrometer could, at times, result in noticeable error in the magnitude of the indicated critical acceleration. This error depends upon the relation between the gap clearance and the amplitude of the impressed acceleration. A theoretical discussion of this effect may be found in the November, 1925 issue of the Journal of the Society of Automotive Engineers (pp. 433-435). Empirical correction factors determined under conditions of simple harmonic motion are given in a subsequent paragraph.

The instrument is calibrated statically by determining the load-deflection rate $s$ of the spring and determining the mass effective as at the center of gravity. It is convenient to express the load-deflection rate in pounds (applied at the center of gravity of the weight) per inch of deflection (measured at and by the micrometer). This is done by suspending known weights at the point G and measuring the micrometer intervals required to restore contact BC or to break contact MC, as the case may be. The mass $m$ of the weight and the position of the center of gravity are obtained by computation. The calibration rate $C$ in feet per second per second per inch may be determined from the equation,

$$
C=\frac{s}{m}
$$

DYNAMIC CALIBRATION TESTS MADE ON SIMPLE HARMONIC MOTION MACHINE
Dynamic calibration tests were made on the machine for producing simple harmonic motion which had been installed at the Bureau of Standards. The ranges in stroke, speed, and acceleration of the machine corresponded to the amplitude, period, and acceleration of motor truck impact encountered through a range in tire equipment from pneumatic to partly worn solid tires. Tests were made with the telephone detector of the instrument arranged in both the break (upper contact) and the make (lower contact) circuits. In order to determine the influence of gap clearance when using the make contacts this condition was varied through a fairly wide range.

A general view of the instrument installed on the simple harmonic motion machine is given in Figure 8. With the simple harmonic machine in motion the instrument moved in accordance with the motion of the crosshead and the micrometer could not be read in ordinary light. A neon lamp (shown attached to the extension cord) was arranged to flash at the culmination of each down stroke of the platen, within the shadow of a hood covering the apparatus. (Fig. 8.) The micrometer was illuminated at only one position of its path (stroboscopic effect) and the operator was enabled to read it quite easily at any speed obtained in the tests.
For each test condition, the acceleration was computed from the known values of stroke and speed of the harmonic motion machine. For any given test,


Figure 8.-The Single Element Contact Accelerometer and Neon Lamp Arranged for Tests on the Simple Harmonic Motion Machine
the reading of the micrometer was subtracted from its zero reading to obtain the spring deflection $\Delta M$. The upper or "break" contact can not ordinarily be used in motor-truck impact tests, and it is sufficient to state that in the case of all strokes of from 0.40 to 3.00 inches the points fall very closely about a straight line for each of the two instruments tested. Some apparent discrepancies occurred at strokes of 0.10 and 0.20 inch. The "break" calibration of one of the instruments (No.1), is given, as an example, in Figure 9 , the rate being 5.28 feet per second per second per 0.001 inch.

As stated previously, it has been found that the instrument can be satisfactorily used in motor-truck impact tests when the influence of parasitic vibrations is cut out by a gap of 0.002 inch used in conjunction with the make contact telephone circuit. Calibration tests involving this gap resulted in a series of curves which yield, when the amplitude of a simple harmonic cycle is known, the acceleration corresponding to a given deflection of the accelerometer spring. It is probable but by no means certain that in motor-truck impact the accelerometer is influenced by the amplitude and period in much the same way as it is in simple harmonic motion. On this assumption the curves afford a means of reading acceleration (due to motortruck impact) directly, if the spring deflection and amplitude of motion are known; and, if the assumption is correct, the accuracy is high (within the limits of the test conditions). The "make" calibration for
instrument No. 1, using 0.002-inch gap, is given in Figure 10, which shows curves of constant spring deflection plotted against acceleration and stroke. The circles are points interpolated from work sheets of the test data, the crosses are extrapolated or interpolated points and the dotted lines indicate that the curvature was estimated. It may also be noted that, in general, the "break" calibration rate could also be used under these "make" conditions without introducing serious errors so far as the measurement of acceleration due to motor-truck impact is concerned

Make calibration tests were also made using gaps of $0.001,0.004$, and 0.007 inch. The curve for the shorter gap adheres somewhat more closely to that for the break calibration, while those for the wider gaps showed greater divergence than the curve for 0.002 -inch gap. Since these gaps were not used in subsequent tests by the committee, they have been omitted from this report.

## MEASUREMENT OF ACCELERATION BY ANALYSIS OF DISPLACE MENT-TIME CURVES COMPARED WITH MEASUREMENT BY CONTACT ACCELEROMETER

In the reaction wheel tests the accelerations produced were measured by means of both the displacement-time apparatus and the contact accelerometer. In analyzing the results obtained by these two instruments it is not sound to assume that either gives the correct acceleration, and the arithmetical mean between the two values was taken as the most probable acceleration for each test. Some conception of the agreement between the two methods may be had by referring to Figure 11, which contains 154 plotted points.

The difference between the two measurements may be studied by use of a quantity analogous to the percentage of error used in other discussions in this report.

Let $A_{D T}=$ acceleration as given by the displacementtime analysis,

$$
\begin{gathered}
A_{C}=\begin{array}{c}
\text { acceleration as given by the contact ac- } \\
\text { celerometer, }
\end{array}
\end{gathered}
$$

Then $A_{M}=\frac{A_{D T}+A_{C}}{2}=$ arithmetical mean,
Let $\quad e=100\left(\frac{A_{D T}-A_{C}}{A_{M}}\right)$ (absolute value).
The quantity $e$ may be defined as the deviation of one value of acceleration from the other expressed as a percentage of the mean between them. In Figure 12 these data are given in the form of a frequency pattern, the number within each rectangle being the number of points falling between the respective coordinate limits. The size of rectangle varies because of the sparsity of points at high accelerations.

In Figure 13 curves are drawn to represent zones in $e$, including 50 per cent and 90 per cent of the data. For each column of rectangles in Figure 12 an ordinate was determined such that the frequency of points having a greater value of $e$ is equal to the frequency of points having a less value of $e$. Similarly, an ordinate was determined such that the ratio of frequencies is nine to one. These two sets of points are plotted in Figure 13, the abscissa of each point being equal to that of the midpoint of the corresponding column of rectangles in Figure 12. The curves $e_{50}$ and $e_{90}$ are drawn through the points so plotted with due regard to the frequencies represented. It will be observed that, except in the case of low accelerations, the $e_{50}$ curve lies in the neighborhood of a value of $e$ equal to 4 per cent, while the $e_{90}$-curve varies between 8 and 12 per cent.


Figure 9.-Calibration of Contat Accelerometer No. 1 Under Break Contact Condition

The $e_{57}$ curve may be regarded as a curve of probable variation between the accelerations given by the two instruments. If $e$ had been computed on the basis of variation from the mean acceleration $A_{M}$, the ordinates of the points and the curve determined by them would have been only half as great. If, therefore, it is assumed that $A_{M}$ is the most probable value of acceleration, it follows that the probable error of either instrument lies between 2 and 3 per cent for a wide range of acceleration values. These facts lead to the conclusion that both the displacement-time recorder and the single element contact accelerometer are satisfactory for the measurement of impact accelerations under laboratory conditions.
the coll spring accelerometer described and theory of operation discussed

Figure 14 shows a single-element coil spring accelerometer mounted on a test truck for use in road tests. Figure 15 is a diagrammatic representation of such an instrument. The sensitive element consists of a weight mounted on a helical spring. Under the impulse of an upward acceleration the weight moves downward relative to the frame of the instrument, compressing the spring. The relative motion is recorded on sensitized paper which moves horizontally in the frame. In its rest position the spring is under no compression save that due to the weight, and the weight is in contact with a backstop which prevents it from moving


Figure 10.-Calibration Curves for Contact Accelerometer No. 1, Under Conditions of Make Contact and a Contact Gap of 0.002 Inch at the Center of Gravity of the Sensitive Element. Accelerations are Plotted Against Stroke, Each Curve Representing a Given Value of the Spring Deflection ( $\Delta M$ )
upward beyond that position. Because of this backstop the instrument will not respond to a downward acceleration.

