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MECHANICS OF PROGRESSIVE CRACKING IN CONCRETE PAVEMENTS

By H. M. WESTERGAARD, Professor of Theoretical and Applied Mechanics, University of Illinois

WHEN cracks form in a concrete pavement, larger slabs of the pavement are divided into smaller ones. The fact that a beam is strengthened by shortening the span, suggests that the smaller slabs of the pavement may be less liable to overstress than were the original larger ones; that is, one may expect that in some cases the stresses will be relieved by reduction of the size of the slabs. The stresses to be considered are due to wheel loads, changes of temperature, and deformations of the subgrade.

In general one does not expect a concrete pavement to remain uncracked. The essential structural requirement is that the cracks, or the cracks and joints, shall not divide the pavement into excessively small units. With this peculiar structural requirement in view, one may ask the following questions:

To what extent are the stresses relieved when the size of the slabs is reduced either by cracking or by introduction of joints or planes of weakness? What is the influence of the size of the slabs upon their strength? If wheel loads, and accompanying impacts large enough to cause a crack in an unbroken slab, occur occasionally, how far will the cracking progress in the course of time, and what is the size of the pieces into which the cracks will divide the pavement ultimately? In the light of the influences of loads, changes of temperature, and settlements or other distortions of the subgrade, when are the pieces small enough to be strong enough?

The purpose of the analysis which follows is to answer these questions.

CTION OF A BEAM ON UNIFORM ELASTIC SUBGRADE LOADED AT THE MIDDLE ANALYZED

A panel of pavement acts as a slab and not as a beam. Yet, one may form an idea as to the influence of size by considering a concrete beam on an elastic subgrade, loaded at the middle, as shown at the top of Figure 1. Let it be assumed that the reaction of the supporting subgrade at each point is proportional to the deflection of the beam, and that the beam acts as if it were homogeneous and elastic.[1] Then it becomes a matter of routine to compute the deflections and bending moments of the beam.[2] By the derivations described below one obtains for different lengths of the beam the moment diagrams shown in Figure 1. The dotted curve shows the relation between the length of the beam and the greatest bending moment.

EQUATIONS DERIVED FOR A BEAM ON UNIFORM SUBGRADE

Let x = horizontal coordinate measured from the middle of the beam;
z = deflection;
a and h = dimensions as shown in Figure 1;
E = modulus of elasticity of the concrete;
k = modulus of subgrade reaction—that is, reaction of the subgrade in pounds per square inch per inch of deflection;
p = load at the middle of the beam per unit of width; p is the total load when, as indicated in Figure 1, the beam is one unit wide;
M = bending moment per unit of width (total bending moment when the beam is one unit wide).

In the present case of a beam which is not wide, Poisson's ratio of the concrete, μ, need not be considered. The flexure of the beam, then, is governed by the equation,[3]

$$\frac{Eh^3}{12}\frac{d^4z}{dx^4}+kz=0 \quad\text{------}\quad (1)$$

It is expedient to introduce the distance,

$$\lambda=\sqrt[4]{\frac{Eh^3}{3k}} \quad\text{------}\quad (2)$$

This quantity is closely related to the distance, l, used elsewhere in studies of the flexure of concrete pavements, under the name "radius of relative stiffness."[4]

In fact,

$$\lambda=\sqrt{2}\,\sqrt[4]{1-\mu^2}\,l \quad\text{------}\quad (3)$$

With λ introduced, equation 1 becomes

$$\frac{\lambda^4}{4}\frac{d^4z}{dx^4}+z=0 \quad\text{------}\quad (4)$$

This equation is satisfied by the following general solution:

$$z=A\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}+B\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda}+C\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}+D\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} \quad\text{------}\quad (5)$$

The four integration constants, A, B, C, and D, are determined by four conditions applying at special points:

$$\text{At } x=0:\ \frac{dz}{dx}=0,\ \frac{Eh^3}{12}\frac{d^3z}{dx^3}=\frac{p}{2} \quad\text{------}\quad (6)$$

$$\text{At } x=a:\ \frac{d^2z}{dx^2}=\frac{d^3z}{dx^3}=0 \quad\text{------}\quad (7)$$

The first two conditions give:

$$-C=D=\frac{p}{2k\lambda} \quad\text{------}\quad (8)$$

[1] Concerning these assumptions, see the paper by the writer, Stresses in Concrete Pavements Computed by Theoretical Analysis, PUBLIC ROADS, vol. 7, No. 2, April, 1926 (or Proceedings of the Highway Research Board, meeting of Dec. 3–4, 1925, Pt. I, p. 90).

[2] See, for example, K. Hayashi, Theorie des Trägers auf elastischer Unterlage, Berlin, 1921, or H. Müller-Breslau, Die graphische Statik der Baukonstruktionen, vol. 2, subvolume 2, second edition, 1925, p. 195. Examples may also be found in the paper by the writer, Analysis of Stresses in Concrete Roads Caused by Variations of Temperature, PUBLIC ROADS, vol. 8, No. 3, May, 1927 (or Proceedings of the Highway Research Board, meeting of Dec. 2–3, 1926, p. 201).

[3] Compare equation 10 in the paper, Analysis of Stresses in Concrete Roads Caused by Variations of Temperature.

[4] See the two papers by the writer referred to in the preceding footnotes.

With the substitution, $\alpha = \frac{a}{\lambda}$ ------------------- (9)

one finds by using the third and fourth conditions:

$$A = \frac{p}{2k\lambda} \frac{\cosh 2\alpha + \cos 2\alpha + 2}{\sinh 2\alpha + \sin 2\alpha},$$

$$B = -\frac{p}{2k\lambda} \frac{\cosh 2\alpha - \cos 2\alpha}{\sinh 2\alpha + \sin 2\alpha} \quad \text{---------- (10)}$$

By differentiating equation 5 twice, and substituting the values of the constants, one finds the following expression for the bending moment at any point between $x=0$ and $x=a$:

$$M = \frac{1}{4} p\lambda \left[\frac{\cosh 2\alpha - \cos 2\alpha}{\sinh 2\alpha + \sin 2\alpha} \cos\frac{x}{\lambda} \cosh\frac{x}{\lambda} + \frac{\cosh 2\alpha + \cos 2\alpha + 2}{\sinh 2\alpha + \sin 2\alpha} \sin\frac{x}{\lambda} \sinh\frac{x}{\lambda} - \cos\frac{x}{\lambda} \sinh\frac{x}{\lambda} - \sin\frac{x}{\lambda} \cosh\frac{x}{\lambda} \right] \quad \text{------- (11)}$$

The greatest bending moment becomes:

$$M_{\max} = \frac{1}{4} \frac{\cosh 2\alpha - \cos 2\alpha}{\sinh 2\alpha + \sin 2\alpha} p\lambda \quad \text{------------------- (12)}$$

The greatest stress becomes:

$$\sigma = \frac{3}{2} \frac{\cosh 2\alpha - \cos 2\alpha}{\sinh 2\alpha + \sin 2\alpha} \sqrt[4]{\frac{E}{3k}}\, p h^{-\frac{5}{4}} \quad \text{------ (13)}$$

Tables are available by which the numerical computations, leading to the curves in Figure 1 can be made conveniently.[5]

USE OF SCALES DISCUSSED

The scales below the curves in Figure 1 interpret, for different values of the depth of the beam, the horizontal distances in terms of feet. The scales on the right interpret the vertical distances as bending stresses in pounds per square inch, produced by a load of 1,000 pounds per foot of width of the beam. For example, in the case of a beam 7 inches deep, the horizontal distances are measured in the horizontal scales on the second line from the bottom, and the stresses are measured in the vertical scales on the second line from the left. The lengths of the units in the scales are defined by the expression for λ (equation 2) and by the assumed numerical values:

$$E = 3 \cdot 10^6 \frac{\text{pounds}}{\text{inches}^2},\ k = 50 \frac{\text{pounds}}{\text{inches}^3},\ p = 1{,}000 \frac{\text{pounds}}{\text{feet}} \quad \text{- (14)}$$

For example, with $h=7$ inches, equation 2 gives $\lambda = 51.2$ inches $= 4.27$ feet. One finds the same value by measuring the distance representing λ on the horizontal scale for $h=7$ inches, that is, on the second line from the bottom, and thereby one verifies the length of the unit of this scale. With $a=\lambda$, that is, with the length

[5] K. Hayashi, Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen und deren Produkte sowie der Gammafunktion, Berlin, 1926. See also the table on pages 200–201 in the work by H. Müller-Breslau referred to in footnote 2.

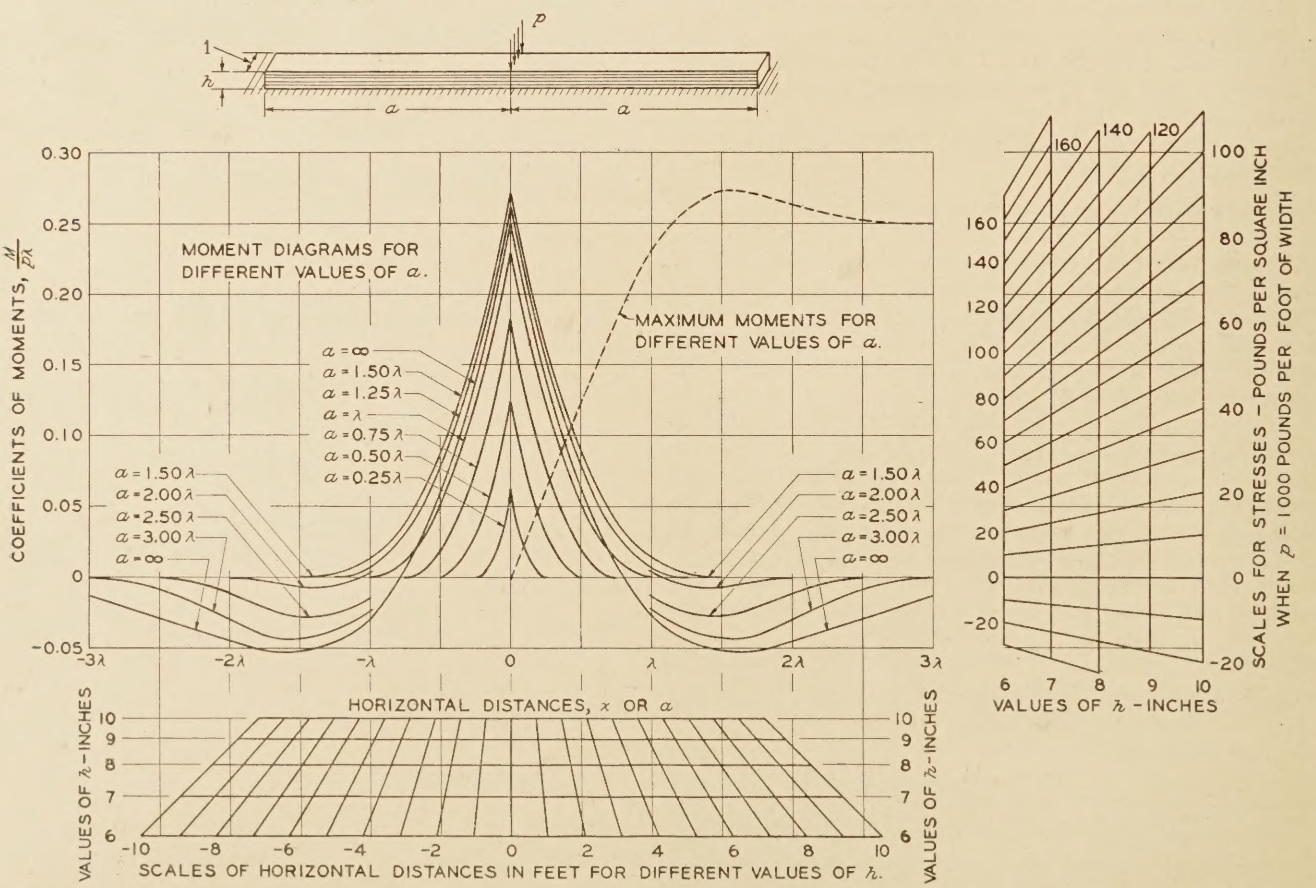

FIGURE 1.—DIAGRAMS OF MOMENTS AND STRESSES IN BEAMS OF DIFFERENT LENGTHS, RESTING ON UNIFORM ELASTIC SUBGRADE, AND LOADED AT THE MIDDLE. NUMERICAL VALUES ASSUMED IN CONSTRUCTING THE SCALES: MODULUS OF ELASTICITY OF CONCRETE, $E=3{,}000{,}000$ POUNDS PER SQUARE INCH; MODULUS OF SUBGRADE REACTION, $k=50$ POUNDS PER CUBIC INCH

of the 7-inch beam equal to $2\lambda=8.54$ feet, one finds the maximum bending moment by measuring either the ordinate of the peak in the corresponding moment diagram or the ordinate of the point of the dotted curve having the abscissa λ. By use of the scale on the left, one finds this moment to be $M=0.230\ p\lambda$, or, with $p=1{,}000$ pounds per foot of the width of the beam, $M=982$ foot-pounds per foot of width $=982$ inch-pounds per inch of width. Dividing by the section modulus, $\frac{1}{6}\cdot 7^2$ inch^3 per inch $=8.167$ inch^2, one finds the maximum stress, $\sigma=120$ pounds per square inch. One obtains the same value, without intermediate computations, by measuring the ordinate on the scale on the right for $h=7$ inches, and one verifies thereby the length of the unit of this scale. By locating a point on the same moment diagram, say, 2 feet from the center, one measures on the vertical scale for $h=7$ inches the corresponding stress, 31 pounds per square inch. These examples show the use of the scales, as well as a method of verifying the lengths of the units.

If k were changed from the value, 50 pounds per cubic inch, representing a rather deformable subgrade, to the value 200 pounds per cubic inch, representing a subgrade four times as stiff, the numerical scales, both vertical and horizontal, would be stretched in the ratio of $\sqrt{2}$ to 1, as illustrated by the scales at the bottom of Figure 4.

DIAGRAMS SHOW CONSPICUOUS INCREASE OF STRENGTH ONLY WHEN THE BEAM BECOMES VERY SHORT

The parts of the moment diagrams near the middle of the beam lie close together for all values of a larger than λ; that is, according to the scales, when a is larger than from 4 to 6 feet. When a decreases below the value, λ, the moment diagram drops, and the stresses are relieved.

When the length of the beam, $2a$, is greater than 5.5λ, the maximum moment is practically constant, equal to $0.250\ p\lambda$. The dotted curve in Figure 1 shows that when the length of the beam is decreased gradually from the value of 5.5λ, the moment at the center increases slightly until it reaches a maximum of $0.273p\lambda$. Thereafter the bending moment decreases, becoming again equal to $0.250\ p\lambda$ when the length is reduced to 2.3λ, and equal to one-half of that amount when the length is λ. The following conclusions are drawn. If a load is just large enough to break a very long beam when applied at the center, the same load is capable of breaking any beam of length greater than 2.3λ, the cross-section and subgrade remaining the same. If a weight of twice that amount be placed first on a very long beam, breaking it, then on each of the pieces into which it breaks, and so on, the breaking up will continue until the pieces have a length of between 0.5λ and λ. If the load moves gradually over the length of the long beam, it will break it into pieces of about this size. In relation to the size of slabs built in concrete pavements, these pieces are small.

By use of the dotted curve in connection with the scales, one may determine, for a given modulus of rupture, the loads, p, that the different beams can carry. Figure 2 shows results of this sort. The assumed modulus of rupture is 600 pounds per square inch. The modulus of subgrade reaction, k, is still assumed to be 50 pounds per cubic inch. The curves show that the strength, measured by p, does not vary greatly with the length so long as a remains greater than 6 feet.