The detailed report includes a discussion of the theory underlying the operation of the coil-spring accelerometer. The theory is based on the assumption that the force exerted by the spring is proportional to its deflection. This assumption, which is very nearly correct for springs tested statically, leads to the development of the following approximate formula for the free period of vibration, $T_{2}$, of the spring-weight element:

$$
T_{2}=2 \pi \sqrt{\frac{w}{g s}}
$$

where $w=$ the effective weight, taken as the weight resting on the spring plus one-half the weight of the spring,
$s=$ the loading rate of the spring.
The differential equation for the motion of the sensitive element takes the form,

$$
\begin{equation*}
\frac{d^{2} r}{d t^{2}}=-\frac{d^{2} y}{d t^{2}}-\frac{4 \pi^{2}}{T_{2}{ }^{2}} r=-\frac{d^{2} y}{d t^{2}}-\frac{g_{s}}{w} r . \tag{1}
\end{equation*}
$$

where $r=$ the displacement of the element relative to the frame at time $t$,
$d^{2} y$
$d t^{2}$${ }^{2}$ the acceleration acting on the frame at time $t$.
In order to obtain a simple expression for the term $d^{2} y$
$d t^{2}$ , the assumption was made that the motion of the frame is that of simple harmonic motion and may be expressed by the equation

$$
y=a \sin \frac{2 \pi}{T_{1}} t
$$

where
$y=$ the displacement of the frame at time $t$,
$a=$ amplitude of the motion,
$T_{1}=$ period of the motion.
On this assumption equation (1) takes the form,

$$
\begin{equation*}
\frac{d^{2} r}{d \dot{t}^{2}}=\frac{4 \pi^{2}}{T_{1}^{2}} a \sin \frac{2 \pi}{T_{1}} t-\frac{4 \pi^{2}}{T_{2}^{2}} r r_{-} \tag{2}
\end{equation*}
$$

The initial conditions of the motion are as follows:

$$
\begin{array}{cl}
t=0 & y=0 \\
\frac{d y}{d t}=\frac{2 \pi}{T_{1}} a & \frac{d r}{d t}=0 \\
\frac{d^{2} y}{d t^{2}}=0 & \frac{d^{2} r}{d t^{2}}=0
\end{array}
$$

The following integral of equation (2) was found to satisfy the conditions governing the operation of the coil spring accelerometer.

$$
r=\frac{a}{n^{2}-1}\left(\sin \frac{2 \pi}{T_{1}} t-\frac{1}{n} \sin \frac{2 \pi}{T_{2}} t\right)
$$

Case I--- (3) where

$$
n=\frac{T_{1}}{T_{2}}
$$

In the special case where $n=1$, i. e. $T_{1}=T_{2}=T$, the integral takes the form,

$$
\begin{equation*}
r=\frac{a}{2}\left(\sin \frac{2 \pi}{T} t-\frac{2 \pi}{T} t \cos \frac{2 \pi}{T} t\right), \quad \text { Case II } \ldots . \tag{4}
\end{equation*}
$$



Figure 11.-Reaction Wheel Tests. Acceleration Indicated by Single-Element Contact Accelerometer Plotted Against Acceleration Computed by Analysis of Displacement-Time Curve

The coil spring accelerometer differs from other instruments involving spring-weight elements in the fact that values of $n$ are generally low, lying in most cases between 1 and 3.5 . Values less than 1 are found occasionally, but values greater than 5 seldom if ever occur in motor truck impact tests. The peculiar properties of the instrument depend upon this range in the values of $n$, which gives rise to a variation of the
calibration factor with the period of the impressed motion.
For values of $n$ lying between 0 and 5 , with the exception of the value $n=1$, the maximum deflection of the element is given by the equation,

$$
\begin{equation*}
R=\frac{a}{n(n-1)} \sin \frac{2 \pi}{n+1}, \text { Case I_... } \tag{5}
\end{equation*}
$$



Figure 12.-Frequency Distribution Chart for the Relation Between Values of e $e=100 \times \frac{A_{d t}-A_{C}}{A_{M}}$ and $A_{M}$


Figure 13.-Curves Showing Probable Variation Between Accelerations ( $A_{c}$ ) Given by Contact Accelerometer and Accelerations ( $A_{D T}$ ) Given by Displacement-Time Analysis. (See Fig. 12)

When $n=1$,

$$
\begin{equation*}
R=\frac{\pi}{2} a, \text { Case II } \tag{6}
\end{equation*}
$$

The calibration factor, $C$, is defined by the formula,

$$
C=-\frac{A}{R}
$$

where $A=$ the maximum acceleration acting upon the frame (negative when $R$ is positive).

In the case of simple harmonic motion,

$$
A=\frac{-4 \pi^{2}}{T_{1}^{2}} a,
$$

and the equations for $C$ take the forms,

$$
\begin{align*}
& C=\frac{n-1}{n} \frac{4 \pi^{2}}{T_{2}^{2}} \operatorname{cosec} \frac{2 \pi}{n+1} \\
& =\frac{n-1}{n} \frac{g s}{w} \operatorname{cosec} \frac{2 \pi}{n+1}, \text { Case I_ }  \tag{7}\\
& C=\frac{4 \pi^{2}}{T^{2}} \frac{2}{\pi}=\frac{g s}{w} \frac{2}{\pi}, \text { Case II_... } \tag{8}
\end{align*}
$$

The quantity $c=\frac{C}{g s}$ is defined as the relative calibra-
tion factor. Substituting this formula in equations (7) and (8) we have

$$
\begin{gather*}
c=\frac{n-1}{n} \operatorname{cosec} \frac{2 \pi}{n+1}, \quad \text { Case I. }  \tag{9}\\
c=\frac{2}{\pi}=0.6366, \quad \text { Case II_-- } \tag{10}
\end{gather*}
$$

THE $n$ vs. $c$ RELATION USED AS BASIS OF ANALYSIS
It is evident that the relative calibration factor, $c$, is a function of $n$ alone. The relation between $n$ and $c$ is shown in Figure 16 for values of $n$ from 0.4 to 5.0 . The special case, $n=1, c=0.6366$, lies on this curve. In subsequent pages this function is referred to as the theoretical $n$ vs. $c$ relation, or $n$ vs. $c$ curve. A notable characteristic of this function is that within a wide range in values of $n$ the value of $c$ varies but little. For example, a value of 0.62 could be used for $c$ with errors no greater than 10 per cent for all values of $n$ between 0.87 and 3.20. This characteristic of the coil spring accelerometer is very important, as it makes


Figure 14.-A Test Truck Equipped for Road Tests With the Coil Spring Accelerometer


Figure 15.-Diagram of Essential Elements of Coil Spring Accelerometer
possible the use of a constant calibration factor without great error, even though the periods of the accelerations to be measured vary between fairly wide limits.

The development of the $n$ vs. $c$ relation opened two avenues of approach to the problem of the coil spring accelerometer. Since it is based upon simple harmonic motion, it made possible a direct check between the theory and the actual performance of the instrument in tests made on the simple harmonic machine at the Bureau of Standards. It also served as the basis for a


Figure 16.-Functional Relation Between the Ratios $c=\frac{C}{\frac{g}{w}}$ and $n=\frac{T_{1}}{T_{2}}$ as Determined Theoretically


Figure 17.-Four-Element Coil Spring Accelerometer Installed on the Simple Harmonic Motion Machine
systematic analysis of the data obtained in the reaction wheel tests.

## SPECIAL FOUR-ELEMENT COIL SPRING ACCELEROMETER

 CONSTRUCTHDFor use in tests described in this report a special accelerometer carrying four spring-weight elements was constructed. By means of this instrument simultaneous records of four elements having different physical characteristics were obtained in each test. The quantity $\frac{g s}{w}$ was used as a criterion in the selection of springs. Values of $s$ were determined by static calibration and values of $w$ by weighing on a beam balance. A different group of elements was selected for each type of tire tested on the reaction wheel. The selection was made so as to include two springs which would normally be used with the given type of tire, one spring having a value of ${ }_{w}^{g s}$ lower than normal and one spring having a value higher than normal.

For convenience in operation on the reaction wheel and the simple harmonic motion machine, the recording styli were arranged so as to bear upon the record paper by the application of pneumatic pressure. Figure 17 shows the instrument mounted on the simple harmonic motion machine. Figure 2 shows its installation for use in the reaction wheel tests.

## FOUR GROUPS OF TESTS MADE ON THE SIMPLE HARMONIC MOTION MACHINE

The tests conducted on the simple harmonic motion machine were designed to simulate, with respect to amplitude and period, the impacts obtained in tests with four different types of tire. For this reason the tests were divided into four groups and for each group the four spring-weight elements were selected which
were used in the reaction wheel tests, the type of tire being simulated by the given group of simple harmonic tests. Table 7 gives the test conditions for each group, together with the spring-weight elements used and their characteristics. It will be observed that tests were run at speeds ranging from 299 to 2,020 revolutions per minute. These speeds correspond to periods of cycle ranging from 0.2007 to 0.0297 second and include all drop periods encountered in motor-truck impact tests except those obtained on tests with badly worn solid tires.

For each test condition a series of autographic records was obtained from each of the four accelerometer elements. The lengths of these records were measured, and the mean of the series of records for each element at a given test condition; designated as $R_{m}$, was taken as the basis for the computations.

Table 7.-Tests with the coil spring accelerometer on the simple harmonic motion machine

| Tire type simulated | Test conditions |  | Accelerometer elements |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Stroke | Revolutions per minute | No. | $\underline{g s}$ | $T_{2}$ |
| Pneumatic--.--------- | Inches | $\begin{aligned} & 299 \text { to } 599 \\ & 299 \text { to } 599 \\ & 299 \text { to } 599 \end{aligned}$ |  | Feet per second per second per inch | Seconds |
|  |  |  | 51 A | 1,653 | 0.0446 |
|  | 1. 006 |  | 102B | 2,784 | . 0344 |
|  | 1. 499 |  | 52 C | 908 | $.0602$ |
|  | 1. 992 |  | 53 D | 588 |  |
| Cushion. |  |  | 51 A | 1,653 | . 0446 |
|  | 1. 006 | 299 to 932 | 102B | 2,784 | . 0344 |
|  | 1. 499 | 599 to 882 | 52 C | 908 | . 0602 |
|  | 1. 992 | 299 to 799 | 401 D | 4,497 | . 0270 |
| Solid. | $\begin{aligned} & 0.500 \\ & \text { 1. } 000 \end{aligned}$ | $\begin{aligned} & 745 \text { to } 1,570 \\ & 745 \text { to } 1,420 \end{aligned}$ | 51 A | 1,653 | . 0446 |
|  |  |  | 102B | 2, 784 | . 0344 |
|  |  |  | 401 C | 6,718 | $.0221$ |
|  |  |  | 402D | 4,526 | . 0270 |
| Worn solid. | 0. 500 | 1,345 to 2, 020 | 401A | 11,606 | . 0168 |
|  |  |  | 601 B | 15, 841 | . 0144 |
|  |  |  | 402C | 6. 760 | . 0221 |
|  |  |  | 201 D | 2, 401 | . 0370 |

EMPIRICAL $n$ vs. $c$ RELATION FOR SIMPLE HARMONIC MOTION DEVELOPED
In order to obtain an empirical function analogous to the $n$ vs. $c$ relation derived theoretically, values of $n$ and $c$ were computed from the formulas $n=\frac{T_{1}}{T_{2}}$ and $c=-\frac{A}{R_{m}} \div \frac{g s}{w}$. For each test there are, in general, four values of $\mathrm{R}_{m}$, and therefore four sets of $n$ vs. $c$ points, or coordinates. These points, 214 in number, were plotted and the plotting was used as the basis of the frequency distribution diagram of Figure 18. The figure within a given square indicates the number of points lying within the same coordinate limits on the original plot of $n$ vs. $c$ coordinates. A point lying on a boundary line was divided equally between the adjacent squares. It will be observed that there are a number of points for which the value of $n$ is greater than 5. Such values do not occur in motor-truck impact and for that reason these points are neglected in the subsequent portion of the discussion.

Figure 19 shows the method followed in obtaining an average curve to represent the empirical $n$ vs. $c$ relation for simple harmonic motion. In this figure are plotted the median value and the arithmetical mean value of $c$ for each column of squares in Figure 18. Little distinction is shown between the median and the


Figure 18.-Frequency Distribution Chart for the $n$ vs. $c$ Relation Indicated by Tests on the Simple Harmonic Motion Machine. The Number Within Each Square Indicates the Number of Points Falling Within its Coordinate Limits


Figure 19.-Development of Average $n$ vs. $c$ Curve From Tests on the Simple Harmonic Motion Machine
mean, although the former tends to reduce the effect of the more widely scattered points. The average curve (fig. 19) was drawn through these points, weight being given according to the frequency represented by each point.

From an inspection of Figures 18, 19, and 16, the following conclusions may be drawn:
(1) There is a noticeable dispersion in the values of $c$.
(2) The empirical relation is similar to the theoretical relation, with respect to both magnitude and manner of variation, even for high values of $n$, where the dispersion is greatest.
(3) Within the proper working range of the instrument (from $n=1.0$ to $n=3.5$ ) the points tend to lie above the theoretical curve. For high values of $n$ they tend to lie below it.

## DISPERSION OF DATA STUDIED

To serve as the basis for a study of the dispersion of the data presented in Figure 18, the quantity, $e$, is defined by the formula $e=100\binom{c}{c^{\prime}}$, where
$c=$ the experimental value of $c$ obtained from a given test.
$c^{\prime}=$ the value of $c$ obtained from the average curve for the given value of $n$.
The quantity $e$, therefore, may be defined as the percentage deviation of any given experimental value of $c$ from the average value given by the curve.

Values of $e$ were computed for all the experimental points except those lying beyond the limits of the average curve (fig. 19). In all, there are 204 such values. The results of this computation are shown in Figure 20 in the form of a frequency pattern, showing the distribution of values of $e$, without distinction of sign, against variation in length of record. There is a clear indication that the dispersion decreases as the length of record increases.

For each column of squares in Figure 20, ordinates were chosen to include 50 per cent and 90 per cent of the points, respectively. The resulting data are shown plotted in Figure 21, and the curves $e_{50}$ and $e_{90}$ are drawn through them. The significance of the curve $e_{50}$ may be stated as follows: If the value of $n$ is known the corresponding value of $c^{\prime}$ may be used in computing


Figure 20.-Frequency! Distribution Chart for the Absolute Values of the Error, e, With Respect to the Accelerometer Reading, $R_{m}$, for Tests on the Simple Harmonic Motion Machine. The Number Within Each Square Indicates the Number of Points Falling Within its Coordinate Limits


Figure 21.-Relation Between Accelerometer Error, e, and Accelerometer Reading, $R_{m}$, Indicated by the tests on the Simple Harmonic Motion Machine. (See Fig. 20.)
accelerations by means of the formula,$-A=c^{\prime} \frac{g s}{w} R$, with an even chance, or 0.5 probability, that the true acceleration lies within a percentage of the acceleration so computed, equal to the ordinate on the $e_{50}$ curve corresponding to the value of $R$ obtained in the test for which the computation is made. There is, similarly, a 9 to 1 chance, or 0.9 probability, that the true acceleration lies within a percentage of the computed value equal to the corresponding ordinate on the curve $e_{90}$. These statements, naturally, apply only to the case of simple harmonic motion.

## THE REACTION WHEEL TESTS DISCUSSED

The method used in analyzing the data obtained from the reaction wheel tests is closely patterned after that used in the case of simple harmonic motion. This procedure was adopted because it was believed that, in motor truck impact as in simple harmonic motion, the duration and the intensity of the impressed acceleration are the chief factors determining the length of the coil
spring acceleromotor record. If this were not so it would be practically impossible to achieve a systematic calibration of the instrument. The addition of other variables to the system would complicate the problem beyond all hope of a satisfactory solution. It is recognized, however, that other factors must necessarily influence the deflection of the spring to some degree. The displacement-time curves, although sinusoid in character, are not simple sine curves nor can they, as a group, be represented by any other simple function. If we should assume that they may be represented by a large number of functions varying in different degrees from the sine we would expect to obtain an equal number of $n$ vs. $c$ relations. These taken together would form a band, the width of the band and the distribution of the curve within it indicating the extent to which the functions differed among themselves. From this consideration alone we should anticipate a dispersion band of some width in the $n$ vs. $c$ relation exhibited by the reaction wheel data. In addition, we have the natural dispersion of the instrument as shown by the tests on the simple harmonic motion machine.