THE PAVEMENT WILL BREAK UP, ULTIMATELY, INTO SMALL UNITS IF THE WHEEL LOADS ARE HEAVY ENOUGH TO BREAK A LARGE SLAB WITHOUT THE AID OF CHANGES OF TEMPERATURE AND CHANGES IN THE SUBGRADE

Figure 3 shows moments and stresses along the edge of a large slab on elastic subgrade loaded at the edge. This diagram[6] has a peak at the middle. This peak is rounded off, as shown by four examples. The resultant maximum value depends on the area of contact between the load and the pavement, and the thickness of the slab. The curves rounding off the peak are drawn so as to indicate the proper values of the stresses at the bottom of the slab. The vertical scales define stresses in pounds per square inch produced in slabs

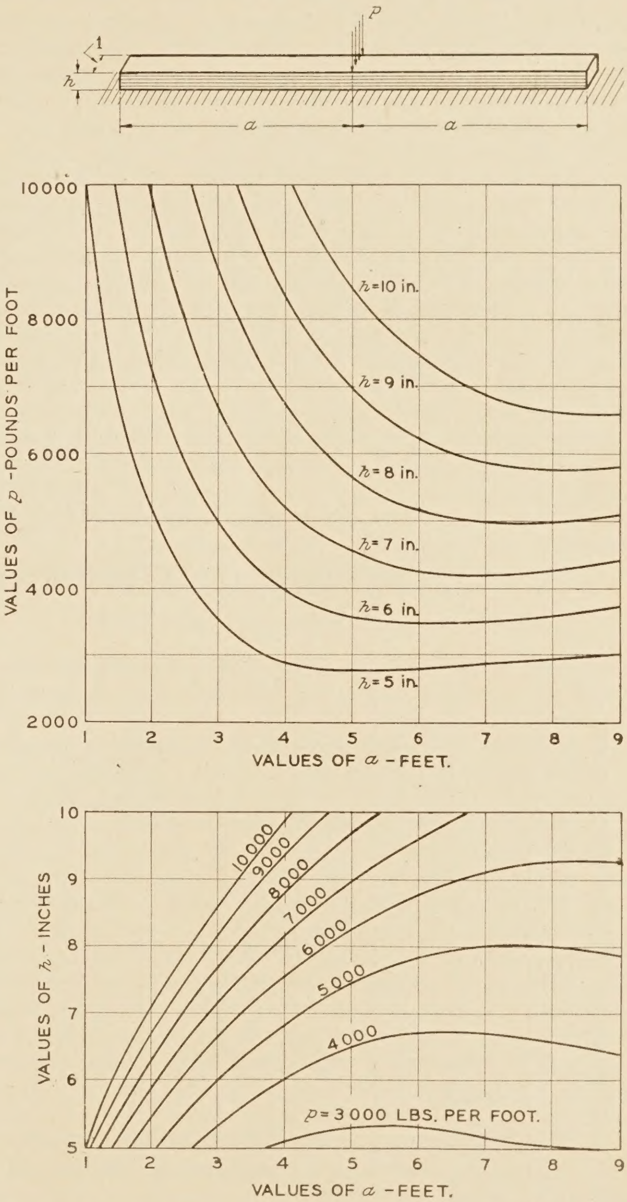

Figure 2.—Relations Between Size and Strength of Beams on Uniform Elastic Subgrade. Numerical Values Assumed: $E=3{,}000{,}000$ Pounds per Square Inch; $k=50$ Pounds per Cubic Inch; Modulus of Rupture of Concrete $=600$ Pounds per Square Inch. The Value of p Measures the Strength

[6] The curve is taken from Figs. 10 and 11 in the paper, Stresses in Concrete Pavements Computed by Theoretical Analysis, referred to previously. The curve was obtained by interpolation between the numerical values given for $\mu=0$ and $\mu=0.25$, respectively, so as to obtain values for $\mu=0.15$. The maximum values are defined by equation 12 or Table 4 in that paper.

of different thicknesses by a load of 10,000 pounds. The horizontal scales are like those in Figure 1. A very similar diagram, though with smaller ordinates, may be drawn representing the moments and stresses produced by a wheel load applied at an interior point of the area of a large slab, at a considerable distance from the edges.[7]

The diagram in Figure 3, representing moments and stresses at the edge of a large slab, shows a general similarity to the corresponding diagram in Figure 1 for the long beam (with $a=\infty$). This similarity warrants the conclusion that if the length of the slab in the direction of the loaded edge be reduced gradually without much change of the dimension perpendicular to the edge, the law of relief of stress by the reduction of size will be about the same as that which was found for the beam. If the width is reduced greatly while the length is reduced, the relief of stress, naturally, either will be delayed or will fail to appear.

The distance between the points of zero bending moments serves to interpret horizontal distances in general. In case of the slab this distance is 0.75 times the corresponding distance in the case of the beam. One may conclude that wheel loads that would overstress a large slab without the aid of changes of temperature and changes in the subgrade are capable of breaking up the pavement ultimately into pieces of even smaller dimensions than were found in the case of the beam.

[7] By use of Figure 5 and Table 3 in the paper referred to in the preceding footnote.

CORNER BREAKS DISCUSSED

In discussing the size of the pieces produced by cracking, attention must be given also to the case of the corner break; that is, the case in which a load at a corner causes a triangular piece to break off. The shortest distance from the corner to the danger section has been computed to be approximately,[8]

$$x_1=2\sqrt{a_1 l} \qquad (15)$$

where a_1 is the distance from the corner to the center of the load, which is assumed to be at the same distance from the two intersecting edges. For example, with $E=3{,}000{,}000$ pounds per square inch, $k=50$ pounds per cubic inch, $\mu=0.15$, and $h=8$ inches, giving $l=40.2$ inches, and with $a_1=4$ inches, one finds $x_1=25.4$ inches. The corresponding distance measured along the edge is $x_1\sqrt{2}=36$ inches.

NARROW CRACKS MAY ACT TO SOME EXTENT AS HINGED JOINTS TRANSMITTING VERTICAL FORCES BETWEEN THE SLABS

While no appreciable bending moments can be transmitted across a crack in a pavement, it is conceivable that vertical forces may be transmitted if the crack remains narrow, so as to permit the rough edges of the adjoining slabs to interlock. The crack then acts to some extent as a hinged joint. A wide open crack, on the other hand, represents free edges of the slabs.

[8] Equation 4 in the paper, Stresses in Concrete Pavements Computed by Theoretical Analysis, referred to previously. Values of the stresses are given in Table 2 and by equation 6 in that paper.

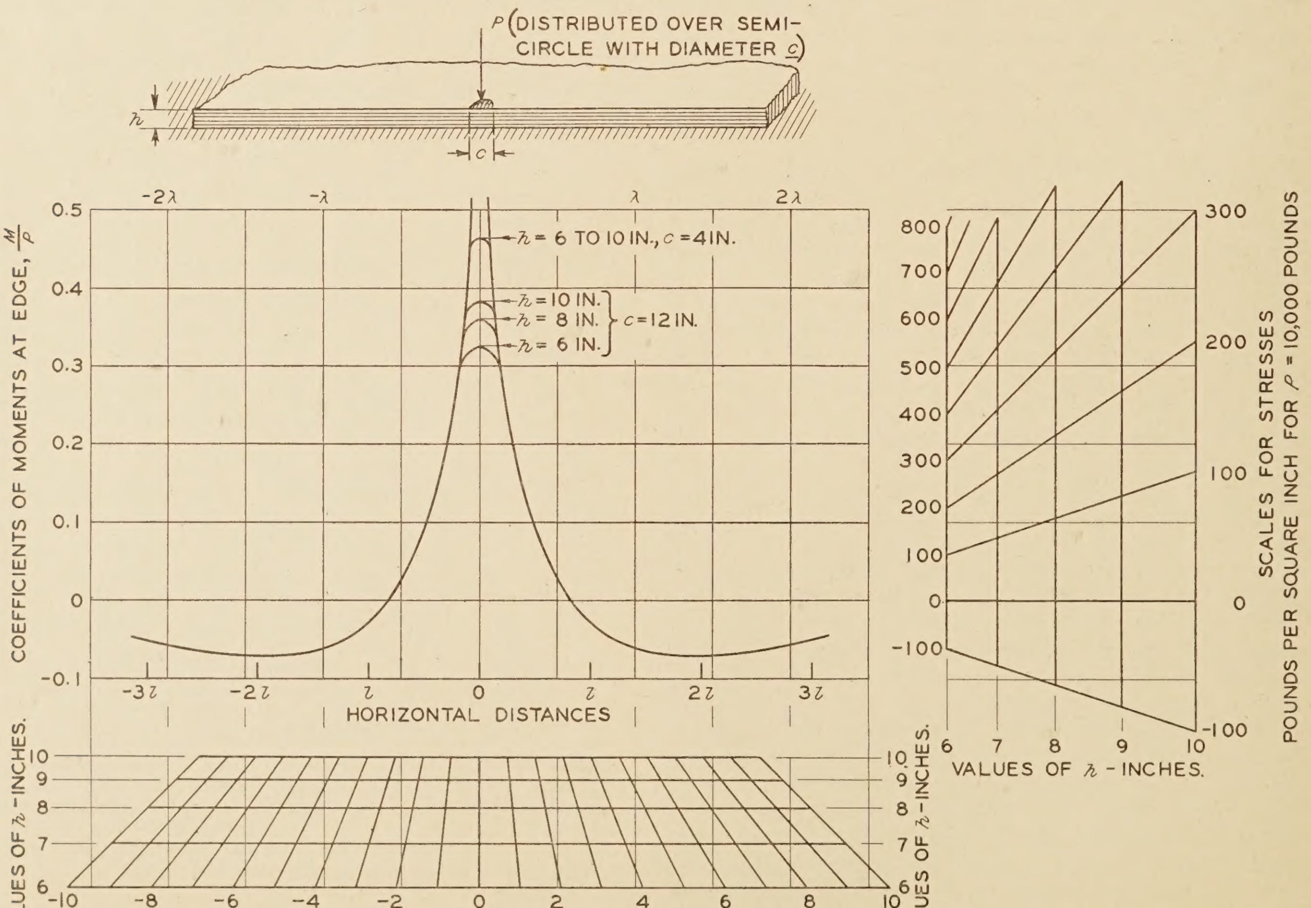

FIGURE 3.—DIAGRAM OF MOMENTS AND STRESSES ALONG THE EDGE OF A LARGE SLAB, RESTING ON UNIFORM ELASTIC SUBGRADE, AND LOADED AT THE EDGE. NUMERICAL VALUES ASSUMED: $E=3{,}000{,}000$ POUNDS PER SQUARE INCH; POISSON'S RATIO OF THE CONCRETE, $\mu=0.15$; $k=50$ POUNDS PER CUBIC INCH

Narrow cracks do not always act as hinged joints. For example, if a crack, separating slabs A and B, follows a sloping surface through the thickness of the slab, it is possible that slab A may rest on B at the crack, but that B will be free to depart from A by deflecting downward. Thus, with A loaded, the crack represents a hinged joint, but with B loaded, it represents a free edge. Even with this possibility, it is probable that the statistical average of narrow cracks acting to some extent as hinged joints is fairly high. This action of the cracks then becomes an important feature in the mechanics of the pavement.

An observation may be obtained from the tables which have been computed for the stresses produced in a large slab by a wheel load, applied either at an interior point of the area or at a free edge.[9] Consider, for example, a 7-inch slab, with the values of E, μ, and k as in Figure 3. With the load $P = 10,000$ pounds applied in the interior of the area, and distributed uniformly over a circle with a radius of 4 inches, the stress given by the table (Table 3 in the paper referred to) is 319 pounds per square inch. With the same load acting at a free edge, for example, at an open crack, the load being distributed uniformly over a semicircle with a radius of 4 inches and with the center at the edge, and the thickness being the same, the stress given in the corresponding table is 494 pounds per square inch. Now consider a narrow crack, with the load applied on one side of it. If this crack acts perfectly as a hinged joint, one may assume that one-half of the vertical force is transmitted through the joint. One might assume further that the strees is approximately cut in two; that is, reduced from 494 to about 247 pounds per square inch, but on account of the character of the local action right under the load, even with the joint acting ideally, the reduction would probably be less than one-half of the original amount. If the stress should be reduced to, say, 300 pounds per square inch, it would still be less than the stress, 319 pounds per square inch, which was produced when the load acted in the interior, distributed over the area of a full circle with radius of 4 inches. The numerical example chosen is typical. An inspection of the tables shows that the most significant feature applies generally; namely, that the stress at a free edge is appreciably less than twice the corresponding stress in the interior, the thickness being the same.

One must not expect that even a narrow crack will be as fully effective as a hinged joint, as was assumed in the numerical example. Still, since it is probable and normal that the narrow crack acts to some extent in this manner, being partly effective as a hinged joint, one may conclude, first, that under normal circumstances wheel loads produce less stresses next to a narrow crack than next to a wide one, and that accordingly a new crack is less likely to start from a narrow crack than from a wide one; secondly, one may conclude that it is probable and normal that the maximum bending stress at the narrow crack will not exceed greatly the stress that the same load produced at the same point before the crack was formed.

Similar considerations apply to the stresses tending to produce a corner break. If a corner is formed in the interior by one crack starting from another, and if the cracks remain narrow, it is probable that the adjoining slabs will be capable of supporting one another to some extent by transfer of vertical forces across the cracks. One may expect a corresponding reduction of the stresses due to a load at the corner, as compared with the stresses that would occur if the edges forming the corner were free.

[9] Tables 3 and 4 in the paper referred to in footnote 8.

REINFORCEMENT KEEPS CRACKS NARROW

In the case of subgrades which are not favorable relatively small amounts of steel reinforcement placed at the top, the middle, or the bottom of the pavement have proved structurally effective. The foregoing discussion shows the advantage of keeping the cracks narrow. It appears that small amounts of reinforcement are capable of keeping cracks narrow, as a rule, and that it is this performance of the reinforcement that accounts for its effectiveness.

LONGITUDINAL JOINTS REDUCE THE FREQUENCY OF WIDE CRACKS

If transverse cracks occur in a fairly large number, only a few of them can open widely, since within a given length of the pavement the total distance covered by concrete can not vary greatly, and thus a limit is set for the sum of the widths of all the cracks. On the other hand, if a longitudinal crack occurs in an unreinforced pavement, it is likely to open up widely, since the slabs may creep transversely. In view of this relative significance of the longitudinal cracks, it appears especially desirable to anticipate the tendency for these cracks to form by introducing properly designed longitudinal joints. Thickened edges of the two slabs, or dowels, or a tongue-and-groove design are well-established means of bringing about the desired increased strength of the joint, as compared with the strength of the free edges at the wide crack.

THICKENED OUTER EDGES DELAY CRACKING

The thickened outer edge may have an important function in keeping the subgrade material in place. The thickened edge may be desirable because the chance of defective support is greater at the edge than in the interior. In the case of an even subgrade, if one considers either a new pavement, not yet cracked, or a pavement in which all the cracks are narrow and capable of acting to some extent as hinged joints, a balanced design requires thickened outer edges. It is true that if some of the cracks open up widely, or if by chance some narrow cracks, as is possible, fail to function as hinged joints, these cracks will represent unthickened free edges, and when these edges are considered, the design is no more completely balanced. Still, the thickening of the outer edge means that one certain weak edge has been eliminated. One may expect, accordingly, a delay in the progress of cracking. This delay may be counted in years if the big loads, the fatal impacts, occur only as rare accidents.

It seems rational to design the pavement with sufficient thickness everywhere to make rare and improbable the accident of an impact load causing a new crack to start anywhere, the case being included in which the new crack starts from a free edge at an existing crack. Regardless of whether the design is balanced under all circumstances or not, the thickening of the outer edges doubtless remains good strategy, and is a rational method of prolonging the life of the pavement by reducing the statistical average of new cracks per year.

STRESSES DUE TO CHANGES OF TEMPERATURE ARE RELIEVED MATERIALLY WHEN A 20-FOOT SLAB BREAKS IN TWO

The slab of length $2a$, shown at the top of Figure 4, tends to curl under the influence of a difference of temperature between the top and the bottom. If loads bring about complete contact between the slab and the subgrade, and if the temperature varies uniformly from the top to the bottom, the maximum bending stresses, contributed at some distance from the edge of length $2a$ by the change of temperature alone, will be as defined by the curve in Figure 4, marked "Case I," in connection with the scales below and on the right.[10] The assumed numerical values are: Coefficient of expansion, $\epsilon_t = 0.000006$ per degree Fahrenheit; modulus of elasticity of concrete, $E = 3{,}000{,}000$ pounds per square inch; difference between temperatures at top and bottom, $t = 10$ degrees Fahrenheit. The scales are used as in Figure 1. The curve shows clearly the relief of stress occurring when a is changed, for example, from 10 feet to 5 feet; that is, when the length, $2a$, is changed from 20 feet to 10 feet. The conclusion is drawn that in the process of breaking up, the slabs profit from reduction of size at a much earlier stage so far as the stresses due to changes of temperature are concerned than so far as the stresses due to the loads are concerned.

SETTLEMENTS IN THE SUBGRADE MAY PRODUCE EFFECTS SIMILAR TO THOSE DUE TO CHANGES OF TEMPERATURE

One may describe changes in the subgrade in terms of the shape that an originally plane upper surface of the subgrade would assume with the pavement unloaded if the resistance of the pavement to bending were destroyed so that the pavement would exert only an even pressure equal to its weight. In Case I, represented at the top of Figure 4, this surface is assumed to be cylindrical, with the constant small curvature,[11] c. It is observed that the same tendency to separation between the slab and the subgrade exists in this case as in the case of curling due to a difference of temperature between the top and the bottom. The curve marked "Case I" applies, therefore, to the present case of

[10] The curve is the same as the lower curve in Fig. 5 in the paper, Analysis of Stresses in Concrete Roads Caused by Variations of Temperature, as presented in PUBLIC ROADS, vol. 8, No. 3, May, 1927. The stresses are defined by equations 7 and 20 in that paper.

[11] The curvature is the reciprocal of the radius of curvature.