Figure 22.-Frequency Distribution Chart for the $n$ vs. $c$ Reladen Indicated by the Reaction Wheel Tests. The Number within Each Square Indicates the Number of Points Falling within Its Coordinate Limits

EMPIRICAL $n$ vs. c RELATION DEVELOPED FOR REACTION WHEEL DATA

In this analysis the same quantities are involved as in the case of simple harmonic motion with certain changes in definition. We have:
$T_{1}=$ twice the period of duration of the impact as measured on the displacement-time curve.
$T_{2}=$ free period of vibration of the accelerometer element.
$n=\frac{T_{1}}{T_{2}}$.
$A=$ The maximum acceleration of the impact. The value used is the mean of the values given by the contact accelerometer and the displace-ment-time recorder.
$R=$ length of the accelerometer record.
$C=\frac{-A}{R}$.

$$
c=C \div \frac{g^{s}}{w} .
$$

For each test there are four values of $R$, and therefore four sets of $n$ vs. $c$ coordinates except in a few cases where one or more elements failed to record properly. Five hundred and eighty-seven such coordinates were obtained from the computations, and their distribution is shown in the frequency pattern of Figure 22. An examination of this frequency distribution reveals the following facts.

1. There is a rather wide dispersion of the points.
2. Despite this dispersion, a systematic variation of $c$ as a function of $n$ is strongly indicated. The mean position of the points is displaced so as to lie in general somewhat above the theoretical curve, but is of similar shape.
3. The dispersion tends to be normal, i. e., as the center of the dispersion band is approached the frequency of the points increases rapidly.


Figure 23.-Development of Average $n$ vs. $c$ Curve for Reaction Wheel Data
4. There is some scattering of points at a great distance from the mean position. It is evident that such points represent conditions highly unfavorable to the use of the coil spring accelerometer.

In Figure 23 the median value and the arithmetical mean value of $c$ are plotted for each column of squares in Figure 22. As in the case of the tests on the simple harmonic motion machine, a mean curve has been drawn. Greater weight was given to the median than to the mean in drawing this curve in order to discount the effect of widely scattered points. In subsequent pages of this report this curve is used as the basic $n$ vs. $c$ relation for motor-truck impact. In Figure 24 the three $n$ vs. $c$ relations derived in this investigation are given together for the purposes of intercomparison.
RELATION FOUND BETWEEN DISPERSION OF REACTION WHEEL
DATA AND LENGTH OF ACCELEROMETER RECORD
Upon examination of the reaction wheel data it was found that nearly all the widely dispersed points in the frequency pattern of Figure 22 correspond to accelerometer records of small length, a great many of them measuring less than 0.05 inch. The same procedure was adopted as in the case of tests on the simple harmonic motion machine. Values of $c$ taken from the average curve of Figure 23 are given the symbol $c^{\prime}$.

The quantity $e=100\left(\frac{c}{c^{\prime}}-1\right)$ is defined as the percentage deviation of any given experimental value of $c$ from the average value $c^{\prime}$ for the corresponding value of $n$. Values of $e$ were computed for all experimental points except those for which the values of $n$ lie outside the limits of the average $n$ vs. $c$ curve. The distribution of the values of $e, 568$ in all, is shown in the frequency pattern of Figure 25. As in Figure 20, absolute values of $e$ are given. This diagram indicates very clearly that the dispersion increases rapidly as the length of record approaches zero. The tendency toward concentration near the zero line persists, even for very low values of $R$. It is plain, however, that results obtained from records less than 0.1 inch in length are unreliable. This fact does not seriously affect the value of the coil spring accelerometer. If the spring-weight element is properly chosen such extremely short records will be obtained only when the accelerations are very low and therefore unimportant.

As in the case of the tests on the simple harmonic motion machine, ordinates were determined including 50 per cent and 90 per cent of the values of $e$ in each column of squares in Figure 25. These ordinates and the corresponding curves, $e_{50}$ and $e_{93}$. are shown in


Figure 24.-The Calibration Coffficient $c$, in the Equation $C=c \frac{g s}{w}$, as a Function of the Ratio $n=\frac{T_{1}}{T_{2}}$


Figure 25.-Frequency Distribution Chart for the Absolute Values of the Error, e, with Respect to Accelerometer Reading, $R$, as Given by the Reaction Wheel Tests. The Number within Each Square Indicates the Number of Points Fatling within its Coordinate Limits


Figure 26.-Relation Between Accelerometer Error, e, and Accelerometer Reading, $R$, Indicated by the Reaction Wheel Tests. (See Fig. 25.)

Figure 26. We may again define the significance of these curves, as applied to motor-truck impact tests, by the following statements. If, in any impact test, the value of $n$ is known with sufficient accuracy for use with the $n$ vs. $c^{\prime}$ curve of Figure 23, the corresponding value of $c^{\prime}$ may be used in the formula,

$$
-A=c^{\prime} \frac{g s}{w} R,
$$

with an even chance, or 0.5 probability, that the true acceleration lies within a percentage of the acceleration so computed equal to the ordinate on the curve $e_{57}$ corresponding to the value of $R$ obtained in the test. In the case of the curve $e_{97}$ there is a 9 to 1 chance, or 0.9 probability, that the true acceleration lies within a percentage of the computed value equal to the corresponding ordinate on that curve. Although the procedure adopted in obtaining these functions is not rigorous, it is believed that the curves $e_{55}$ and $e_{99}$ are fair indices of the type of dispersion to be expected when the $n$ vs. $c^{\prime}$ relation of Figure 23 is used as a basis for computing motor truck impact accelerations from records of the coil spring accelerometer.

It will be noted that the curves $e_{50}$ and $e_{90}$ are drawn through the bulk of the points on Figure 26. These curves were drawn to express the general relation between dispersion and the length of accelerometer record and are used in subsequent computations. It will be observed that, for each of these curves, several points representing frequencies which should not be neglected lie definitely above the curve as drawn. For the purpose of making general deductions, two additional curves have been drawn in broken line, at values of e equal to 8 and 15 per cent, which, within the significant range of record lengths, definitely include more than 50 per cent and more than 90 per cent of the data, respectively.


Figure 27.-Graph of Mean Drop Period as a Function OF $T_{s i}$

## THE USE OF THE $n$ vs. $c$ RELATION IN IMPACT_COMPUTATIONS DISCUSSED

The empirical $n$ vs. $c^{\prime}$ relation developed in the preceding discussion and illustrated in Figure 23 may be regarded as the basis of the "most probable" calibration system for the coil spring accelerometer. In order to make use of this relation it is necessary to determine the period of impact $T_{1}$ for use in computing the ratio $n=\frac{T_{1}}{T_{2}}$. It is not required that the period be known with great accuracy, since the relative calibration factor $c^{\prime}$ varies very slowly within a wide range in values of $n$. Except for very low or very high values of $n$, an error of 10 or even 20 per cent in $T_{1}$ would not produce a large error in the value of $c^{\prime}$. The probable error due to the dispersion of the instrument under impact conditions is the chief limitation upon the accuracy which may be expected from the use of the $n$ vs. $c^{\prime}$ relation.

In order to find a method for estimating the period of impact from known test conditions it is necessary to determine the relations existing between that period and those variables (such as load, speed, unsprung weight, type of obstruction and type of tire) which may be supposed to influence its magnitude. In the reaction wheel tests shock and drop periods were measured for a wide variety of test conditions. These data were analyzed with respect to the variables listed above and methods of estimating the value of $T_{1}$ were devised which give values in good agreement with the shock and drop periods obtained in the reaction wheel tests.

## DROP PERIOD COMPUTED BY SIMPLE PROCEDURE

The analysis of drop periods presented a relatively simple problem. It was found that the period of drop is virtually independent of truck speed and height of obstruction, with a slight variation due to load. The chief sources of variation proved to be tire type and unsprung weight. A relation was found to exist between the drop period $T_{1}$ and a quantity $T_{s i}$, defined as the instantaneous period at static load. $T_{s i}$ is given by the formula $T_{s i}=2 \pi \sqrt{\frac{W}{g S_{d}}}$, where $W=$ the unsprung
weight and $S_{d}=$ the loading rate of the tire or the tangent to the load-deflection curve at static load. This relation is shown in Figure 27, where mean drop periods are plotted against the corresponding values of $T_{s i}$, computed from the known unsprung weight and the value of $S_{d}$ obtained from the load-deflection curve of the tire. Each condition of tire and load used in the reaction wheel tests is represented by a circle and a cross, as indicated in the figure. The ordinate of each circle is equal to the median value of the drop periods for the given tire-load combination; the ordinate of each cross is equal to the arithmetical mean of the periods. A curve has been drawn to express the relation between mean drop period and $T_{s i}$ indicated by the points. It will be observed that the value of $T_{s i}$ could be used for $T_{1}$ without great error for all tires except the worn solids. It is believed, however, that the curve drawn in Figure 27 offers a more reliable method of estimating the period of drop impact when the wheel load, the unsprung weight, and the load-deflection characteristics of the tire are known.

## FUNCTION DEVELOPED FOR DETERMINING SHOCK PERIOD

The data regarding the period of shock proved more difficult of analysis. It was found necessary to go more deeply into the mechanics of the problem than in the case of drop to obtain a function expressing logically the manner in which the controlling test conditions influence the duration of the shock reaction. From an examination of the data the following facts were learned regarding the variation of shock period:
(1) It decreases as the truck speed increases.
(2) It decreases with the stiffness of the tire used.
(3) It increases with the height of obstruction.
(4) The effect of load is practically negligible, although there is some indication that the period varies inversely with the load.

In order to account for these sources of variation an equation of motion for the shock reaction was developed. This equation is identical in form with that derived for the motion of the accelerometer (equation 3). With altered coefficients the equation takes the form

$$
\begin{equation*}
y=\frac{a p^{2}}{q^{2}-p^{2}}\left(\sin p t-\frac{p}{q} \sin q t\right) \tag{11}
\end{equation*}
$$

Two types of shock reaction are to be considered: Case I, that occurring in reaction wheel tests; and Case II, that occurring in tests on a road surface. These two conditions are illustrated in Figures 28 and 29, respectively. The significance of the variables and coefficients in equation 11 is made plain by these figures and by the following list of definitions:
$y=$ vertical displacement of the truck wheel at time $t$, measured from the point when upward motion begins.
In Figures 28 and 29-
$R_{2}=$ effective radius of the truck wheel, taken as equal to the overall radius of the tire, less its static deflection.
$l=$ width of obstruction. Rectangular obstructions only are considered.
$h=$ height of obstruction.
$\psi=$ angle between a vertical line through the center of the truck wheel and a radius drawn to the edge of the obstruction at the instant of contact.

In Figure 28 -
$R_{1}=$ radius of the reaction wheel .
$\alpha=$ the angle about the center of the reaction wheel subtended by the obstruction,

$$
=\frac{l}{R_{1}+h}
$$

$\beta=$ the angle $O_{2} O_{1} P$. The value of $\beta$ is obtained by solution of the triangle $O_{1} P O_{2}$, of which the three sides are known.


Figure 28.-Diagram of Tire Encountering Obstruction on Reaction Wheel


Figure 29.-Diagram of Tire Encountering Obstruction on Road Surface


Figure 30.-Graph of $T_{p q}$ as a Function of $p$ and $q . \quad p$ sin $p T_{p q}=q$ sin $q T_{p q}$

For Case I, on the reaction wheel,

$$
p=\frac{\pi}{2} \frac{1}{\xi} \frac{V_{t}}{R_{1}}
$$

where
$V_{t}=$ "truck speed," i. e., tangential speed of truck wheel and reaction wheel.

## $\xi=\beta+1 / 2 \alpha$

For Case II, on the road surface,

$$
p=\frac{\pi V_{t}}{l+2 \sqrt{2 R_{2} h-h^{2}}},
$$

where $V_{t}=$ truck speed.
For both Case I and Case II,
where

$$
\begin{aligned}
& q=\sqrt{\frac{K+S}{m}} \\
& a=\frac{K h}{m p^{2}}
\end{aligned}
$$

$S=$ loading rate of truck spring,
$m=$ unsprung mass,
$K=$ a coefficient characteristic of the tire.
It will be observed that all the terms defined above with the exception of the coefficient $K$ are definitely fixed by the test conditions. The problem of analysis consisted in finding an approximate or average value of $K$ for each of the tires used in the reaction wheel tests and determining the relation between the values of $K$ so
obtained and the known static characteristics of the tires.

The shock impact ends at the first point of inflection of the displacement-time curve (see fig. 5, arc AB), i. e., at the instant upward acceleration ceases. The relation between $p, q$, and $t$ at this instant is expressed by the following equation, derived from equation 11,

$$
\begin{equation*}
p \sin p t=q \sin q t . \tag{12}
\end{equation*}
$$

This equation can not be solved for $t$ by rational methods. A graphical solution was obtained, however, by means of a system of curves giving the half-period ${ }^{4}$ of shock, $\frac{T_{1}}{2}$, as a function of $p$ and $q$. This system of solutions is shown in Figure 30. Since the coefficient $K$ appears in the formula for $q$ but not in the formula for $p$, all values of the shock period for a given tire-truck combination should be satisfied by an equation of the form of equation 12 , the value of $q$ being constant. On this basis values of $\frac{T_{1}}{2}$ were plotted against values of $p$ computed from the known test conditions, and superimposed upon the sustem of curves in Figure 30. By this means an average value of $q$ was obtained for each tire-truck combination used in the reaction wheel tests, and the corresponding values of $K$ were computed from the equation

$$
q=\sqrt{\frac{K+S}{m}}
$$

[^3]

Figure 31.-Relation Between $K$ and $S_{L}$

In Figure 31 the values of $K$ so obtained are plotted against $S_{\mathrm{L}}$, the loading rate of the tire at capacity load. It is plain that a relation exists between these two quantities. The curve drawn in this figure is taken as a basis for estimating shock periods when the test conditions are known.
Estimation of the period of impact by the methods described above makes possible the use of the $n$ vs. $c^{\text {' }}$ relation of Figure 23 in computing inpact accelerations from the records of the coil spring accelerometer. It is desirable also to take account of the probable dispersion of results obtained from this instrument as shown in Figure 26. For this purpose the curves $e_{50}$ and $e_{97}$ are used. From the $n$ vs. $c^{\prime}$ relation is obtained the most probable value of acceleration, the probable error of which is given by the ordinate on the curve $e_{57}$ corresponding to the value of $R$ obtained in the given test. The curve $e_{93}$ indicates the approximate extreme range of error to be expected.

CONSTANT VALUE OF $\boldsymbol{c}$ CAN BE USED UNDER CERTAIN CONDITIONS
The fact has been noted that for values of $n$ between 1.0 and 3.5 the value of $c$ varies but little. This fact has led to the adoption of a constant value, $c=0.65$, for use under conditions where there is reasonable assurance that values of $n$ to be encountered lie within those limits. Effective use of this constant requires a knowledge of the range of periods to be expected and a careful choice of the accelerometer element to be used. The dispersion of the values will be similar to that encountered when the $n$ vs. $c$ relation is used, but somewhat greater, because of the small error in $c$.

## STUDY MADE OF ACCURACY OF THE DATA OBTAINED IN THE

 PREVIOUSLY CONDUCTED MO'TOR TRUCK IMPACT TESTSOne of the primary objectives of this instrument investigation was to determine the accuracy of the motor-truck impact data upon which the published report and the two reports withheld from publication,
are based. It has been found that the magnitude of the reaction depends upon four major variables (tire equipment, wheel load, vehicle speed, and road roughness conditions) and a consideration of the accuracy of the data should comprehend suitable ranges of these variables. Such a consideration should also include a fair range in accelerometer elements which were used in gathering the data. The best available material which fulfills these requirements is found in the tests which formed the basis for Figures 12 to 16 in the report entitled "Motor Truck Impact as Affected by Tires, other Truck Factors, and Road Roughness," published in the June, 1926, issue of Public Roads. A comparison of the original curves with curves plotted from recomputations made in the full light of this accelerometer investigation will, it is believed, be a fair measure of the general accuracy of the motor truck impact data. Table 8 gives the variation in the test conditions for the data selected for recomputation.