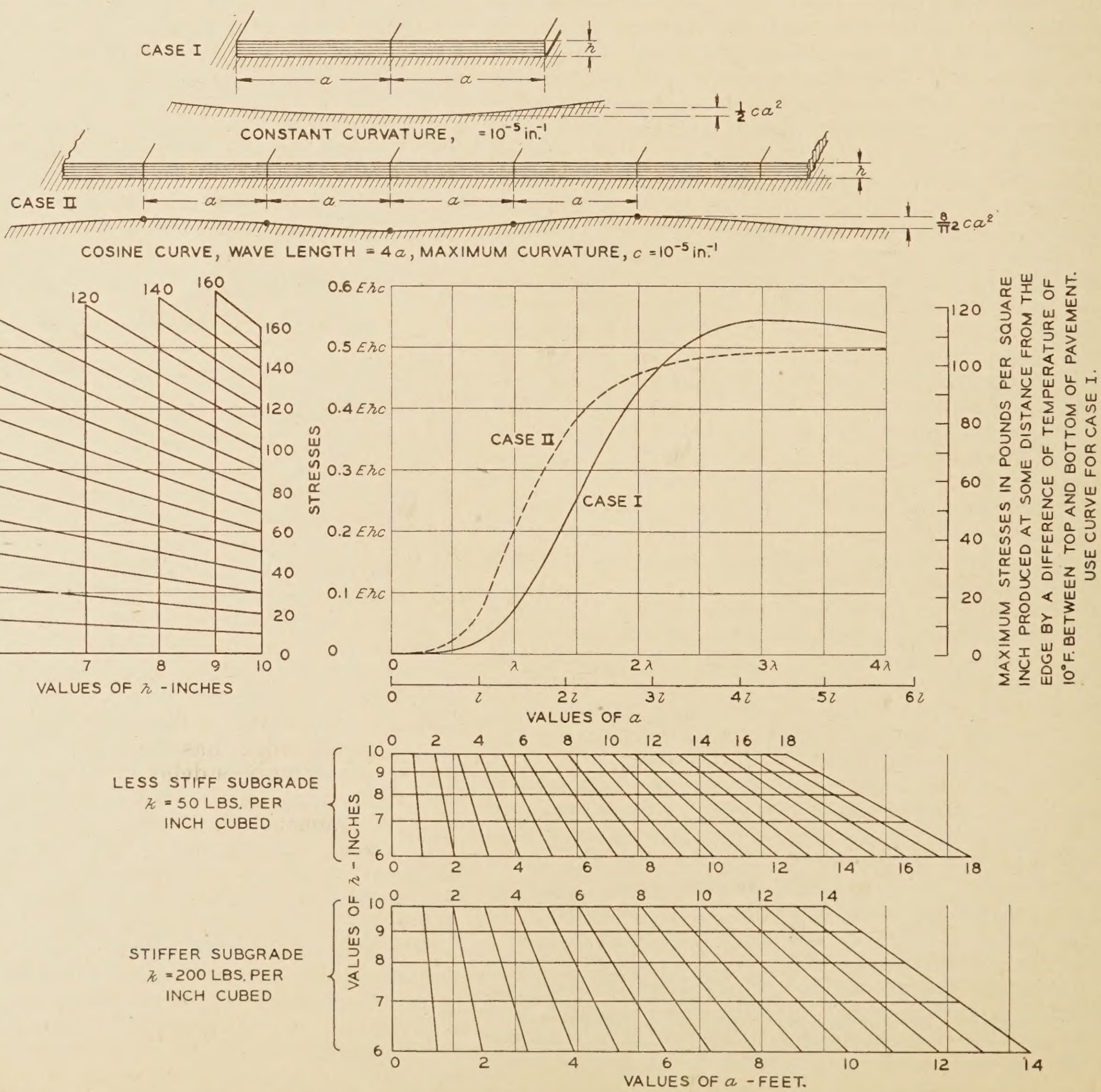

FIGURE 4.—MAXIMUM BENDING STRESSES DUE TO CHANGES OF TEMPERATURE AND SETTLEMENTS IN THE SUBGRADE. NUMERICAL VALUES ASSUMED: $E = 3{,}000{,}000$ POUNDS PER SQUARE INCH, $\mu = 0.15$; COEFFICIENT OF TEMPERATURE EXPANSION, $e_t = 0.000006$ PER DEGREE FAHRENHEIT

settlements in the subgrade as well as to the case of changing temperatures. The scales on the left are drawn so that they define the maximum stress at the edge corresponding to $E=3{,}000{,}000$ pounds per square inch, $c=10^{-5}$ in.$^{-1}$ (radius of cylindrical surface $=\frac{1}{c}=10^5$ inches $=1.6$ miles).

In Case II, in Figure 4, the unloaded top of the subgrade is assumed to follow a cosine wave with wave length $4a$. When the maximum curvature is the same as in the preceding case, equal to c, the equation of this surface becomes

$$z_0=\frac{4}{\pi^2}ca^2\cos\frac{\pi x}{2a} \qquad (16)$$

With the slab continuous over several wave lengths, and with loads maintaining contact between the slab and the subgrade, one obtains, by the derivations given below, the dotted curve marked "Case II." The same scales, below and on the left, apply to both curves.

FORMULAS DERIVED FOR COMPUTING STRESSES RESULTING FROM DEFORMATIONS IN THE SUBGRADE

Let z_0 denote the deformation at any point of the unloaded surface of the subgrade. With contact maintained between the slab and the subgrade, the reaction of the subgrade per unit of area becomes $k(z-z_0)$. So long as z and z_0 are functions of x only (independent of the other horizontal coordinate, y), the following equation governs the flexure (replacing equation 1):

$$\frac{Eh^3}{12(1-\mu^2)}\frac{d^4z}{dx^4}+k(z-z_0)=0 \qquad (17)$$

Poisson's ratio, μ, enters here because the slab is assumed to be broad in the direction of y. By introducing the quantity

$$\lambda=\sqrt[4]{\frac{Eh^3}{3(1-\mu^2)k}}=l\sqrt{2} \qquad (18)$$

which differs only slightly from the corresponding value defined by equation 2, equation 17 becomes:

$$\frac{\lambda^4}{4}\frac{d^4z}{dx^4}+z=z_0 \qquad (19)$$

Because of Poisson's ratio, the deflections actually are not independent of y at and close to the edges which are parallel to the axis of x. No great inaccuracy is introduced, however, by assuming the maximum value of the curvature, $-\frac{\partial^2 z}{\partial x^2}$ to be the same at the edge as farther in. The maximum bending stress at the edge is computed then, both in Case I and Case II, by the formula,

$$\sigma=\pm\frac{Eh}{2}\left[\frac{d^2z}{dx^2}\right]_{x=0} \qquad (20)$$

with z determined from equation 19.

In Case I, with $z_0=-\frac{1}{2}cx^2$, one arrives at the solution equivalent to that which was obtained in the study of curling.[12] Then, by use of equation 20, the maximum stress at the edge is computed to be

$$\sigma=Ehc\left[\frac{1}{2}-\frac{\cos\alpha\sinh\alpha+\sin\alpha\cosh\alpha}{\sinh 2\alpha+\sin 2\alpha}\right] \qquad (21)$$

where $\alpha=\frac{a}{\lambda}$.

In Case II, with z_0 defined by equation 16, the following solution satisfies equation 19:

$$z=\frac{1}{1+\frac{\pi^4}{64}\frac{\lambda^4}{a^4}}\frac{4ca^2}{\pi^2}\cos\frac{\pi x}{2a} \qquad (22)$$

The maximum stress at the edge becomes, according to equation 20,

$$\sigma=\frac{Ehc}{2+3.044\frac{\lambda^4}{a^4}} \qquad (23)$$

SO FAR AS STRESSES DUE TO DEFORMATIONS OF THE SUBGRADE ARE CONCERNED, LARGE SLABS ARE PENALIZED, BUT SMALLER SLABS PROFIT BY INCREASE OF THE THICKNESS

Both in Case I and Case II, when a is large, the stresses due to deformations of the subgrade are approximately proportional to the thickness of the slab. Writing $a=\lambda\alpha$, $\sigma=Ehc\beta$, one may interpret α and β as coordinates of either one of the curves. One may observe that the rate of increase of stress, $\frac{d\sigma}{dh}$, is positive or negative, that is, the slab is penalized or profits, respectively, by an increase of the thickness, according to whether $\frac{\beta}{\alpha}$ is greater than or smaller than $\frac{3}{4}\frac{d\beta}{d\alpha}$.

In Case I, these two quantities are approximately equal when $a=2\lambda$. With smaller values of a—that is, smaller size—the slab profits from an increase of h. For example: $2a=12$ feet, $k=50$ pounds per cubic inch, $h=6$ inches, gives according to the diagram: $a=1.57\lambda$, $\sigma=51$ pounds per square inch; changing h to 8 inches gives $\sigma=39$ pounds per square inch.

A GIVEN DEFORMATION IS THE MORE UNDESIRABLE THE STIFFER THE SUBGRADE

A glance at the horizontal scales in Figure 4 shows that the stiffer the subgrade, the more severe on the pavement is a given deformation of the subgrade of the type of Case II (so long as a is less than 3λ), and the smaller is the value of a corresponding to a given stress. Thus one observes the virtue of a subgrade which is capable of adjusting itself under the pavement after a deformation has occurred.

CONCLUSIONS

Conclusions are summarized as follows:

The pavement breaks into pieces of small size, possibly 4 feet, or less, if the loads are capable of producing cracks without the aid of changes of temperature or settlements in the subgrade.

The bending stresses contributed by changes of temperature and the deformation of the subgrade are expected to be relieved materially when the size of the pieces has become about 10 feet. If the deformations in the subgrade remain moderate for a period of years, there is reason to expect that the pavement will not break up further during this period of years, provided that the loads do not produce excessive stresses.

The desirability of thick pavements is indicated by a consideration of the probable frequency of cracks after a period of years.

[12] Equations 18 and 20 in the paper, Analysis of Stresses in Concrete Roads Caused by Variations of Temperature.

EFFECT OF TYPE AND GRADATION OF COARSE AGGREGATE UPON THE STRENGTH OF CONCRETE

Reported by W. F. KELLERMANN, Assistant Materials Engineer, Division of Tests, United States Bureau of Public Roads

THE compression test has been employed almost universally in the past for measuring the quality of Portland cement concrete. Applied to structural concrete for general purposes, this measure of quality has proved quite adequate, because in such cases the crushing strength is the property of greatest interest. When the concrete is to be used in the construction of pavements, however, the crushing strength, although still important, is no longer the critical factor. Concrete pavements should be designed to support, without cracking, heavy concentrated loads which subject them to high bending stresses.

Flexural strength thus becomes the most important strength characteristic, and the factors which affect it become of interest to the highway engineer, even though they may not be of primary importance in other types of concrete construction. To illustrate this distinction: It has been demonstrated through research that within quite wide limits the kind of coarse aggregate empoyed has relatively little effect on the crushing strength of concrete, provided the mixture is workable and the aggregates sound. This is not true in the case of flexure, however, all of the test data so far accumulated indicating that such factors as surface texture, angularity of fragments, and other characteristics of the aggregates affect the flexural and tensile strength of the concrete to a marked degree. The tests which are reported in this paper substantiate earlier experiments along the same lines and indicate that the character of aggregate must be given consideration in the design of concrete for pavements.

These tests also indicate that the compressive strength of concrete may be affected by the character of the coarse aggregate to a considerably greater extent than has been shown by prior investigations.

A VARIETY OF COARSE AGGREGATES AND AGGREGATE GRADINGS TESTED

The tests were conducted primarily for the purpose of determining the effect of type of coarse aggregate upon the flexural strength of concrete. Auxiliary data regarding resistance to direct tension as well as crushing strength and yield of concrete were also obtained. Seventeen typical coarse aggregates, including seven gravels, seven crushed stones and three blast-furnace slags were selected from various sources so as to give as wide a range in physical characteristics as possible. These were tested in concrete, using four typical coarse aggregate gradings as well as four paving mixes. The sand used was a typical high-grade concrete sand. Its physical properties as well as those of the Portland cement employed in the tests are given in Table 1. The physical properties of the various coarse aggregates are given in Tables 2 and 3.

Each coarse aggregate was separated into four sizes at the laboratory and recombined into four definite gradings as shown below. These gradings will be referred to by number and it should be noted that they range from coarse to fine in numerical order of designation.

Grading No.	Percentage passing—square openings				
	⅛-inch	¼-inch	¾-inch	1¼-inch	2-inch
1	0	0	15	40	100
2	0	0	30	55	100
3	0	5	45	70	100
4	0	5	55	100	100

TABLE 1.—*Physical properties of cement and fine aggregate used in all tests*

CEMENT

Fineness, percentage retained on 200-mesh sieve, 11.5.
Time of set (Gillmore):
  Initial, 3 hours 10 minutes.
  Final, 5 hours 35 minutes.
Steam test for soundness, O. K.
Normal consistency, 23.1 per cent.
Tensile strength (pounds per square inch, 1:3 Ottawa sand mortar):

	Pounds
7 days	285
28	380

FINE AGGREGATE

Sieve analysis:	
Total retained on ¼-inch screen, per cent	1
Total retained on No. 10 sieve, per cent	12
Total retained on No. 20 sieve, per cent	25
Total retained on No. 30 sieve, per cent	42
Total retained on No. 40 sieve, per cent	72
Total retained on No. 50 sieve, per cent	93
Total retained on No. 100 sieve, per cent	100
Silt and clay, per cent	0. 3
Apparent specific gravity	2. 66
Weight in pounds per cubic foot (dry rodded)	104
Organic matter (color test)	Satisfactory.
Strength ratio:	
7 days	117
28 days	120

Description: Sand consists essentially of subangular grains of quartz, sandstone and shale, slate and feldspar.

Table 4 gives the weight per cubic foot (dry rodded) and the percentage of voids for each coarse aggregate and for each grading.

All aggregates are identified by number rather than by source of supply.

In outlining this series of tests it was thought advisable to include more than one proportion, and for this reason four nominal mixes (based on dry-rodded volumes) were included, with the following proportions:

Mix No. 1—1 : 1.6 : 3.
Mix No. 2—1 : 1.6 : 4.
Mix No. 3—1 : 2 : 4.
Mix No. 4—1 : 2 : 4½.

Mixes No. 1 and No. 2 were designed to correspond approximately to 1 : 2 : 3 and 1 : 2 : 4 field mixes, respectively.

It will be noted that mix No. 4 (1 : 2 : 4½) is the only one which does not conform to present practice. This proportion was used in an endeavor to determine the manner in which the lower sanded mixes behave in flexure as compared to mixes containing smaller amounts of coarse aggregate, but with the same sand-cement ratio.

FABRICATION AND TESTING OF SPECIMENS DESCRIBED

The procedure followed was to make up specimens for mix No. 1 and grading No. 1, using all aggregates, on the same day. Gradings No. 2, No. 3, and No. 4 were then made in turn on following days. This was repeated for mixes No. 2, No. 3, and No. 4, in the order named, so that the first round of tests consisted of the four mixes and the four gradings for each of the gravel and stone aggregates and two mixes and four gradings of each for the slag aggregates, the harsher mixes, No. 2 and No. 4, not being used with the slags. This required 16 working days to complete one round of tests and, where the quantity of material permitted, four complete rounds were made, making a total of 64 batches of concrete for each aggregate. Thus the general averages given in Table 5 are in

TABLE 2.—*Physical properties of coarse aggregates*

Aggregate No.	Per cent wear	Per cent absorption	Specific gravity	Per cent crushed material
40 (trap)	[1] 2.4	0.09	2.91	100
44 (siliceous gravel)	[2] 9.9	.58	2.59	[3] 13 to 28
46 (siliceous limestone)	[1] 3.0	.04	2.76	100
50 (siliceous gravel)	[2] 10.8	.98	2.60	45
60 (granite)	[1] 2.0	.17	2.61	100
61 (sandstone)	[1] 13.2	4.55	2.23	100
62 (cherty limestone)	[1] 3.5	.16	2.66	100
63 [4] (argillaceous limestone)	[1] 6.8	4.42	2.30	100
63a [5] (argillaceous limestone)	[1] 10.4	6.56	2.18	100
64 (siliceous gravel)	[2] 14.5	.50	2.65	20
65 (siliceous gravel)	[2] 13.4	.36	2.61	[3] 46 to 76
66 (limestone gravel)	[2] 9.8	1.23	2.64	[3] 33 to 39
67 (limestone gravel)	[1] 15.1	2.17	2.57	[3] 67 to 92
68 (siliceous gravel)	[2] 12.8	.32	2.58	15
69 (slag)	[1] 22.5		1.83	100
70 (slag)	[1] 13.7		2.06	100
71 (slag)	[1] 12.2		2.27	100
72 (shell limestone)	[1] 7.0	1.87	2.47	100

[1] Test made on crushed material.
[2] Standard test for gravel.
[3] The percentage of crushed material in different gradings was as follows:

Aggregate No.	Grading No. 1	2	3	4
44	13	14	28	20
65	50	46	76	67
66	33	33	35	39
67	71	67	92	88

[4] First shipment of material.
[5] Second shipment of material.

TABLE 3.—*Mineral composition of coarse aggregates*

Aggregate No.	Mineral composition
40	Trap—Diabase and basalt.
44	Gravel—Quartzite, 65 per cent; quartz, 30 per cent; ironstone concretions, 2 per cent; chert, 3 per cent.
46	Siliceous limestone—Massive limestone, 98 per cent; lime, calcite veins, or pure calcite pieces, 2 per cent.
50	Gravel—Quartzite, 60 per cent; gneiss, 20 per cent; slate, 15 per cent; chert, 1 per cent; basic igneous, 4 per cent.
60	Granite—Dark, 25 per cent; gray, 10 per cent; pink, 65 per cent.
61	Sandstone—Rounded fragments.
62	Cherty limestone—Pure limestone, 30 per cent; chert limestone, 70 per cent.
63	Argillaceous limestone—Light, 50 per cent; dark, 50 per cent.
64	Gravel—Milk quartz, 10 per cent; gneiss, 80 per cent; badly weathered gneiss, 10 per cent.
65	Gravel—Quartz, 90 per cent; gneiss, schist, and basic minerals, 10 per cent.
66	Gravel—Cherty dolomite (3 colors).
67	Gravel limestone—Thin rounded limestone, 75 per cent; chert, 5 per cent; basic rocks, granites, 20 per cent.
68	Gravel—Granular milk quartz, 80 per cent; rotten chert, 15 per cent; gneiss, 5 per cent.
69	Slag.
70	Do.
71	Do.
72	Limestone—Shell limestone.

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TABLE 4.—*Weight per cubic foot and percentage of voids of coarse aggregates*

Aggregate No.	Grading No. 1		Grading No. 2		Grading No. 3		Grading No. 4	
	Weight per cubic foot	Voids	Weight per cubic foot	Voids	Weight per cubic foot	Voids	Weight per cubic foot	Voids
	Pounds	*P. ct.*	*Pounds*	*P. ct.*	*Pounds*	*P. ct.*	*Pounds*	*P. ct.*
40 (trap)	105	42	105	42	106	42	105	42
44 (siliceous gravel)	110	32	111	31	113	30	110	32
46 (siliceous limestone)	101	41	101	41	102	41	102	41
50 (siliceous gravel)	105	35	106	35	108	33	108	33
60 (granite)	99	39	100	39	101	38	100	39
61 (sandstone)	88	37	88	37	90	35	90	35
62 (cherty limestone)	99	40	100	40	100	40	100	40
63 [1] (argillaceous limestone)	88	39	88	39	90	38	90	38
63a [2] (argillaceous limestone)	85	38	86	37	87	36		
64 (siliceous gravel)	109	34	111	33	111	33	112	32
65 (siliceous gravel)	109	33	110	33	110	33	108	34
66 (limestone gravel)	107	35	108	35	109	34	108	35
67 (limestone gravel)	105	34	107	33	108	33	106	34
68 (siliceous gravel)	108	33	110	32	110	32	110	32
69 (slag)	65	43	67	41	66	42	68	40
70 (slag)	74	43	76	41	77	40	78	40
71 (slag)	82	42	83	42	83	42	85	40
72 (shell limestone)	90	42	90	42	90	42	89	42

[1] First shipment of material.
[2] Second shipment of material.

TABLE 5.—*Water-cement ratio, cement factor and strength tests—general average of all mixes and all grading* [1]

Aggregate No.	$\frac{W}{C}$	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. inch	*Lbs. per sq. inch*	*Lbs. per sq. inch*
40 (trap)	0.85	6.08	525	220	2,850
44 (siliceous gravel)	.84	5.53	475	195	2,860
46 (siliceous limestone)	.86	5.99	590	235	2,940
50 (siliceous gravel)	.85	5.63	530	215	2,980
60 (granite)	.88	5.84	520	225	3,020
61 (sandstone)	1.08	5.73	530	235	3,470
62 (cherty limestone)	.88	5.91	590	235	2,940
63 (argillaceous limestone)	1.01	5.92	495	240	3,140
64 (siliceous gravel)	.85	5.63	465	190	2,920
65 (siliceous gravel)	.83	5.64	495	195	2,860
66 (limestone gravel)	.86	5.69	585	250	3,310
67 (limestone gravel)	.89	5.68	580	250	3,300
68 (siliceous gravel)	.84	5.61	525	210	2,920
72 (shell limestone)	.96	5.95	550	255	3,140

[1] Flexure and tension specimens broken at 29 days, compression specimens broken at 33 days.

practically all cases based on tests on 64 specimens. Each batch of concrete was large enough to make one 6 by 6 by 30 inch beam and one 6 by 21 inch tension cylinder with some excess. The volume of concrete in in each batch was measured for yield determination.

All mixing was done by hand in dry pans, the amount of water used being that required to produce a consistency corresponding to a flow of 150.[1] Steel forms resting on steel plates were used for the beams, while the cylinders also rested on steel plates. The concrete was placed in the beam molds in two layers, each layer being rodded about thirty-five times with a ⅝-inch steel rod, bullet-shaped on the end. The sides were then spaded and the top struck off with a wooden float, the final finishing being done with a steel float. In making the tension cylinders the concrete was placed in three layers, each layer being rodded about thirty times with the same rod. Due to the limited facilities for handling the large number of specimens involved it was found necessary to keep them in moist

[1] Percentage of original diameter of the mass after 15 drops of ½-inch in ten second on a 30-inch flow table.

air for 28 days after 1 day in the molds, so that the age at test was 29 days instead of the conventional 28 days.

The beams were tested with a portable cantilever device with an extension arm and dynamometer for applying the load. A view of this machine is shown in Figure 1. The dynamometer was fastened directly to the extension arm so that no friction due to pulleys was included in the measured load. Two breaks were made on each beam. The tension cylinders were broken in a 100,000-pound Universal testing machine

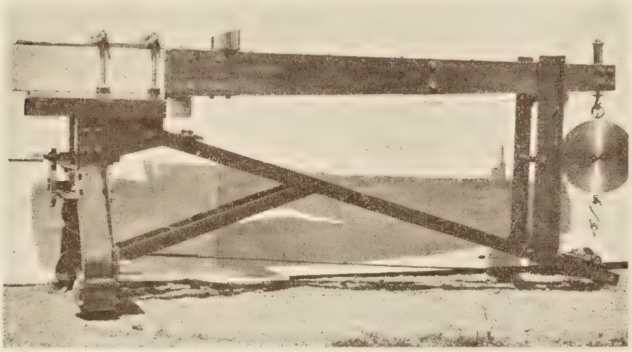

Figure 1.—Portable Device for Testing Beams as Cantilevers Showing a Beam Ready for Test

equipped with a hand wheel for slow application of load. A set of grips similar to those designed in the research laboratory of the Portland Cement Association was used for gripping the specimens. They may be briefly described as two pieces of 6-inch steel pipe lined with leather and split part way along four elements so as to slip over the ends of a straight cylindrical concrete specimen. The segments are drawn tight on the specimen by means of tangential bolts, thus developing enough friction between concrete and leather to prevent slipping during the test. Bolts passing through the head of each grip are provided with ball and socket joints both in the grip and at the centering plate on the testing machine, thus making the specimen and grips self aligning. A view of the device is shown in Figure 2.

Remnants from the tension specimens were capped with a neat cement paste made with a calcium chloride solution and broken in compression at 33 days. They varied in height and a correction factor was applied to make the ultimate load correspond to a specimen having a height twice its diameter. Results were therefore obtained in flexure and tension on the same batch of concrete cured in exactly the same manner while results were also obtained in tension and compression on exactly the same specimen but tested at slightly different ages.

RESULTS SHOW WATER-CEMENT RATIO NOT THE ONLY FACTOR AFFECTING STRENGTH

Figure 3 gives the average results of all strength tests for all aggregates except the three slags. They were omitted because they were not included in mixes No. 2 and No. 4 (1:1.6:4 and 1:2:4½) and naturally could not be included in these grand averages. This same information, together with the corresponding water-cement ratios and cement factors, is given in Table 5. The unit values for the tensile strength and modulus of rupture were calculated to the nearest pound. These figures were carried through to the final averages. In the tables, however, the unit values are given to the nearest 5 pounds. The water-cement ratios reported

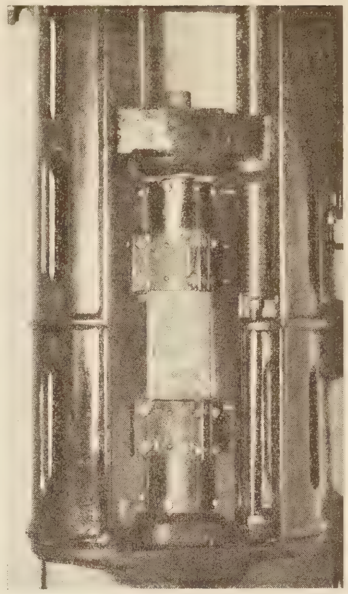

Figure 2.—A Tension Specimen Ready for Testing

in this and other tables were based on the total amounts of water added to each batch without correction for absorption of aggregates.

The modulus of rupture curve in Figure 3 shows that the aggregates arrange themselves in three general groups. For the purpose of discussion those showing a modulus of rupture of more than 550 pounds per square inch have been placed in the first group, those between 550 and 500 in the second group, and those less than 500 in the third group. In the first group there are four aggregates, Nos. 46, 62, 66, and 67 (one siliceous limestone, one cherty limestone, and two limestone gravels), with but 10 pounds per square inch difference among them. In the second group there are six aggregates, Nos. 40, 50, 60, 61, 68, and 72 (one each of trap, granite, sandstone, shell limestone, and two siliceous gravels), while in the third or low group are four aggregates, Nos. 44, 63, 64, and 65 (three siliceous gravels and one argillaceous limestone). Figure 4 shows how the average strengths in tension, compression, and bending for each aggregate vary from the average for all aggregates. The highest value for modulus of rupture exceeds the lowest value by 27 per cent. A question naturally arises as to the reason for this difference, and it is explained by a consideration of such factors as water content, absorption, angularity of particles, and structural soundness of the aggregates. Since all of the concrete was made to the same consistency and the proportions and gradation for a given condition were constant, the amount of water used depended to a large extent upon the shape and absorption of the aggregates.

We may assume first, that, other factors remaining the same, the amount of mixing water or the water-cement ratio used would be the cause of this difference. Comparing the siliceous gravel aggregate No. 64 with a water-cement ratio of 0.85 and an absorption of 0.50 per cent with the siliceous limestone aggregate No. 46 with a water-cement ratio of 0.86 and an absorption of 0.04 per cent, we would expect the siliceous gravel to absorb a considerable amount of water, while the siliceous limestone would not, thereby lowering the net water-cement ratio in the case of the siliceous gravel. Under these circumstances the siliceous gravel should give the higher flexural strength, but on the contrary the siliceous limestone gave a strength 27 per cent higher than the siliceous gravel. Following the same line of thought, the siliceous gravel (No. 64) should give higher strengths than a number of the other materials, but reference to the data will show that this is not the case. Comparing other aggregates on the same basis will show that in so far as this investigation is concerned the water-cement ratio is not the only factor which controls the strength.

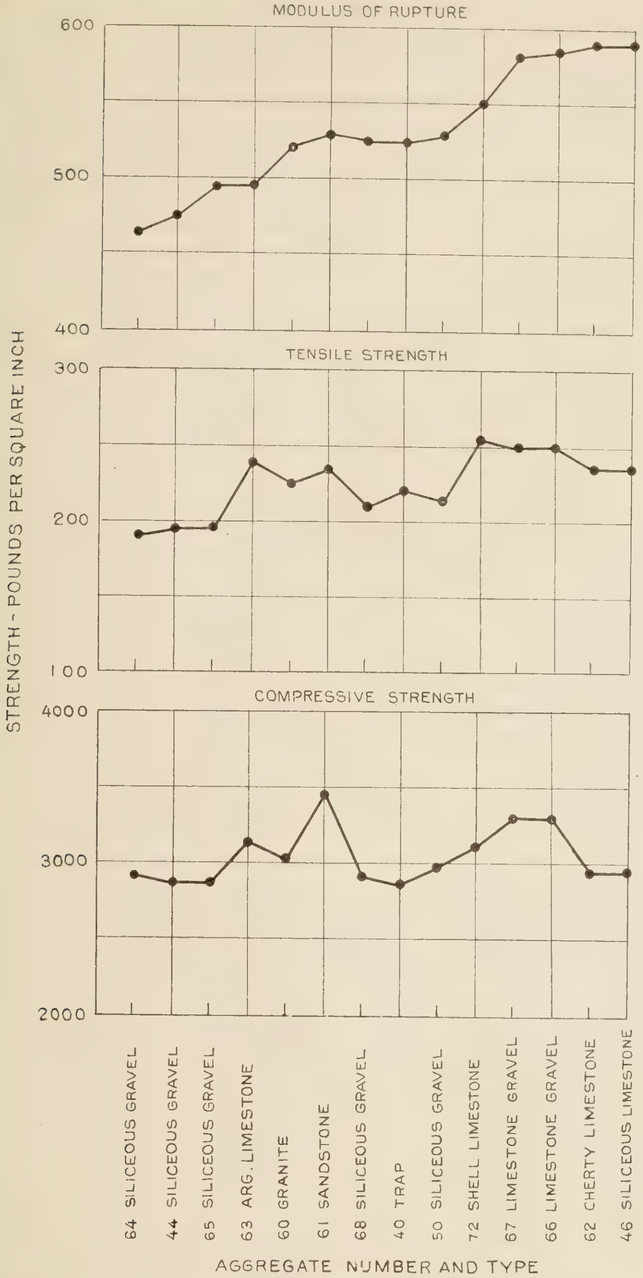

FIGURE 3.—AVERAGE STRENGTH TESTS FOR ALL MIXES AND ALL GRADINGS

Considering next the angularity of the particles of the coarse aggregate, we see from Table 2 that the siliceous limestone and cherty limestone aggregates in the high group designated as Nos. 46 and 62, consisted entirely of crushed particles, while the limestone gravel designated as No. 66 had from 33 to 39 per cent crushed and No. 67, a limestone gravel, had from 67 to 92 per cent, depending upon the particular gradation considered. The two limestone gravels differ from the two limestones mentioned first in that they are crushed gravels and have some surfaces rounded, while the limestones are 100 per cent crushed. In the low group, the siliceous gravels, Nos. 44 and 64 both had a very low percentage of crushed material, although the siliceous gravel, No. 65, also of the low group, had a greater percentage than the limestone gravel, No. 66, in the high group. The trap, siliceous limestone, and granite (Nos. 40, 46, and 60), all had 100 per cent

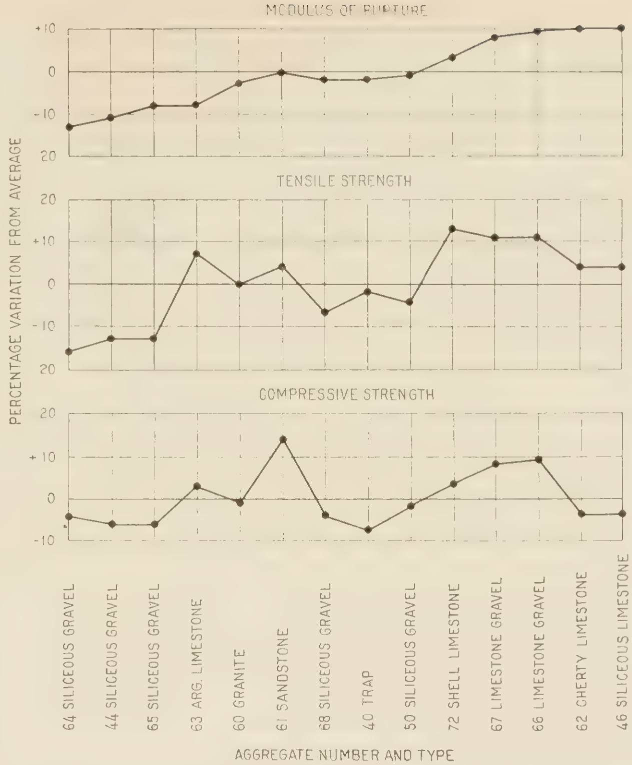

FIGURE 4.—VARIATION IN STRENGTH OF EACH AGGREGATE FROM THE GENERAL AVERAGE FOR ALL AGGREGATES EXCEPT SLAG

crushed material, but this siliceous limestone showed a strength about 13 per cent higher than the other two.

MINERAL COMPOSITION OF AGGREGATE FOUND TO BE IMPORTANT

Since these facts do not explain the strength variations described let us examine the physical characteristics of the coarse aggregates still further. Taking first the high group, we see from Table 3 that No. 46 is a siliceous limestone, No. 62 is a cherty limestone, No. 66 is a cherty dolomitic crushed gravel, and No. 67 is a crushed limestone gravel with some chert. Two of these materials are crushed stones, one is a crushed gravel with about one-third crushed pieces, and the fourth is a gravel with about three-fourths crushed pieces. All of these materials are from different sources, two of them falling into the general class of crushed stone aggregates and two into the general class of gravel aggregates. From a mineralogical standpoint, however, they are practically the same, all four being essentially calcareous.