Table 8.-Schedule of the test conditions for which the data were recomputed

| Tire | Accelerometer characteristics |  | Tests over artificial obstructions |  |  | Road tests |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\frac{g s}{w}$ | $T_{2}$ | Speed variation | Load variation | Obstructions ${ }^{1}$ | Road, speed, and load ${ }^{2}$ |
|  | Feet per second per second per inch | Seconds | Miles per hour | Pounds | 1d, 7d, 2d | R and T , |
| $\begin{aligned} & \text { Dual pneumatic } \\ & (\mathrm{No} .47) \text {. } \end{aligned}$ | 832 | 0.0629 | 5-223/2 | $\left\{\begin{array}{l} 2,365- \\ 6,600 \end{array}\right.$ | $\left\{\begin{array}{r}4 s, 8 \mathrm{~s}, 3 \mathrm{~s} \\ 4 \mathrm{~d}, 8 \mathrm{~d}, 3 \mathrm{~d}\end{array}\right.$ | 12 miles per hour, <br> 4,400 pounds |
| Dual new cushion (Nos. 13 and 16). | 1,758 | . 0434 | 5-221/2 | $\left\{\begin{array}{c} 2,445- \\ 5,100 \end{array}\right.$ | $\left\{\begin{array}{c} 1 \mathrm{~d}, 7 \mathrm{~d}, 2 \mathrm{~d} \\ 4 \mathrm{~s}, 8 \mathrm{~s}, 3 \mathrm{~s} \\ 4 \mathrm{~d}, 8 \mathrm{~d}, 3 \mathrm{~d} \end{array}\right.$ | R and T , 12 miles per hour, <br> 3,400 pounds |
| Dual new solid (Nos. 40 and 41). | $\left\{\begin{array}{l}2.382 \\ 3,594 \\ 4.892\end{array}\right.$ | $\begin{array}{r} .0372 \\ .0303 \\ .0259 \end{array}$ | 5-20 | $\left\{\begin{array}{l} 2,349- \\ 6,000 \end{array}\right.$ | $\left\{\begin{array}{r}1 \mathrm{~d}, 7 \mathrm{~d}, 2 \mathrm{~d} \\ 4 \mathrm{~s}, 8 \mathrm{~s}, 3 \mathrm{~s} \\ 4 \mathrm{~d}, 8 \mathrm{~d}, 3 \mathrm{~d}\end{array}\right.$ | R and T , <br> 12 miles per hour, <br> 4,000 pounds |
| Dual worn solid (Nos. 40a and 41a). | $\left\{\begin{array}{l}2,382 \\ 4,892 \\ 7,824\end{array}\right.$ | $\begin{aligned} & .0372 \\ & .0259 \\ & .0205 \end{aligned}$ | 5-115/2 | $\left\{\begin{array}{l} 2,317- \\ 6,000 \end{array}\right.$ | ( $\begin{aligned} & 1 \mathrm{~d}, 7 \mathrm{~d}, 2 \mathrm{~d} \\ & 4 \mathrm{~s}, 8 \mathrm{~s}, 3 \mathrm{~s} \\ & 4 \mathrm{~d}, 8 \mathrm{~d}, 3 \mathrm{~d}\end{aligned}$ | R and T , 12 miles per hour, <br> 4,000 pounds |

${ }^{1} 1 \mathrm{dd}, 7 \mathrm{~d}$, 2d: Drop from 30 -inch inclined planes, $0.81,1.50,1.94$ inches high, respectively. $4 \mathrm{~s}, 8 \mathrm{~s}, 3 \mathrm{~s}$; and $4 \mathrm{~d}, 8 \mathrm{~d}, 3 \mathrm{~d}$ : Respectively shock at and drop after 3-inch rectangular obstructions, $0.56,0.88,1.12$ inches high, respectively
${ }_{2} \mathrm{R}$ and T : Rough stone and smooth concrete road sections, respectively

W TIRE EQUIPMENT
W = DUAL WORN SOLID
$S$ = DUAL NEW SOLID
C = DUAL NEW CUSHION
P = DUAL PNEUMATIC
$P=$ DUAL PNEUMATIC
COMPUTATION BASED ON


The period $T_{1}$ for each of the test conditions involving artificial obstructions was computed by the methods outlined in preceding paragraphs. It has been found that, within the operating conditions of the impact tests, the drop impact is more severe than the shock. Since the road tests were analyzed so as to show the magnitude with respect to the frequency of occurrence, the greater significance was given to the more severe impacts. For the reason given above these were assumed to be drop impacts and the period $T_{1}$ used in recomputing the data for road tests was therefore computed on the drop basis.

## DATA RECOMPUTED, WITH DISPERSION ZONES INDICATED

The values of $T_{1}$ having been estimated, the values of the ratio $n=\frac{T_{1}}{T_{2}}$ were computed for each test condition. For each value of $n$, the calibration coefficient $c=\frac{C}{g s}$ was obtained from the curve given in Figure 23 $w$
and the calibration factor $C$ for each test condition was computed. Using these individual calibration factors, the recomputations were made by substituting the new values for $C$ in the equation for total impact reaction,

$$
\begin{equation*}
F=m C R+m g+S, \tag{13}
\end{equation*}
$$

wherein $m=$ unsprung mass,
$R=$ accelerometer reading, ${ }^{s}$
$g=$ acceleration due to gravity,
$S=$ truck spring component.

The impact reaction is expressed as a percentage of the static wheel load by dividing by the corresponding static loads and multiplying by 100 .

[^4]\[

$$
\begin{aligned}
& \text { 2-TON TRUCK } \\
& \text { I2 MILES PER HOUR } \\
& \text { CAPACITY LOAD }
\end{aligned}
$$
\]

DROP FROM $30^{\prime \prime}$ INCLINED PLANE

RECOMPUTATION BASED ON $c \approx 0.65$

 |  |  |
| :--- | :--- |
| $\ldots \ldots$ | $\ldots$ |



Figure 33.-Comparison of Original and Recomputed Impact Data


Figure 34.-Comparison of Original and Recomputed Impact Data

In a preceding paragraph it was shown that there is an instrumental error due to dispersion and that the error bears a distinct relation (fig. 26) to the magnitude of the accelerometer reading. The indicated dispersion zones applicable to the recomputed impact data were computed for each test condition using actual values of the reading, $R$, to obtain values of the error $e_{50}$ and $e_{90}$. It will be noted that these errors affect only the term $m C R$ in equation 13.

The study of the $n$ vs. $c$ relation also showed that a constant value of $c$ equal to 0.65 would reasonably apply throughout the range in $n$ ordinarily encountered in motor-truck impact tests. The use of such a constant calibration factor greatly simplifies the computation. The values of impact reaction were also recomputed on this basis ( $c=0.65$ ) and the results are shown in con-
junction with the results by the original computation method and the results of the computation method involving an individual calibration factor for each test condition, in order to compare the impact relations obtained when the same field data are computed according to the three methods.
Figures 32 to 35 have been selected to show the order of variation caused by the three computations. In each figure the left-hand panel represents the data as originally computed. The center panels represent recomputed values obtained with calibration factors based on a constant value of $c$ equal to 0.65 . The right-hand panels represent the recomputed values obtained with individual values of $c$ according to the $n$ rs. $c$ relation shown in Figure 23. The 50 per cent and 90 per cent dispersion zones are shown in vertical


Figure 35.-Comparison of Original and Recomputed Impact Data
and horizontal shading, respectively, superimposed on the curves in the last named panels. It is quite evident that general conclusions as to the influence of various factors on motor truck impact reactions could be determined equally well by the curves according to any of the three methods of computation.

There is a tendency, however, for the recomputed impact reactions to be higher than the originally computed values. This is better shown in Figures 36 and 37 , in which the original data have been plotted against corresponding recomputed values according to the two methods of recomputation ( $c=0.65$ and $c$ having individual values, respectively). In these figures all the data available have been plotted, 56 shock conditions and 112 drop conditions being represented. Figure 36 shows that the recomputed values obtained on the basis of $c=0.65$ are about $12 \frac{1}{2}$ per cent higher than the original values while the recomputed values obtained by the use of individual calibration factors (see fig. 37) averaged about $11 \frac{1}{2}$ per cent higher than the original.