Considering the low group, we find Nos. 44 and 65 are essentially quartz gravels, while No. 64 is a gravel composed mostly of gneiss and quartz. No. 63 is an entirely different material, being a soft argillaceous limestone. This particular material was of two varieties and varied considerably throughout the series. The quartz gravels, Nos. 50 and 68, may be placed in the same mineral group with Nos. 44 and 65, but they show a considerably higher flexural strength and were placed in the intermediate strength group. Since in the material found in any one of these aggregates weathering may have progressed farther than in similar material in another, we might expect a difference in structural soundness which would affect the flexural strength.

Of the aggregates in the intermediate group, No. 40 is a very hard and heavy trap rock; No. 60 is a sound granite; No. 61 is a very soft sandstone, practically every piece of which fractured in the flexure tests; No. 68 is a material somewhat similar to No. 50, but from an entirely different source; and No. 72 is a shell limestone containing many soft pieces but which gave much higher strength than any other aggregate listed in the intermediate group.

Considering all factors, the tests clearly indicate that the mineral composition of the coarse aggregate is of prime importance and must be considered along with other factors when the question of resistance to bending arises.

Referring again to Figure 3 and Table 5, and examining the tensile strength values, we find a difference of 34 per cent between the highest and lowest results. In this particular case the shell limestone aggregate No. 72 shows the highest value, with the limestone gravel aggregates Nos. 66 and 67 only slightly lower. It will be remembered that the latter two aggregates showed very high strength in the flexure tests. The siliceous limestone, cherty limestone, sandstone and argillaceous limestone (Nos. 46, 62, 61, and 63) also show high tensile strength. In fact, the difference among eight aggregates, beginning with No. 68 at 210 pounds per square inch and ending with No. 63 at 240 pounds per square inch is very slight. However, the values are rather small, which makes the percentage variations large. There is a distinctive low group composed of three siliceous gravels (Nos. 64, 44 and 65), the same aggregates showing low flexural strength. It is readily apparent that the aggregates do not arrange themselves in exactly the same order in tensile strength that they do in flexure, the main difference occurring in the soft sandstone and limestone aggregates, Nos. 61, 63 and 72, which show a higher relative strength with respect to the other aggregates in tension than in flexure.

In the compressive tests also shown in Figure 3 and Table 5, we see that with the exception of the sandstone and two limestone gravels Nos. 61, 66 and 67, the average crushing strength runs fairly close to 3,000 pounds per square inch for all aggregates. Comparing the compression curve with that for flexure and tension shows that the two limestone gravel aggregates Nos. 66 and 67 are outstandingly high for all three types of test, while three of the siliceous gravels Nos. 44, 64, and 65 are low in all cases. For the balance of the materials, however, there appears to be no consistent relation between compression and either modulus of rupture or direct tension. These values in general show the fallacy of attempting to use direct ratios between compression, tension and flexure in a general way, and without taking into account the particular material at hand.

Proceeding now to Figure 5, we shall examine the strength values for each mix, individually and collectively. These data, as well as the corresponding water-cement ratios and cement factors for each mix, are given in detail in Table 6.

In Figure 5, the three slag aggregates, Nos. 69, 70, and 71, are included for the 1:1.6:3 and 1:2:4 mixes. It will be observed that each of the three slags shows relatively high values for modulus of rupture for the 1:1.6:3 mix, but that the corresponding values for 1:2:4 mix are somewhat low as compared to the calcareous materials comprising the high-strength group. Examining the four modulus of rupture curves collectively we see that in general they are parallel, the greatest discrepancy being sandstone aggregate No. 61, which shows quite erratic results. It is also interesting to note from the graphs that the curves for the 1:1.6:3 and 1:1.6:4 mixes dip down in the case of the argillaceous limestone aggregate No. 63, while for the two

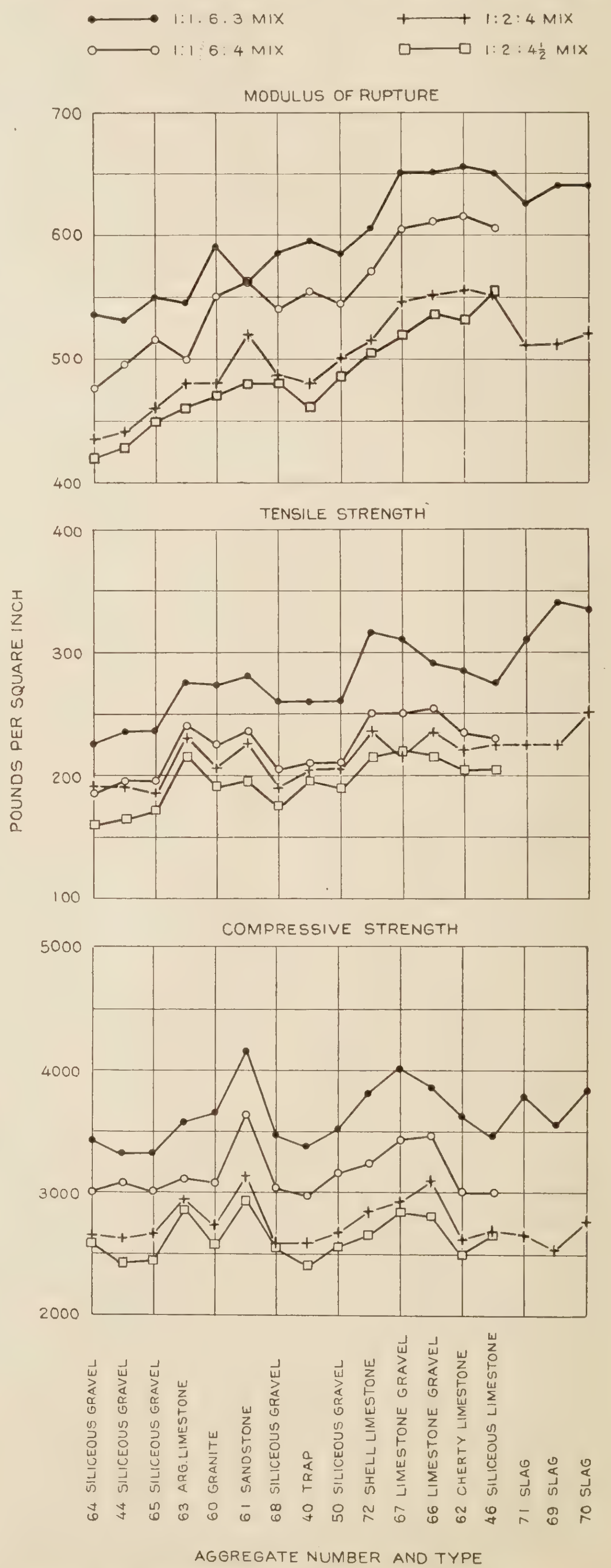

Figure 5.—Strength Tests for Each Proportion, Average of Four Gradings

leaner mixes they go up. For the trap aggregate No. 40, the curves dip down for the leaner mixes, while they go up for the richer ones. Considering the two materials, No. 40 is a very hard trap rock which we would naturally expect to show to better advantage in a rich mix, while No. 63, being a softer material, the opposite is of course true. Note also the relatively high strengths for the granite aggregate No. 60 in the two richer mixes.

TABLE 6.—*Average water-cement ratio, cement factor, and results of strength tests (includes four gradings) of all mixes* [1]

Aggregate No.	$\frac{W}{C}$	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*
1:1.6:3 MIX					
40 (trap)	0.75	7.08	595	260	3,390
44 (siliceous gravel)	.74	6.49	530	235	3,320
46 (siliceous limestone)	.75	6.97	650	275	3,450
50 (gravel)	.75	6.59	585	260	3,520
60 (granite)	.76	6.80	590	275	3,650
61 (sandstone)	.93	6.71	560	280	4,150
62 (cherty limestone)	.76	6.90	655	285	3,610
63 (argillaceous limestone)	.87	6.92	545	275	3,590
64 (gravel)	.75	6.57	535	225	3,420
65 (gravel)	.74	6.57	550	235	3,320
66 (gravel)	.75	6.68	650	290	3,850
67 (gravel limestone)	.78	6.69	650	310	4,010
68 (gravel)	.74	6.57	585	260	3,460
69 (slag)	.86	7.17	640	340	3,570
70 (slag)	.81	7.08	640	335	3,850
71 (slag)	.80	7.11	625	310	3,780
72 (shell limestone)	.83	6.94	605	315	3,810
1:1.6:4 MIX					
40 (trap)	0.81	6.16	555	210	2,980
44 (siliceous gravel)	.79	5.54	495	195	3,070
46 (siliceous limestone)	.81	6.03	605	230	3,000
50 (siliceous gravel)	.80	5.69	545	210	3,150
60 (granite)	.83	5.92	550	225	3,080
61 (sandstone)	1.04	5.81	560	235	3,650
62 (cherty limestone)	.83	5.97	615	235	3,000
63 (argillaceous limestone)	.98	5.98	500	240	3,130
64 (siliceous gravel)	.80	5.68	475	185	3,020
65 (siliceous gravel)	.78	5.69	515	195	3,010
66 (limestone gravel)	.81	5.75	610	255	3,460
67 (limestone gravel)	.85	5.74	605	250	3,430
68 (siliceous gravel)	.80	5.65	540	205	3,030
72 (shell limestone)	.92	6.06	570	250	3,250
1:2:4 MIX					
40 (trap)	0.91	5.74	480	205	2,590
44 (siliceous gravel)	.90	5.22	440	190	2,630
46 (siliceous limestone)	.92	5.64	550	225	2,680
50 (siliceous gravel)	.91	5.31	500	205	2,680
60 (granite)	.94	5.51	480	205	2,740
61 (sandstone)	1.15	5.40	520	225	3,140
62 (cherty limestone)	.95	5.56	555	220	2,620
63 (argillaceous limestone)	1.07	5.58	480	230	2,940
64 (siliceous gravel)	.91	5.30	435	190	2,650
65 (siliceous gravel)	.88	5.29	460	185	2,660
66 (limestone gravel)	.92	5.35	550	235	3,100
67 (limestone gravel)	.95	5.33	545	215	2,930
68 (siliceous gravel)	.91	5.27	485	190	2,600
69 (slag)	1.09	5.71	510	225	2,540
70 (slag)	1.02	5.70	520	250	2,790
71 (slag)	1.00	5.72	510	225	2,660
72 (shell limestone)	1.03	5.57	515	235	2,850
1:2:4½ MIX					
40 (trap)	0.94	5.36	460	195	2,430
44 (siliceous gravel)	.91	4.86	430	165	2,430
46 (siliceous limestone)	.94	5.27	555	205	2,650
50 (siliceous gravel)	.94	4.94	485	190	2,570
60 (granite)	.97	5.12	470	190	2,590
61 (sandstone)	1.21	5.01	480	195	2,930
62 (cherty limestone)	.96	5.21	530	205	2,520
63 (argillaceous limestone)	1.11	5.21	460	215	2,880
64 (siliceous gravel)	.93	4.94	420	160	2,580
65 (siliceous gravel)	.91	4.99	450	170	2,450
66 (limestone gravel)	.94	4.98	535	215	2,820
67 (limestone gravel)	.99	4.97	520	220	2,840
68 (siliceous gravel)	.92	4.94	480	175	2,590
72 (shell limestone)	1.06	5.24	505	215	2,660

[1] Flexure and tension specimens broken at 29 days, compression specimens broken at 33 days.

RESULTS INDICATE DESIRABILITY OF DETERMINING CONCRETE-MAKING PROPERTIES OF AGGREGATES

The relation between flexural strength and proportions as affected by either changes in the sand-cement ratio or the amount of coarse aggregate may also be studied by reference to Figure 5. For instance, using mix No. 3, 1:2:4, as the starting point, we find that increasing the amount of coarse aggregate to 4½ parts only slightly lowers the strength, the average difference in modulus of rupture being only 15 pounds per square inch. On the other hand, decreasing the sand to 1.6 parts (mix No. 2) and holding the coarse aggregate constant has a marked effect, the average increase for all aggregates being 50 pounds per square inch. Comparing the 1:1.6:3 and 1:1.6:4 mixes (Nos. 1 and 2) likewise shows that decreasing the coarse aggregate one part, with the sand held constant, still further increases the strength about 40 pounds per square inch. It is interesting to note, however, that the maximum variations in strength for a given mix due to type of aggregate is as great as the average difference in strength between the richest and leanest mixes used. The siliceous limestone aggregate No. 46, for instance, used in a 1:2:4½ mix, develops a somewhat higher strength than siliceous gravel aggregate No. 64 in the 1:1.6:3 mix. Reference to Table 6 shows that in the first case 5.27 bags of cement were used per cubic yard of concrete, whereas in the second case 6.57 bags were required. The economic possibilities resulting from a study of the concrete making properties of aggregates should be obvious to any one studying these data.

Referring now to the tensile strength curves in Figure 5, we note that the three slag aggregates are high in strength for the 1:1.6:3 mix while in the 1:2:4 mix, slag No. 70 shows the highest strength of all, with the other two slags, Nos. 69 and 71, in the high group. One noticeable difference between these curves and those for the flexure tests is that the greatest difference in tensile strength was found between the 1:1.6:3 and 1:1.6:4 mixes (Nos. 1 and 2) instead of between the 1:2:4 and 1:2:4½ mixes (Nos. 2 and 3). This is an indication that the tensile and flexural strength does not increase in exactly the same ratio as the mix is changed. This will be discussed more fully later.

In the compression curves in Figure 5, two of the slags, Nos. 70 and 71, are fairly high in strength in the 1:1.6:3 mix, while the other slag, No. 69, is slightly below the average. In the 1:2:4 mix, however, slag No. 70 is about the average, while Nos. 69 and 71 are below the average, showing a slight falling off in strength for the slags in the leaner mix. In all mixes, the sandstone No. 61 is higher than any other aggregate, while the limestone gravels, Nos. 66 and 67, also show consistently high values. All four curves are practically parallel. In general, there is about the same difference in strength between the 1:1.6:3 and 1:1.6:4 mixes as between the 1:1.6:4 and 1:2:4 mix, while the 1:2:4½ mix was close to the 1:2:4 mix, as was the case in tension and flexure.

Figure 6 shows the percentage variations from the average in tension, flexure and compression for each aggregate for each mix.

Figure 7 and Table 7 give the strength values for each individual grading, each value being the average for the four mixes. The modulus of rupture curves show no consistent difference in strength for variations

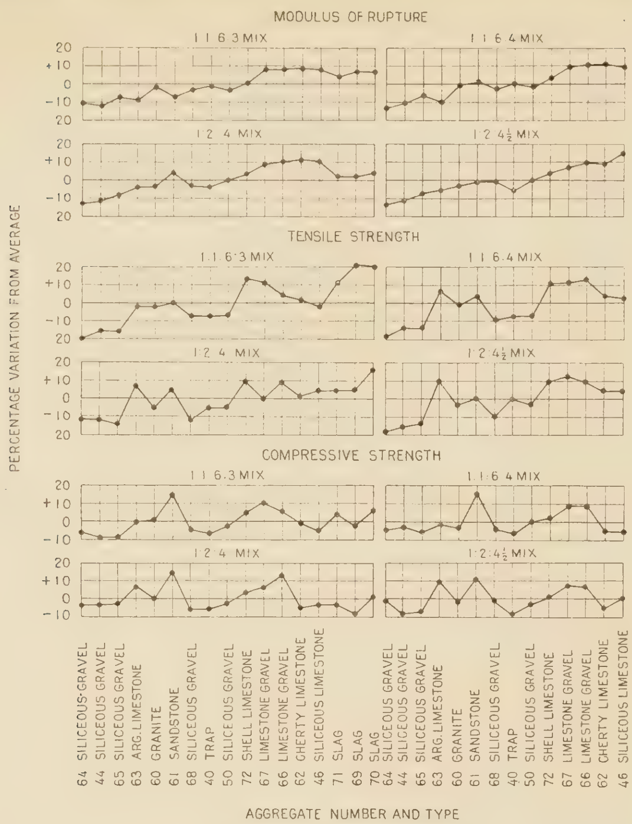

FIGURE 6.—VARIATIONS OF EACH AGGREGATE FROM THE GENERAL AVERAGE FOR EACH MIX

TABLE 7.—*Water-cement ratio, cement factor and results of strength tests (on four mixes)*[1]

GRADING NO. 1

Aggregate No.	$\frac{W}{C}$	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. inch	*Lbs. per sq. inch*	*Lbs. per sq. inch*
40 (trap)	0.82	6.13	515	210	2,830
44 (siliceous gravel)	.80	5.58	475	180	2,810
46 (siliceous limestone)	.82	6.04	595	210	2.900
50 (siliceous gravel)	.82	5.71	540	195	2,990
60 (granite)	.83	5.93	535	210	3,150
61 (sandstone)	1.03	5.82	535	240	3,530
62 (cherty limestone)	.83	6.00	595	230	3,000
63 (argillaceous limestone)	.96	6.02	520	240	3,280
64 (siliceous gravel)	.82	5.71	475	180	2,920
65 (siliceous gravel)	.81	5.68	490	175	2,770
66 (limestone gravel)	.83	5.75	590	240	3,370
67 (limestone gravel)	.85	5.76	585	225	3,210
68 (siliceous gravel)	.82	5.69	520	185	2,870
72 (shell limestone)	.90	6.05	565	240	3,200

GRADING NO. 2

40 (trap)	0.84	6.14	540	220	2,900
44 (siliceous gravel)	.82	5.54	480	200	2,940
46 (siliceous limestone)	.83	6.04	595	235	3,110
50 (siliceous gravel)	.84	5.68	525	215	3,050
60 (granite)	.85	5.86	540	225	3,040
61 (sandstone)	1.06	5.79	535	235	3,620
62 (cherty limestone)	.85	5.90	595	240	3,080
63 (argillaceous limestone)	.98	5.98	505	240	3,220
64 (siliceous gravel)	.84	5.64	460	195	2,950
65 (siliceous gravel)	.82	5.62	505	200	2,870
66 (limestone gravel)	.84	5.69	565	245	3,240
67 (limestone gravel)	.87	5.68	585	250	3,430
68 (siliceous gravel)	.83	5.61	520	205	2,940
72 (shell limestone)	.93	5.96	555	260	3,280

GRADING NO. 3

40 (trap)	0.87	6.04	520	215	2,870
44 (siliceous gravel)	.85	5.48	470	200	2,900
46 (siliceous limestone)	.86	5.98	575	240	2,930
50 (siliceous gravel)	.86	5.58	515	220	3,000
60 (granite)	.90	5.81	500	220	3,030
61 (sandstone)	1.10	5.69	540	230	3,460
62 (cherty limestone)	.89	5.87	575	235	2.950
63 (argillaceous limestone)	1.01	5.92	490	245	3,150
64 (siliceous gravel)	.85	5.62	470	185	2,960
65 (siliceous gravel)	.84	5.61	490	195	2,990
66 (limestone gravel)	.88	5.65	575	255	3,360
67 (limestone gravel)	.91	5.64	575	260	3,330
68 (siliceous gravel)	.85	5.56	530	215	2,970
72 (shell limestone)	.98	5.93	550	260	3,090

GRADING NO. 4

40 (trap)	0.90	6.03	515	225	2,780
44 (siliceous gravel)	.87	5.51	475	210	2,810
46 (siliceous limestone)	.90	5.89	600	250	2,850
50 (siliceous gravel)	.89	5.57	535	235	2,880
60 (granite)	.93	5.76	515	240	2,840
61 (sandstone)	1.14	5.63	505	235	3,260
62 (cherty limestone)	.94	5.86	585	240	2,720
63 (argillaceous limestone)	1.07	5.76	475	235	2,900
64 (siliceous gravel)	.88	5,53	455	195	2,830
65 (siliceous gravel)	.86	5.64	495	215	2,820
66 (limestone gravel)	.88	5.67	620	260	3,270
67 (limestone gravel)	.93	5.66	570	265	3,240
68 (siliceous gravel)	.87	5.56	525	225	2,910
72 (shell limestone)	1.02	5.88	530	255	3,010

[1] Flexure and tension specimens broken at 29 days, compression specimens broken at 33 days.

in grading, although it is noticed that for the softer aggregates, Nos. 61, 63, and 72 (sandstone, argillaceous limestone, and shell limestone), grading No. 4 gave the lowest values. In most cases, gradings No. 1 or No. 2 were high in strength, while No. 3 or No. 4 were low, the most noticeable exception being limestone gravel aggregate No. 66, which gave a high value for grading No. 4, and low value for grading No. 2. In most cases, as for aggregates Nos. 44, 65, 67, and 68, the difference for all four gradings was so slight as to indicate that within the ranges used in this investigation the grading of the coarse aggregate has little direct effect upon flexural strength.

In the tensile-strength tests, grading No. 1 resulted in low values in all but two cases, while grading No. 4 usually gave the highest values. One possible explanation for this is the relation between the size of the cross section of the specimen and the maximum size of aggregate used. In grading No. 1, 60 per cent of the material was retained on the 1¼-inch screen. The tension specimens had approximately 6-inch circular cross sections and it is believed that a larger cross section would probably have given higher strengths for the coarser gradings. One of the conclusions reached by Gonnerman and Shuman in their paper, Compression, Flexural and Tension Tests of Plain Concrete,[2] was that the size of specimen did not affect the tensile strength. However, it must be remembered that in their work the maximum size of aggregate was 1½ inches. The fact that grading No. 4, which had no particles greater than 1¼-inch, gave the highest values in tension but did not do so in flexure or compression is an indication that variations in grading possibly affect the tensile strength to a greater extent than they affect the flexural or compressive strength.

NO RELATION FOUND BETWEEN RESULTS OF ABRASION TEST AND STRENGTH OF CONCRETE

On examination of the compressive-strength curves in Figure 7 we note just the opposite effect; that is, grading No. 4 shows low values generally while grading No. 2 is high in strength. Taking into account the fact that the compression tests were made on the broken tension pieces, the results give indications that, from the standpoint of strength, changing the grading may produce opposite results in different types of tests.

[2] Proceedings of A. S. T. M., vol. 28, pt. 2. p. 551.

A more detailed study of the strength values may be made by referring to Table 8. In this table the water-cement ratios, cement factors, and the results of the three types of strength tests are shown for each grading and each mix separately.

The results of abrasion tests on each of the coarse aggregates are given in Table 2, together with a notation in each case, indicating the particular type of abrasion test made. In Figure 8 these values have been plotted against the results of the three strength tests, for the purpose of ascertaining if any relation exists. It may be concluded from this chart that, within the range of quality here considered, no relation exists between the quality of the coarse aggregate as measured by this test and the strength of the concrete.

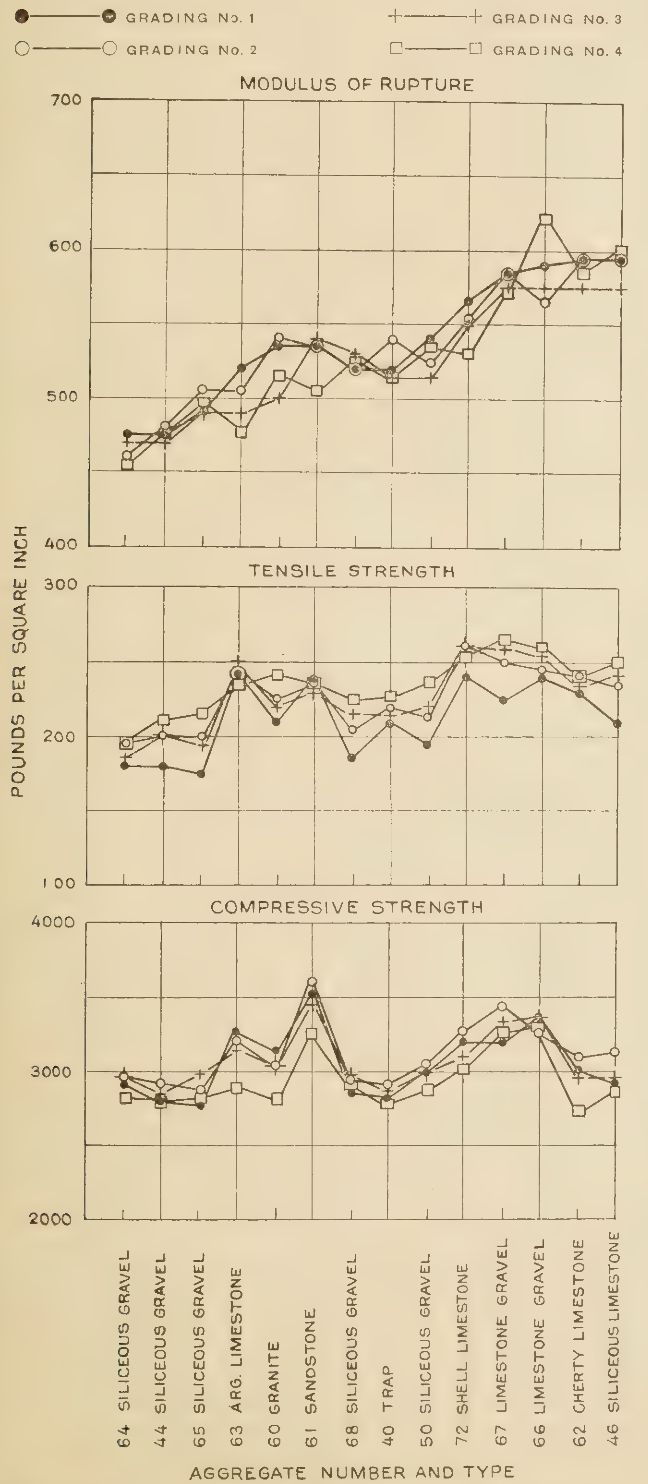

Figure 7.—Results of Strength Tests on Each Grading, Average for Four Mixes

RELATION BETWEEN TYPE OF AGGREGATE AND YIELD STUDIED

The amount of cement required to produce a cubic yard of concrete for each of the aggregates and proportions studied is plotted in Figure 9. In this graph the aggregates are plotted in the order of ascending flexural strength as in Figure 3. The effect of shape of coarse aggregate fragment on yield is very apparent.

It will be noted that the gravel aggregates Nos. 64, 44, 50, 65, 68, 67, and 66, all of which contain rounded fragments, show consistently higher yields or lower cement factors than any of the aggregates consisting of crushed fragments. This of course is merely the effect of variations in void content due to shape of particles as is very clearly brought out by reference to Figure 10, where the relation between cement factor and percentage of voids for each grading is shown. It will be noted that there is a maximum variation of 12 per cent in voids for 14 of the 17 types comprised in this study. This occurs for grading No. 3 and caused a maximum variation in the cement factor of 0.56 bags of cement. (See Table 7.) These values illustrate the effect of shape of aggregate fragment on yield when the proportioning is done by the usual volumetric method.

The effect of grading of coarse aggregate on yield for each type of aggregate may be studied by reference to Table 7. It will be observed that grading No. 1, in general, requires somewhat more cement than the others. However, the maximum variation in cement requirement due to gradation, within the limits used in this investigation is much less than the variation due to type of aggregate. It must be remembered in this connection that only reasonably uniform gradations were employed. For wide variations in grading the differences in the cement content would have been much greater.

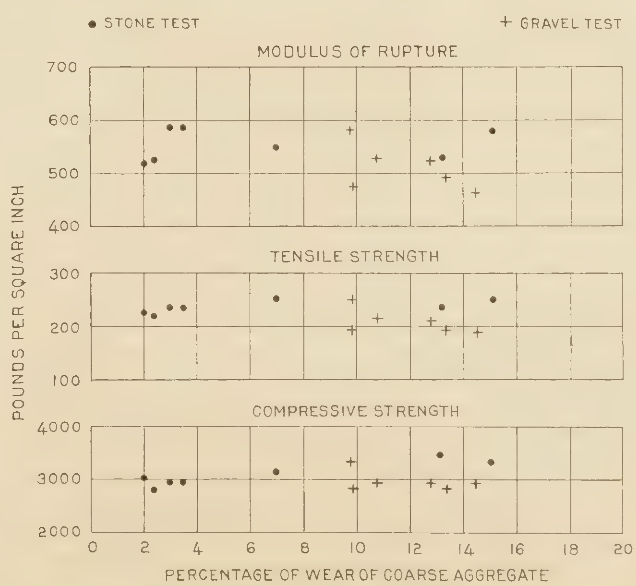

Figure 8.—Relation Between Strength of Concrete and Percentage of Wear of Coarse Aggregate

TABLE 8.—*Water-cement ratio, cement factor, and results of strength tests* [1]

1:1.6:3 MIX, GRADING NO. 1

Aggregate No.	W/C	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*
40 (trap)	0.72	7.15	570	255	3,280
44 (siliceous gravel)	.71	6.52	520	210	3,160
46 (siliceous limestone)	.72	7.00	675	245	3,180
50 (siliceous gravel)	.73	6.60	590	230	3,460
60 (granite)	.73	6.91	605	260	3,750
61 (sandstone)	.90	6.82	550	270	3,990
62 (cherty limestone)	.73	6.98	655	280	3,710
63 (argillaceous limestone)	.84	7.08	560	255	3,650
64 (siliceous gravel)	.73	6.65	560	205	3,390
65 (siliceous gravel)	.73	6.57	530	220	3,370
66 (limestone gravel)	.74	6.71	645	265	3,910
67 (limestone gravel)	.76	6.74	655	285	3,900
68 (siliceous gravel)	.73	6.68	570	230	3,340
69 (slag)	.82	7.31	655	310	3,440
70 (slag)	.77	7.23	640	305	3,930
71 (slag)	.77	7.21	645	280	3,640
72 (shell limestone)	.77	7.05	610	305	3,950

1:1.6:3 MIX, GRADING NO. 2

40 (trap)	0.74	7.15	615	270	3,460
44 (siliceous gravel)	.73	6.50	540	250	3,370
46 (siliceous limestone)	.73	7.03	645	275	3,630
50 (siliceous gravel)	.74	6.65	580	250	3,540
60 (granite)	.74	6.80	620	280	3,640
61 (sandstone)	.92	6.74	540	270	4,150
62 (cherty limestone)	.74	6.90	680	290	3,700
63 (argillaceous limestone)	.85	6.93	545	275	3,640
64 (siliceous gravel)	.74	6.59	520	230	3,390
65 (siliceous gravel)	.72	6.59	560	250	3,170
66 (limestone gravel)	.74	6.68	625	285	3,700
67 (limestone gravel)	.76	6.70	670	310	4,120
68 (siliceous grave)	.73	6.59	610	255	3,430
69 (slag)	.86	7.12	635	345	3,490
70 (slag)	.79	7.15	650	330	3,860
71 (slag)	.78	7.10	615	325	3,970
72 (shell limestone)	.82	6.93	630	320	3,840

1:1.6:3 MIX, GRADING NO. 3

40 (trap)	0.76	6.98	610	255	3,500
44 (siliceous gravel)	.75	6.45	530	235	3,350
46 (siliceous limestone)	.75	6.97	640	285	3,480
50 (siliceous gravel)	.75	6.56	590	285	3,570
60 (granite)	.77	6.76	570	275	3,710
61 (sandstone)	.93	6.69	580	275	4,270
62 (cherty limestone)	.77	6.85	625	275	3,560
63 (argillaceous limestone)	.87	6.93	545	285	3,610
64 (siliceous gravel)	.76	6.54	530	220	3,390
65 (siliceous gravel)	.74	6.53	550	210	3,410
66 (limestone gravel)	.76	6.62	635	300	3,860
67 (limestone gravel)	.80	6.65	655	315	4,110
68 (siliceous gravel)	.74	6.49	595	265	3,520
69 (slag)	.87	7.15	645	355	3,710
70 (slag)	.82	7.03	660	345	3,880
71 (slag)	.81	7.06	630	310	3,820
72 (shell limestone)	.83	6.93	615	310	3,780

1:1.6:3 MIX, GRADING NO. 4

40 (trap)	0.78	7.03	590	265	3,330
44 (siliceous gravel)	.76	6.50	540	250	3,410
46 (siliceous limestone)	.78	6.88	650	300	3,510
50 (siliceous gravel)	.77	6.53	570	275	3,520
60 (granite)	.81	6.74	555	280	3,480
61 (sandstone)	.95	6.57	570	290	4,190
62 (cherty limestone)	.80	6.87	655	300	3,480
63 (argillaceous limestone)	.91	6.74	525	285	3,470
64 (siliceous gravel)	.77	6.51	525	235	3,520
65 (siliceous gravel)	.76	6.59	555	255	3,340
66 (limestone gravel)	.77	6.70	700	315	3,920
67 (limestone gravel)	.81	6.68	615	330	3,910
68 (siliceous gravel)	.75	6.50	570	285	3,540
69 (slag)	.91	7.09	635	345	3,630
70 (slag)	.86	6.91	605	355	3,730
71 (slag)	.85	7.05	615	315	3,680
72 (shell limestone)	.88	6.83	575	330	3,650

[1] Flexure and tension specimens broken at 29 days, compression specimens broken at 33 days.