An inspection of the right-hand panels in Figures 32 to 35 shows that the dispersion apparently varies directly with the magnitude of the impact reaction. This is better illustrated in Figure 38 which was constructed from work sheets similar to Figure 37 but on which the recomputed values were represented by lines defining the respective dispersion bands. The areas enveloping these bands have been shown in Figure 38, the vertical shading representing the 50 per cent dispersion zones and the horizontal shading representing the 90 per cent dispersion zones for the shock and drop conditions. The magnitudes of the dispersion ranges indicated in Figure 38 are expressed below as


Figure 36.-Comparison of Original and Recomputed Impact Data. Recomputed Values on the Basis of $c=0.65$
percentages of the most probable recomputed value, i. e., the value of the center of the dispersion zones:

|  | Shock, per cent | Drop, per cer $t$ |
| :---: | :---: | :---: |
| 50 per cent range | - 6 | 5 |
| 90 per cent range | - 13 | 14 |

In the discussion of Figure 37 it was shown that the recomputed or most probable values of impact reaction differ from the original values by an average percentage equal to 11.5 . We may, therefore, conclude


Figure 37.-Comparison of Original and Recomputed Impact Data. Recomputed Values Based on Individual Calibration Factors
the discussion of the reaction wheel tests with the observation that impact forces based upon the previous calibration are systematically in error by about 12 per cent. The coil spring accelerometer is subject to dispersion errors which decrease in magnitude with an increase in deflection. For the significant accelerations measured with a given spring-weight combination, 50 per cent of the values of measured acceleration lie within 8 per cent of the mean and 90 per cent within 15 per cent of the mean. These values were obtained
by tests on the reaction wheel involving artificial obstructions and the committee interprets them as applying also to motor-truck impact tests on a smooth road with artificial obstructions. For motor-truck tests on naturally rough roads greater uncertainties exist.

## CONCLUSIONS OF ACCELEROMETER COMMITTEE SUMMARIZED

In summarizing its detailed report the committee called attention to the fact that its conclusions are based on a careful analysis of results obtained within limited ranges of speed and other operating conditions and that these conclusions should not be assumed to apply beyond these ranges. The conclusions reached by the committee are as follows:

1. The contact type of accelerometer may be used to obtain highly accurate measurements of acceleration of the order of magnitude and period encountered in motor-truck impact.
(a) Single-element instruments of this type are suitable for use under conditions where a given phenomenon is periodically reproduced such as by a drop machine or a reaction wheel. The cantilever-spring contact accelerometer was investigated and is considered to be satisfactory by this committee.
(b) While this committee has not investigated a multiple-element cantilever-spring contact accelerometer, it is of the opinion that such an instrument may be developed for use in road tests.
2. The coil spring accelerometer may be used to obtain reasonably accurate measurements of acceleration due to motor truck impact when due consideration is given to the relation between the period of the impact reaction and the period of the accelerometer element.
(a) With these restrictions instruments of this type are suitable for road tests. Their operation is not difficult and results are quickly obtained with them.
(Continued on page 111)
\#\# DISPERSION ZONES INCLUDING $50 \%$ OF THE DATA DISPERSION ZONES INELUDING $90 \%$ OF THE DATA


Figure 38.-Agreement Between Originally Computed Impact Reactions of the Motor Truck Tests and Corresponding Recomputed Reactions

# PROGRAM FOR INTERNATIONAL ROAD CONGRESS ANNOUNCED 

Official delegates from 50 countries in all parts of the world have announced that they will attend the Sixth Congress of the Permanent International Association of Road Congresses which will open in Washington on Monday, October 6, and close on Saturday, October 11. Large delegations are expected from England, France, Germany, and Italy, and it is possible that some of these delegations may include as many as 100 engineers. Smaller delegations from other countries, including those as far distant as China, India, and Australia, will make the total attendance large.

This is the first such congress ever to be held outside of Europe, and it is believed that it will be the largest and most important gathering of highway engineers ever held. Contributing to this end, there is now a world-wide interest in highway transportation and a desire by foreign engineers to inspect the methods of construction and the results which have been attained in the United States. The delegations will include the most influental and representative of foreign highway engineers.

In preparation for the congress, 69 papers from engineers in 20 different countries are being published and will be received by the members of the congress before they start for Washington. These papers deal with the following subjects, which have been included in the agenda of the congress:

## FIRST SECTION

## CONSTRUCTION AND MAINTENANCE

First question: Results obtained by the use of(a) Cement.
(b) Bricks or other artificial paving.
(Methods employed for road construction and maintenance in these materials.)
Second question: The most recent methods adopted for the use of tar, bitumen, and asphalt in road construction.

Third question: The construction of roads in new countries, such as colonies and undeveloped regions.

## SECOND SECTION

## TRAFFIC AND ADMINISTRATION

Fourth question: Ways and means of financing highways: (a) Road construction.
(b) Maintenance.

Fifth question: Highway transport: Correlation and coordination with other methods of transport; adaptation to collective (organizations) and individual uses.

Sixth question:

1. Traffic regulation in large cities and their suburbs; traffic signals; design and layout of roads and adaptation to traffic requirements in built-up areas.
2. Parking and garaging of vehicles.

The reports on each question have been reviewed by a general reporter (two general reporters for first question), and these general reports will be read at the meeting of the congress, followed by open discussion and adoption of conclusions. It is the custom to appoint general reporters from the country in which the congress is held, and the following authorities of recognized standing are acting in this capacity: Question 1, part (a), Frank T. Sheets, chief highway engineer, Illinois Department of Public Works; question 1,
part (b), P. J. Freeman, chief engineer, bureau of tests and specifications, department of public works, Allegheny County, Pittsburgh, Pa.; question 2, Roy W. Crum, director, highway research board, National Research Council, Washington, D. C.; question 3, Edwin W. James, chief, Division of Highway Transport, Bureau of Public Roads, United States Department of Agriculture, Washington, D. C.; question 4, A. B. Barber, manager, transportation and communications department, Chamber of Commerce of the United States of America, Washington, D. C.; question 5, Henry R. Trumbower, professor of economics, University of Wisconsin, Madison, Wis., and question 6, Miller McClintock, director, Albert Russel Erskine Bureau for Street Traffic Research, Harvard University, Cambridge, Mass.

## PROGRAM OF THE CONGRESS

The American Organizing Commission has announced the following program of the congress:

## Monday, October 6

Morning: Meeting of Permanent International Commission. Registration of delegates.

Afternoon: First plenary session-opening of congress-Constitution Hall.

Evening: No formal engagement.

## Tuesday, October 7

Morning: Section meetings.
Luncheon: American Road Builders' Association.
Afternoon: Official opening of International Exposition of American Road Builders' Association.

Evening: Visits to International Exposition.
Wednesday, October 8
Morning: Section meetings.
Afternoon: Section meetings.
Evening: Official reception.

$$
\text { Thursday, October } 9
$$

Morning: Section meetings.
Afternoon: Second plenary session for discussion of conclusions.

Evening: American Organizing Commission dinner.
Friday, October 10
Morning: Inspection trip to experiment station of the United States Bureau of Public Roads at Arlington, Va.

Luncheon: Picnic luncheon, auspices of American Organizing Commission, at Mount Vernon, Va.

Afternoon: Inspection of construction of Mount Vernon Memorial Highway and visit to Mount Vernon (the home of George Washington)

Evening: Official closing of the congress and reception.

## Saturday, October 11

Morning: Inspection of United States Naval Academy at Annapolis, Md., under auspices of American Organizing Commission.

Luncheon: Reception and luncheon tendered by the Hon. Albert C. Ritchie, Governor of Maryland.

Afternoon: Recreation.
Evening: No formal engagement.
Note. - The necessity may arise for certain modfications in the program as here given.

Participation in all official excursions and social functions will be by ticket, which will be issued without charge to all delegates from other countries and to official delegates from the United States.

The congress is being held in this country at the invitation of the United States Government and arrangements are being made by the American Organizing Commission with headquarters at 1723 N Street NW., Washington. The commission consists of: President, Roy D. Chapin, National Automobile Chamber of Commerce; Secretary-General, Thomas H. MacDonald, United States Bureau of Public Roads; and the following members, Wilbur J. Carr, Department of State; A. J. Brosseau, Chamber of Commerce of the United States; H. H. Rice, Highway Education Board; Robert Hooper, American Automobile Association; H. G. Shirley, American Association of State Highway Officials; Thomas R. Taylor, Department of Commerce; and Charles M. Upham, American Road Builders' Association. The commission is assisted by Pyke Johnson, National Automobile Chamber of Commerce, and H. S. Fairbank. United States Bureau of Public Roads, as administrative aides, and J. Truman Thompson, highway research specialist, United States Bureau of Public Roads, as manager.