In the construction of concrete pavements the cost of the materials is influenced to a large extent by the amount of cement required to produce a cubic yard of concrete. It has just been shown that the cement factor varied considerably with the different aggregates

TABLE 8.—*Water-cement ratio, cement factor, and results of strength tests*—Continued

1:1.6:4 MIX, GRADING NO. 1

Aggregate No.	W/C	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*
40 (trap)	0.78	6.21	570	200	2,950
44 (siliceous gravel)	.75	5.58	510	185	3,000
46 (siliceous l mestone)	.78	6.13	620	230	3,100
50 (siliceous gravel)	.77	5.80	570	190	3,210
60 (granite)	.78	6.01	530	215	3,270
61 (sandstone)	.99	5.89	565	240	3,860
62 (cherty limestone)	.77	6.04	625	225	3,050
63 (argillaceous limestone)	.92	6.10	525	250	3,220
64 (siliceous gravel)	.77	5.77	485	180	2,930
65 (siliceous grave)	.75	5.73	515	165	2,860
66 (limestone gravel)	.77	5.81	635	255	3,420
67 (limestone gravel)	.81	5.83	610	225	3,180
68 (siliceous gravel)	.77	5.72	540	175	2,820
72 (shell limestone)	.86	6.14	590	235	3,200

1:1.6:4 MIX, GRADING NO. 2

40 (trap)	0.79	6.23	595	225	3,010
44 (siliceous gravel)	.76	5.57	490	205	3,170
46 (siliceous limestone)	.79	6.13	575	230	3,140
50 (siliceous gravel)	.79	5.77	540	215	3,290
60 (granite)	.82	5.98	590	230	3,000
61 (sandstone)	1.01	5.87	590	250	3,890
62 (cherty limestone)	.79	6.01	600	235	3,260
63 (argillaceous limestone)	.95	6.05	525	240	3,380
64 (siliceous gravel)	.78	5.70	470	195	3,140
65 (siliceous gravel)	.77	5.67	495	200	3,080
66 (limestone gravel)	.78	5.75	580	265	3,570
67 (limestone gravel)	.83	5.74	595	245	3,670
68 (siliceous gravel)	.78	5.66	520	200	3,180
72 (shell limestone)	.88	6.00	555	265	3,540

1:1.6:4 MIX, GRADING NO. 3

40 (trap)	0.82	6.14	525	195	3,070
44 (siliceous gravel)	.81	5.49	485	185	3,080
46 (siliceous limestone)	.82	6.08	580	220	2,940
50 (siliceous gravel)	.81	5.61	510	210	3,130
60 (granite)	.85	5.89	520	210	3,130
61 (sandstone)	1.06	5.77	555	240	3,450
62 (cherty limestone)	.84	5.95	625	250	3,180
63 (argillaceous limestone)	.99	5.95	485	240	3,140
64 (siliceous gravel)	.80	5.66	485	175	3,210
65 (siliceous gravel)	.79	5.65	515	215	3,170
66 (limestone gravel)	.84	5.73	590	250	3,520
67 (limestone gravel)	.87	5.68	600	275	3,470
68 (siliceous gravel)	.81	5.60	540	220	3,080
72 (shell limestone)	.95	6.08	585	250	3,160

1:1.6:4 MIX, GRADING NO. 4

40 (trap)	0.86	6.05	535	210	2,890
44 (siliceous gravel)	.82	5.52	495	210	3,010
46 (siliceous limestone)	.86	5.98	635	240	2,810
50 (siliceous gravel)	.84	5.60	565	225	2,950
60 (granite)	.88	5.80	555	250	2,900
61 (sandstone)	1.11	5.71	520	225	3,400
62 (cherty limestone)	.90	5.89	615	225	2,510
63 (argillaceous limestone)	1.05	5.80	475	235	2,860
64 (siliceous gravel)	.84	5.59	465	195	2,810
65 (siliceous gravel)	.81	5.72	540	205	2,940
66 (limestone gravel)	.85	5.71	640	260	3,340
67 (limestone gravel)	.89	5.71	615	260	3,420
68 (siliceous gravel)	.83	5.63	565	220	3,050
72 (shell limestone)	.97	6.01	555	255	3,120

1:2:4 MIX, GRADING NO. 1

40 (trap)	0.87	5.76	475	205	2,750
44 (siliceous gravel)	.86	5.30	455	160	2,560
46 (siliceous limestone)	.87	5.72	560	205	2,850
59 (siliceous gravel)	.87	5.42	510	190	2,750
60 (granite)	.88	5.62	495	185	2,940
61 (sandstone)	1.10	5.48	540	230	3,160
62 (cherty limestone)	.89	5.72	545	220	2,690
63 (argillaceous limestone)	1.02	5.64	510	235	3,100
64 (siliceous gravel)	.87	5.42	430	185	2,840
65 (siliceous gravel)	.86	5.35	430	165	2,570
66 (limestone gravel)	.89	5.44	570	225	3,260
67 (limestone gravel)	.91	5.40	530	190	2,990
68 (siliceous gravel)	.88	5.37	480	165	2,600
69 (slag)	1.04	5.68	490	210	2,440
70 (slag)	.98	5.77	500	230	2,670
71 (slag)	.97	5.75	500	220	2,660
72 (shell limestone)	.98	5.71	520	210	2,960

TABLE 8.—*Water-cement ratio, cement factor, and results of strength tests*—Continued

1:2:4 MIX, GRADING NO. 2

Aggregate No.	$\frac{W}{C}$	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*
40 (trap)	0.89	5.77	495	195	2,640
44 (siliceous gravel)	.88	5.23	430	185	2,770
46 (siliceous limestone)	.91	5.69	555	215	2,710
50 (siliceous gravel)	.91	5.31	485	215	2,780
60 (granite)	.91	5.53	470	200	2,890
61 (sandstone)	1.14	5.46	535	230	3,320
62 (cherty limestone)	.93	5.50	570	205	2,740
63 (argillaceous limestone)	1.04	5.62	485	235	2,990
64 (siliceous gravel)	.90	5.30	445	195	2,660
65 (siliceous gravel)	.88	5.30	490	195	2,690
66 (limestone gravel)	.92	5.35	535	220	3,050
67 (limestone gravel)	.93	5.34	575	205	2,980
68 (siliceous gravel)	.89	5.26	480	195	2,750
69 (slag)	1.07	5.69	515	230	2,630
70 (slag)	.99	5.72	555	255	3,160
71 (slag)	.99	5.73	520	205	2,700
72 (shell limestone)	1.00	5.65	520	240	2,940

1:2:4 MIX, GRADING NO. 3

40 (trap)	0.93	5.72	475	205	2,500
44 (siliceous gravel)	.90	5.16	435	195	2,610
46 (siliceous limestone)	.94	5.62	540	235	2,580
50 (siliceous gravel)	.91	5.26	480	195	2,660
60 (granite)	.97	5.46	480	210	2,660
61 (sandstone)	1.16	5.36	540	225	3,230
62 (cherty limestone)	.96	5.52	555	220	2,580
63 (argillaceous limestone)	1.07	5.57	465	225	3,020
64 (siliceous gravel)	.91	5.31	430	185	2,510
65 (siliceous gravel)	.89	5.26	450	180	2,730
66 (limestone gravel)	.92	5.32	550	235	3,090
67 (limestone gravel)	.96	5.28	530	230	2,950
68 (siliceous gravel)	.92	5.23	500	190	2,540
69 (slag)	1.10	5.76	535	230	2,520
70 (slag)	1.03	5.67	515	250	2,710
71 (slag)	.98	5.75	525	245	2,760
72 (shell limestone)	1.05	5.51	515	250	2,790

1:2:4 MIX, GRADING NO. 4

40 (trap)	0.96	5.70	480	220	2,460
44 (siliceous gravel)	.94	5.17	425	215	2,550
46 (siliceous limestone)	.96	5.53	540	240	2,600
50 (siliceous gravel)	.95	5.26	520	225	2,540
60 (granite)	1.00	5.44	475	215	2,490
61 (sandstone)	1.20	5.28	470	225	2,830
62 (cherty limestone)	1.03	5.49	550	235	2,460
63 (argillaceous limestone)	1.13	5.47	460	225	2,660
64 (siliceous gravel)	.95	5.18	430	190	2,580
65 (siliceous gravel)	.91	5.27	440	205	2,670
66 (limestone gravel)	.94	5.31	555	260	3,010
67 (limestone gravel)	1.00	5.29	535	235	2,810
68 (siliceous gravel)	.95	5.22	470	215	2,530
69 (slag)	1.15	5.71	495	240	2,530
70 (slag)	1.07	5.65	510	255	2,620
71 (slag)	1.05	5.67	490	220	2,520
72 (shell limestone)	1.09	5.47	505	240	2,720

1:2:4½ MIX, GRADING NO. 1

40 (trap)	0.89	5.39	455	175	2,350
44 (siliceous gravel)	.87	4.90	425	155	2,500
46 (siliceous limestone)	.90	5.32	525	165	2,460
50 (siliceous gravel)	.90	5.02	490	170	2,550
60 (granite)	.91	5.19	500	175	2,650
61 (sandstone)	1.14	5.08	495	215	3,110
62 (cherty limestone)	.91	5.27	560	195	2,520
63 (argillaceous limestone)	1.07	5.27	480	215	3,170
64 (siliceous gravel)	.90	5.00	420	145	2,520
65 (siliceous gravel)	.88	5.05	455	150	2,290
66 (limestone gravel)	.90	5.05	505	210	2,870
67 (limestone gravel)	.93	5.05	545	210	2,790
68 (siliceous gravel)	.89	5.00	485	170	2,700
72 (shell limestone)	.98	5.31	535	215	2,680

TABLE 8.—*Water-cement ratio, cement factor, and results of strength tests*—Continued

1:2:4½ MIX GRADING NO. 2

Aggregate No.	$\frac{W}{C}$	Bags of cement per cubic yard	Modulus of rupture	Tensile strength	Compressive strength
			Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*
40 (trap)	0.92	5.39	460	190	2,480
44 (siliceous gravel)	.90	4.87	450	165	2,440
46 (siliceous limestone)	.90	5.31	595	220	2,960
50 (siliceous gravel)	.91	4.98	490	190	2,600
60 (granite)	.93	5.14	480	200	2,600
61 (sandstone)	1.16	5.07	480	195	3,110
62 (cherty limestone	.92	5.19	530	220	2,620
63 (argillaceous limestone)	1.09	5.31	465	220	2,870
64 (siliceous gravel)	.92	4.95	415	165	2,610
65 (siliceous gravel)	.90	4.93	470	165	2,540
66 (limestone gravel)	.92	4.97	525	205	2,650
67 (limestone gravel)	.96	4.94	500	230	2,960
68 (siliceous gravel)	.91	4.93	470	165	2,390
72 (shell limestone)	1.02	5.26	505	225	2,780

1:2:4½ MIX, GRADING NO. 3.

40 (trap)	0.96	5.31	465	195	2,440
44 (siliceous gravel)	.92	4.81	430	175	2,540
46 (siliceous limestone)	.94	5.26	540	215	2,710
50 (siliceous gravel)	.95	4.89	475	200	2,640
60 (granite)	.99	5.11	440	185	2,620
61 (sandstone)	1.23	4.95	480	180	2,880
62 (cherty limestone)	.99	5.17	500	195	2,490
63 (argillaceous limestone)	1.12	5.22	465	225	2,830
64 (siliceous gravel)	.94	4.95	430	170	2,750
65 (siliceous gravel)	.92	4.98	435	175	2,630
66 (limestone gravel)	.98	4.93	530	230	2,950
67 (limestone gravel)	1.00	4.95	515	215	2,800
68 (siliceous gravel)	.93	4.93	480	185	2,740
72 (shell limestone)	1.10	5.18	495	225	2,630

1:2:4½ MIX, GRADING NO. 4

40 (trap)	0.98	5.33	455	210	2,450
44 (siliceous gravel)	.95	4.84	420	160	2,250
46 (siliceous limestone)	1.00	5.18	570	220	2,470
50 (siliceous gravel)	.98	4.87	480	210	2,510
60 (granite)	1.03	5.06	465	205	2,500
61 (sandstone)	1.31	4.95	455	190	2,610
62 (cherty limestone)	1.01	5.18	525	200	2,430
63 (argillaceous limestone)	1.17	5.03	435	190	2,660
64 (siliceous gravel)	.96	4.85	405	165	2,430
65 (siliceous gravel)	.95	4.98	440	190	2,340
66 (limestone gravel)	.96	4.96	575	210	2,820
67 (limestone gravel)	1.03	4.94	515	235	2,820
68 (siliceous gravel)	.94	4.90	495	185	2,510
72 (shell limestone)	1.12	5.19	480	205	2,540

used in this investigation. If our only problem were to construct a slab containing concrete proportioned arbitrarily by volume, we would naturally select aggregates producing the highest yield. If, on the other hand, we had only to design a slab with a specified strength without considering economy, we might select entirely different aggregates. The ideal aggregate is the one giving high strength and also high yield. By dividing the flexural strength of each aggregate by the corresponding cement factor, we obtain a series of values which may be used to compare these aggregates on a strength-yield basis. This has been done for the general average values and the results are shown in Figure 11. Examining the curve showing the relation between modulus of rupture and cement factor, we note that there are four aggregates which stand out above the others. They are Nos. 46, 62, 66, and 67 (two limestones and two limestone gravels), the same four that gave the highest flexural strength. There are four others, Nos. 50, 61, 68, and 72 which are grouped together while the remaining six, Nos. 40, 44, 60, 63, 64, and 65 show a rather low factor. With the exception of the trap and argillaceous limestone aggregates Nos. 40 and 63, the aggregates line up in somewhat the same order as they do in Figure 3.

These facts demonstrate that the increased strength developed by certain aggregates is not due entirely to increased cement content in terms of unit of volume of concrete. If the variations in strength were due to this cause alone, we should expect the strength-yield factors derived in the above manner to produce a horizontal line, instead of an ascending curve.

It is interesting to note that both the tension and compression graphs in Figure 11 present approximately the same order of ascending and descending values as in Figure 3. These values as well as the transverse test

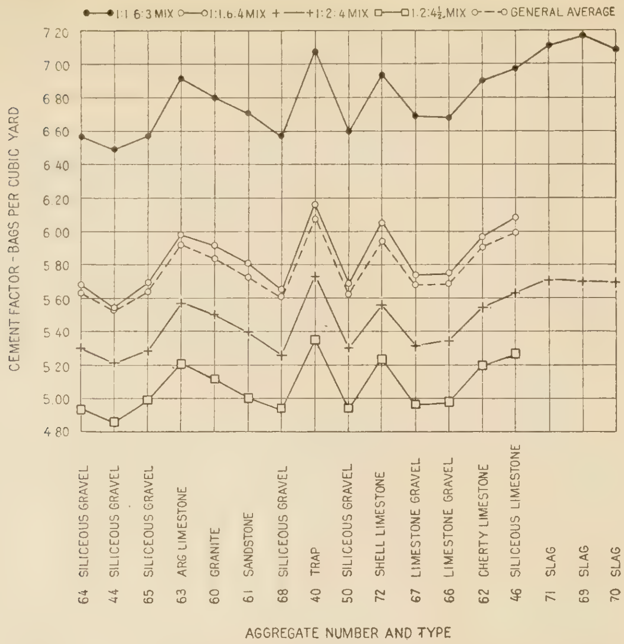

Figure 9.—Quantity of Cement Required for 1 Cubic Yard of Concrete. Aggregates Arranged in Ascending Order of Flexural Strength

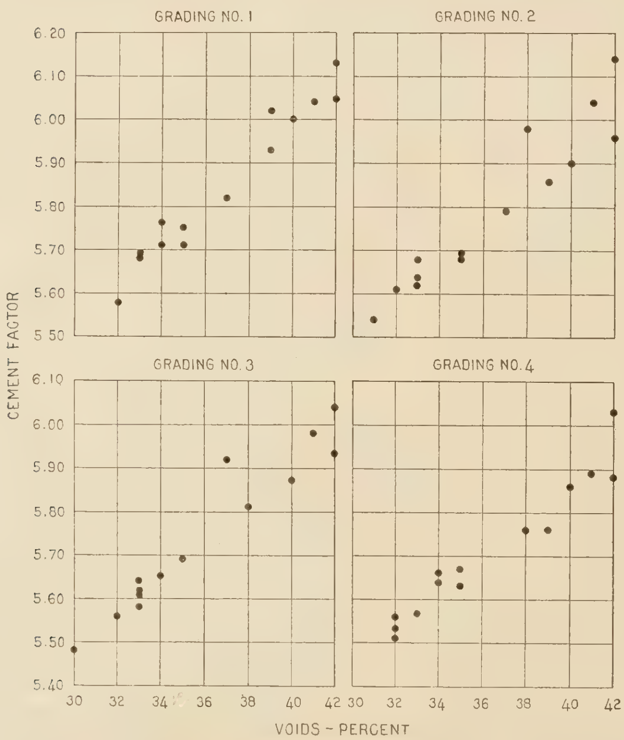

Figure 10.—Relation Between Cement Factor and Voids in Coarse Aggregate, Average of Four Mixes

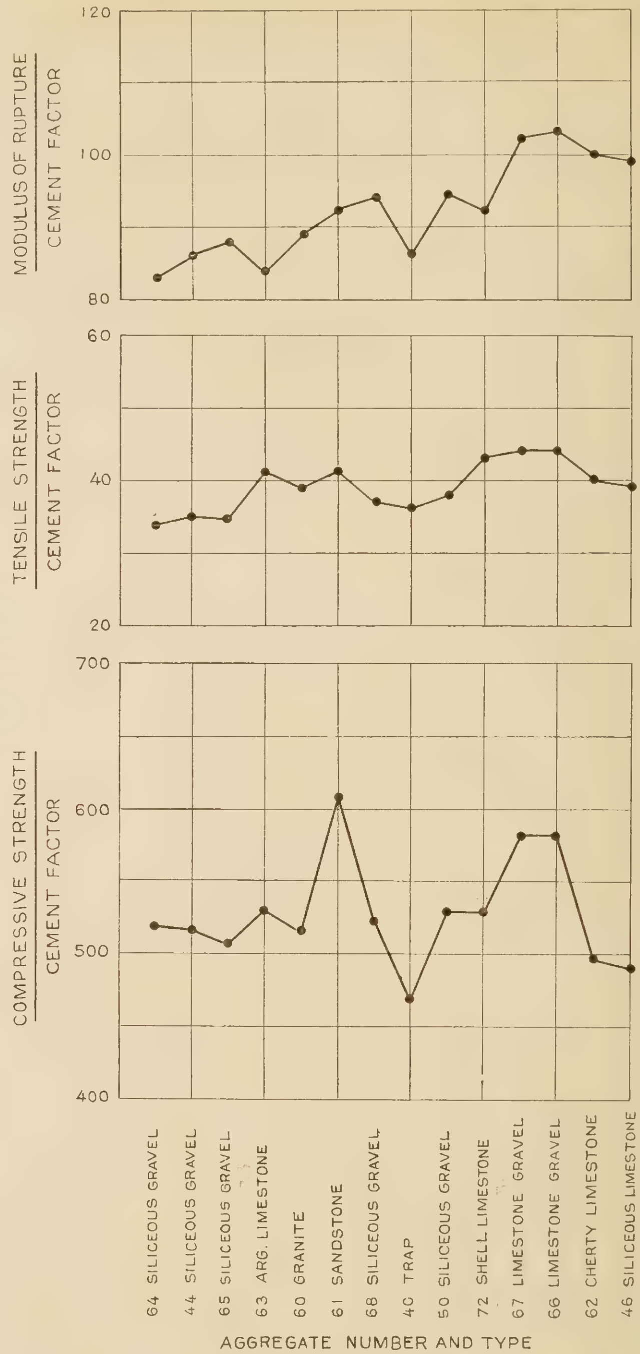

Figure 11.—Relation Between Cement Factor and Flexural, Tensile, and Compressive Strengths, Average of All Tests

results discussed above indicate that the strength of the concretes must have been affected by factors inherent in the aggregates themselves as well as by the actual amount of cement present.