Participation in the proceedings of the congress and receipt of the reports to the congress and the printed proceedings of the congress are possible only through membership in the congress. A temporary membership carrying these privileges may be had upon application to the American Organizing Commission at the above address. The fee for such membership is $\$ 5$.

Simultaneously with the congress, an international road machinery and materials exhibition and demonstration will be held by the American Road Builders Association. The exhibition will be held in the Washington Auditorium, while the demonstration grounds where machinery may be seen in operation will be provided at a near-by point.

## AMERICAN ENGINEERS URGED TO ATTEND CONGRESS

It is hoped that a large number of American highway engineers and representatives of related industries will attend the congress and participate in the proceedings. The attractive program of meetings and trips which has been arranged and the opportunity of meeting the leaders in highway construction and transportation should attract a large attendance. October is one of the most desirable months of the year for a visit to Washington.

## (Continued from p. 109)

(b) The coil spring accelerometer is subject to dispersion errors the magnitude of which decreases as the length of record increases. However, the indicated probable errors in total force resulting from this dispersion are not excessive for the type of tests involved.
3. A recomputation of published data using calibration factors obtained in this investigation indicates that the impact reaction values which are based on the original calibration factors are from 10 to 15 per cent too low because of systematic errors in calibration.

The comparisons made and conclusions drawn in reports based on such data, being general in nature and depending upon many tests rather than upon individual measurements, show with sufficient accuracy the relations and factors which influence the magnitude of the impact reactions.
4. The displacement-time apparatus may be used to determine the magnitude of the acceleration, the period of impact, and other characteristics of motor-truck impact reactions.
(a) Its use is limited to laboratory set-ups.
(b) Good agreement has been secured between acceleration values determined by the analysis of displacement-time curves, those measured with an accelerometer of the contact type, and those computed from a simple harmonic motion machine.

## ROAD PUBLICATIONS OF BUREAU OF PUBLIC ROADS



## ANNUAL REPORTS

Report of the Chief of the Bureau of Public Roads, 1924
Report of the Chief of the Bureau of Public Roads, 1925. Report of the Chief of the Bureau of Public Roads, 1927. Report of the Chief of the Bureau of Public Roads, 1928.

## DEPARTMENT BULLETINS

No. *136D. Highway Bonds. 20c.
220D. Road Models.
257D. Progress Report of Experiments in Dust Prevention and Road Preservation, 1914.
*314D. Methods for the Examination of Bituminous Road Materials. 10 c .
*347D. Methods for the Determination of the Physical Properties of Road-Building Rock. 10c.
*370D. The Results of Physical Tests of Road-Building Rock. 15 c .
386D. Public Road Mileage and Revenues in the Middle Atlantic States, 1914.
387D. Public Road Mileage and Revenues in the Southern States, 1914
388D Public Road Mileage and Revenues in the New England States, 1914.
390D. Public Road Mileage and Revenues in the United States, 1914. A Summary.
407D. Progress Reports of Experiments in Dust Prevention and Road Preservation, 1915.
*463D. Earth, Sand-Clay, and Gravel Roads. 15c.
*532D. The Expansion and Contraction of Concrete and Concrete Roads. 10 c .
*583D. Reports on Experimental Convict Road Camp, Fulton County, Ga. 25 c .
*660D. Highway Cost Keeping. 10c.
*670D. The Results of Physical Tests of Road-Building Rock in 1916 and 1917.
*691D. Typical Specifications for Bituminous Road Materials. 10 c .
*724D. Drainage Methods and Foundations for County Roads. 20c.
1216D. Tentative Standard Methods of Sampling and Testing Highway Materials, adopted by the American Association of State Highway Officials and approved by the Secretary of Agriculture for use in connection with Federal-aid road construction.
1259D. Standard Specifications for Steel Highway Bridges, adopted by the American Association of State Highway Officials and approved by the Secretary of Agriculture for use in connection with Federalaid road work.
1279D. Rural Highway Mileage, Income, and Expenditures 1921 and 1922.
1486D. Highway Bridge Location.

DEPARTMENT CIRCULARS

No. 94 C. T. N. T. as a Blasting Explosive.
331 C.

## TECHNICAL BULLETIN

No. 55. Highway Bridge Surveys.

## MISCELLANEOUS CIRCULARS

No. 62 M . Standards Governing Plans, Specifications, Contract Forms, and Estimates for Federal-Aid Highway Projects.
*93M. Direct Production Costs of Broken Stone. 25 c .
*109M. Federal Legislation and Regulations Relating to the Improvement of Federal-Aid Roads and NationalForest Roads and Trails. 10c.

## SEPARATE REPRINTS FROM THE YEARBOOK

No. 914 Y . Highways and Highway Transportation.
937 Y . Miscellaneous Agricultural Statistics.
1036 Y. Road Work on Farm Outlets Needs Skill and Right Equipment.

## TRANSPORTATION SURVEY REPORTS

Report of a Survey of Transportation on the State Highway System of Connecticut
Report of a Survey of Transportation on the State Highway System of Ohio.
Report of a Survey of Transportation on the State Highways of Vermont.
Report of a Survey of Transportation on the State Highways of New Hampshire.
Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio.
Report of a Survey of Transportation on the State Highways of Pennsylvania.

## REPRINTS FROM THE JOURNAL OF AGRICULTURAL RESEARCH

Vol. 5, No. 17, D- 2. Effect of Controllable Variables upon the Penetration Test for Asphalts and Asphalt Cements.
Vol. 5, No. 19, D- 3. Relation Between Properties of Hardness and Toughness of Road-Building Rock.
Vol. 5, No. 24, D-6. A New Penetration Needle for Use in Testing Bituminous Materials.
Vol. 6, No 6, D- 8. Tests of Three Large-Sized ReinforcedConcrete Slabs Under Concentrated Loading.
Vol. 11, No. 10, D-15. Tests of a Large-Sized Reinforced-Concrete Slab Subjected to Eccentric Concentrated Loads.

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[^0]:    ${ }^{1}$ This report is a summary of a detailed report on a cooperative investigation by the
    U.S.Bureau of Standards and the U.S. Bureau of Public Roads. The purpose of this investigation was to determine the accuracy and other characteristics of such instruments as had been used, or were proposed for use, in the cooperative motor truck impact investigation conducted by the Bureau of Public Roads, the Rubber Association of America, and the Society of Automotive Engineers.
    The work was actively guided by a committee composed of members of the technical staff of each bureau as follows.
    Utaft of each Bureau of Standards: Dr. H. C. Dickinson, Dr. L. B. Tuckerman, Mr. E. F. Mueller, and Mr. W. A. Jacobs.
    E. F. S. Bureau of Public Roads: Messrs. E. F. Kelley, I. W. Teller, J. A. Buchanan, and G. P. St. Clair.

    Dr. B. Liebowitz was retained as research consultant throughout the investigation. The actual conduct of the tests was largely the work of Mr. Jacobs and Mr. St. Clair, while the analysis of the data and the preparation of the detailed and summary reports Were carried out by Mr. Buchanan and Mr. St. Clair. The mathematical work
    involved in the investigation, relative to the coil spring accelerometer, the displacement time recorder, and the phenomena of motor truck impact, was prepared by Mr. St. Clair.

[^1]:    ${ }^{2}$ A discussion of this subject is given in the Encyclopedia Britanica, eleventh edition, under the heading "Differences, Calculus of." A more complete treatment is contained in "The Theory and Practice of Interpolation," by H. L. Rice, the Nichols
    Press, 1899.

[^2]:    ${ }^{3}$ See Jour. S. A. E., March, 1926, pp. 249-250.

[^3]:    - The actual duration of shock impact, as described above.

[^4]:    5 The field data were originally plotted on work sheets so that the accelerometer readings could be interpolated at the speeds selected for computation. The interval in speed between the plotted points was about 3 miles per hour. It is these interpolated values of accelerometer reading which were used in the original computations and in these recomputations.

[^5]:    * Department supply exhausted.