The ratios found between the strength in flexure, tension, and compression are given in Tables 9 and 10. It should be remembered that the compression specimens were broken four days after the flexure and tension of specimens were broken which of course makes the ratio between tension and flexure the only true one. However, the other ratios are useful in making relative comparisons. Table 9 gives these ratios for each mix, while in Table 10 they are given for each grading. There is one point worthy of note in Table 9 and that is the ratios of tension to flexure for the different mixes. The 1:2:4½ mix showed about the same ratios as the 1:1.6:4 mix, both being low, while the 1:1.6:3 mix showed the highest values throughout. The tension-compression ratios are about the same for all four mixes, while the modulus-compression ratios are higher for the lean mixes. These tables, show in general that the same ratios do not exist between the three types of tests for all aggregates.

TABLE 9.—*Ratios between strength in flexure, tension, and compression for each mix (average of four gradings)*[1]

Aggregate No.	1:1.6:3			1:1.6:4			1:2:4			1:2:4½		
	Ten./Mod.	Ten./Comp.	Mod./Comp.	Ten./Mod.	Ten./Comp.	Mod./Comp.	Ten./Mod.	Ten./Comp.	Mod./Comp.	Ten./Mod.	Ten./Comp.	Mod./Comp.
40	0.44	0.077	0.176	0.38	0.070	0.186	0.43	0.079	0.185	0.42	0.080	0.189
44	.44	.071	.160	.39	.064	.161	.43	.072	.167	.38	.068	.177
46	.42	.080	.188	.38	.077	.202	.41	.084	.206	.37	.077	.210
50	.44	.074	.166	.39	.067	.173	.41	.077	.187	.39	.074	.189
60	.47	.075	.162	.41	.073	.179	.43	.075	.175	.40	.073	.181
61	.50	.068	.135	.43	.065	.152	.43	.074	.166	.41	.067	.164
62	.44	.079	.181	.38	.078	.205	.40	.084	.212	.39	.081	.210
63	.50	.077	.152	.48	.077	.160	.48	.078	.163	.47	.075	.160
64	.42	.066	.156	.39	.061	.157	.44	.072	.164	.38	.062	.163
65	.43	.071	.166	.38	.065	.171	.40	.070	.173	.38	.069	.184
66	.45	.075	.169	.42	.074	.176	.43	.076	.177	.40	.076	.190
67	.48	.077	.162	.41	.073	.176	.39	.073	.186	.43	.079	.183
68	.44	.075	.169	.38	.068	.178	.39	.073	.187	.36	.068	.185
69	.53	.095	.179	------	------	------	.44	.089	.200	------	------	------
70	.52	.087	.166	------	------	------	.48	.090	.188	------	------	------
71	.50	.082	.165	------	------	------	.44	.085	.192	------	------	------
72	.52	.083	.159	.44	.077	.175	.46	.082	.181	.43	.081	.190
Average	.47	.077	.165	.40	.071	.175	.43	.078	.183	.40	.074	.184

[1] Flexure and tension specimens broken at 29 days, compression specimens broken at 33 days.

TABLE 10.—*Ratios between strengths in flexure, tension, and compression for each grading (average of four mixes)*[1]

Aggregate No.	Grading No. 1			Grading No. 2			Grading No. 3			Grading No. 4		
	Ten./Mod.	Ten./Comp.	Mod./Comp.	Ten./Mod.	Ten./Comp.	Mod./Comp.	Ten./Mod.	Ten./Comp.	Mod./Comp.	Ten./Mod.	Ten./Comp.	Mod./Comp.
40	0.40	0.074	0.182	0.41	0.076	0.186	0.41	0.075	0.181	0.44	0.081	0.185
44	.38	.064	.169	.42	.068	.163	.43	.069	.162	.44	.075	.169
46	.35	.072	.205	.39	.076	.191	.42	.082	.196	.42	.088	.211
50	.36	.065	.181	.41	.071	.172	.43	.073	.172	.44	.082	.186
60	.39	.067	.170	.42	.074	.178	.44	.073	.165	.47	.085	.182
61	.45	.068	.152	.44	.065	.148	.43	.067	.156	.47	.072	.155
62	.39	.077	.198	.40	.078	.193	.41	.080	.195	.41	.088	.215
63	.46	.073	.159	.48	.075	.157	.50	.078	.156	.49	.081	.164
64	.38	.062	.163	.42	.066	.156	.39	.063	.159	.43	.069	.161
65	.36	.063	.177	.40	.070	.176	.40	.065	.164	.43	.076	.176
66	.41	.071	.175	.43	.076	.174	.44	.076	.171	.42	.080	.190
67	.38	.070	.182	.43	.073	.171	.45	.078	.173	.46	.082	.176
68	.36	.064	.181	.39	.070	.177	.41	.072	.178	.43	.077	.180
72	.42	.075	.177	.47	.079	.169	.47	.084	.178	.48	.085	.176
Average	.39	.069	.177	.42	.073	.172	.43	.074	.172	.45	.080	.180

[1] Flexure and tension specimens broken at 29 days, compression specimens broken at 33 days.

DATA INDICATE DESIRABILITY OF PROPORTIONING CONCRETE BY TRIAL METHOD

The preceding discussion has demonstrated quite conclusively that the water-cement ratio alone does not control the strength of the concrete. It has been shown that the various coarse aggregates which are in common use to-day in the construction of pavements may have certain qualities inherent in the aggregates themselves which may cause a wide variation in strength for a given water-cement ratio. Further analysis of the data, however, shows equally well that for any given aggregate, variations in the water-cement ratio will affect the strength substantially in accordance with the well established fundamental law. This fact is of considerable assistance in connection with the design of concrete paving mixtures to meet certain strength requirements, as will be discussed below.

In Figure 12 there has been plotted for each aggregate the relation between the water-cement ratio and the strengths in flexure, tension, and compression. Before plotting these curves an attempt was made to correct the apparent water-cement ratios, as shown in the tables, for absorption of coarse aggregate. It was found, however, that certain of the aggregates were so nonhomogeneous that it was impossible to obtain reasonable concordance on repeated tests for absorption. These discrepancies were so great in a number of cases as to lead to the conclusion that the present standard method of making absorption tests on small 1,000-gram samples is practically worthless for nonhomogeneous aggregates. Attempts to make mathematical corrections were therefore abandoned and values corresponding to the average water-cement ratios for aggregates having little or no absorption were adopted for each of the four mixes. It was felt that under the circumstances this was the most logical method of showing this relationship because it assumes that, if it had been possible to make an accurate correction for absorption, the net water-cement ratios for all the aggregate for a given mix would have been approximately the same. The only other variable which might affect this assumption is the shape of the aggregate fragments. It seems reasonable to suppose that, other things being equal, an aggregate having rounded surfaces would require less water than one having angular surfaces. However, inspection of Table 6 fails to reveal any significant differences which may be attributed to this factor. It is felt, therefore, that the water-cement ratios assumed for use in the charts are reasonably close to the true values, with the possible exception of aggregates Nos. 61 and 63, both of which were very highly absorptive.

Referring now to Figure 12, it will be observed that for all three types of test the strength decreases proportionately with increases in the water-cement ratio, resulting in a series of substantially parallel curves each of which represents the strength-water ratio relation for a given aggregate. For purposes of studying the

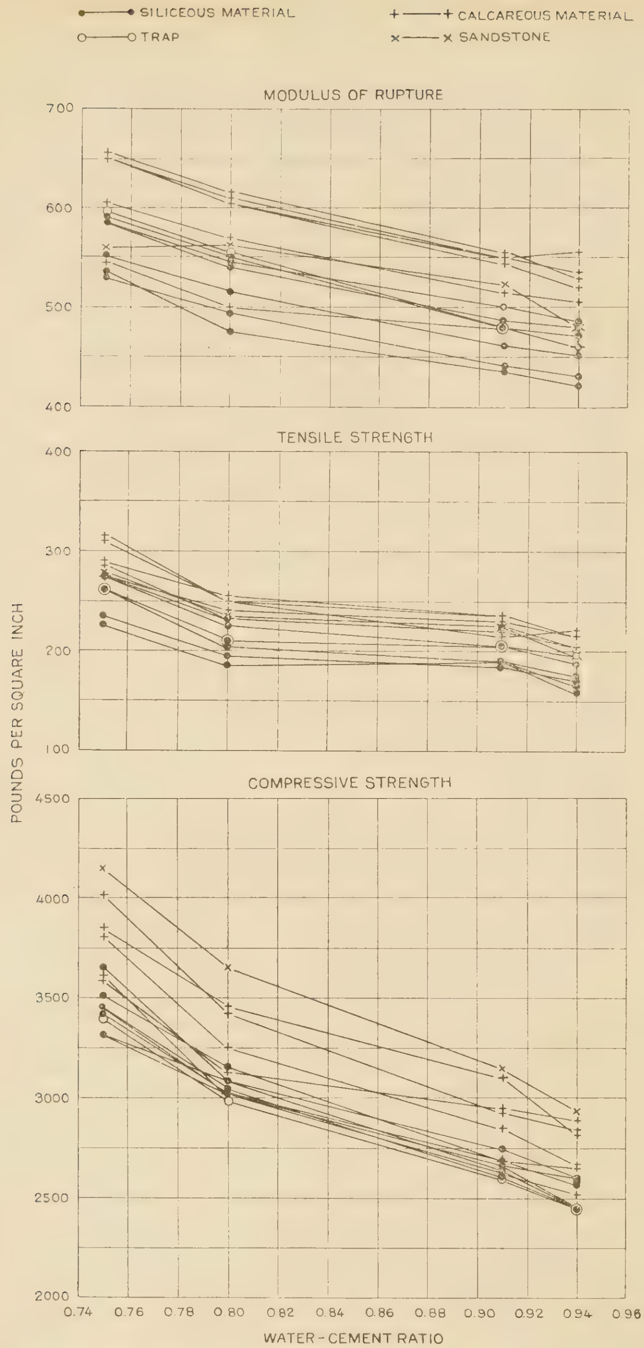

Figure 12.—Relation Between Water Cement Ratio and Flexural, Tensile, and Compressive Strength

effect of mineral composition the aggregates have been grouped into four divisions, the six calcareous materials being indicated by crosses, the single sandstone by an X, the trap by an open circle, and the various materials which are essentially siliceous, such as the quartz gravels and the granite, by solid circles. The grouping of materials by types as previously discussed is at once apparent. Of interest, also, is the fact that, in so far as tensile strength is concerned, the curves are practically horizontal for water-cement ratios between 0.8 and 0.94. Aside from this the relation between strength and water-cement ratio follows the well known law fairly closely for each type of test.

The fact that these water-cement ratio-strength curves are practically parallel gives us a method of designing concrete paving mixtures by trial as described in the paper, The Design of Concrete Paving Mixtures by the Water-Cement Ratio Method, which appeared in the August, 1928, issue of Public Roads. This method consists essentially in determining for any given water-cement ratio, say 0.8, and for each combination of aggregates under consideration, the transverse strengths at 28 days under standard laboratory conditions. The resulting values are then plotted and curves drawn through each parallel to the basic water-cement ratio-strength curve.

By noting the water-cement ratio at which each of these curves cuts the design strength line, the proportions which must be used in the case of each aggregate to secure the design strength with the desired workability may be determined by trial, from which in turn may be determined the quantities of materials in each case for a unit volume of concrete. A study of the relative costs of materials will then fix, in general, the most economical combination to use.

CONCLUSIONS

Based on the variables included in this investigation and the resulting test data, the following conclusions are drawn:

1. That the tensile, flexural and compressive strength of concrete are affected appreciably by the character of the coarse aggregate used.
2. That the tensile and flexural strength are affected to a greater extent than the compressive strength.
3. That for a given aggregate there is a fairly well defined relation between the strength of the concrete and the water-cement ratio.
4. That variations in the character of the coarse aggregates, other things being equal, may result in a difference in flexural strength equal to that produced by an appreciable change in the water-cement ratio with any given aggregate. (In this study, for instance, aggregate No. 46, with a water-cement ratio of 0.94, produced concrete of somewhat higher flexural strength than aggregate No. 44, with a water-cement ratio of 0.74—a difference of 0.2.)
5. That there is a fairly definite relation between certain mineralogical characteristics of the coarse aggregate and the strength of concrete, calcareous aggregates in general giving consistently higher flexural and tensile strength than siliceous aggregates.
6. That, in general, aggregates having rounded fragments produce concrete of lower flexural and tensile strength than aggregates which are composed wholly or in part of crushed fragments.
7. That, within the limits of this study, variations in grading of coarse aggregates have no consistent effect upon the strength of concrete. (It is not to be inferred from this statement, however, that control of grading is not important. Variations in grading occurring during construction not only affect yield when measurements are made by volume but also affect the workability and and therefore the uniformity of the concrete).
8. That, within the range in quality covered by this study, there is no relation between the quality of the coarse aggregate, as measured by the abrasion test, and the strength of the concrete.

ROAD PUBLICATIONS OF BUREAU OF PUBLIC ROADS

Applicants are urgently requested to ask only for those publications in which they are particularly interested. The Department can not undertake to supply complete sets nor to send free more than one copy of any publication to any one person. The editions of some of the publications are necessarily limited, and when the Department's free supply is exhausted and no funds are available for procuring additional copies, applicants are referred to the Superintendent of Documents, Government Printing Office, this city, who has them for sale at a nominal price, under the law of January 12, 1895. Those publications in this list, the Department supply of which is exhausted, can only be secured by purchase from the Superintendent of Documents, who is not authorized to furnish publications free.

ANNUAL REPORTS

Report of the Chief of the Bureau of Public Roads, 1924.
Report of the Chief of the Bureau of Public Roads, 1925.
Report of the Chief of the Bureau of Public Roads, 1927.
Report of the Chief of the Bureau of Public Roads, 1928.

DEPARTMENT BULLETINS

No. *136D. Highway Bonds. 20c.
220D. Road Models.
257D. Progress Report of Experiments in Dust Prevention and Road Preservation, 1914.
*314D. Methods for the Examination of Bituminous Road Materials. 10c.
*347D. Methods for the Determination of the Physical Properties of Road-Building Rock. 10c.
*370D. The Results of Physical Tests of Road-Building Rock. 15c.
386D. Public Road Mileage and Revenues in the Middle Atlantic States, 1914.
387D. Public Road Mileage and Revenues in the Southern States, 1914.
388D. Public Road Mileage and Revenues in the New England States, 1914.
390D. Public Road Mileage and Revenues in the United States, 1914. A Summary.
407D. Progress Reports of Experiments in Dust Prevention and Road Preservation, 1915.
463D. Earth, Sand-Clay, and Gravel Roads.
*532D. The Expansion and Contraction of Concrete and Concrete Roads. 10c.
*537D. The Results of Physical Tests of Road-Building Rock in 1916, Including all Compression Tests. 5c.
*583D. Reports on Experimental Convict Road Camp, Fulton County, Ga. 25c.
*660D. Highway Cost Keeping. 10c.
*670D. The Results of Physical Tests of Road-Building Rock in 1916 and 1917.
*691D. Typical Specifications for Bituminous Road Materials. 10c.
*724D. Drainage Methods and Foundations for County Roads. 20c.
1216D. Tentative Standard Methods of Sampling and Testing Highway Materials, adopted by the American Association of State Highway Officials and approved by the Secretary of Agriculture for use in connection with Federal-aid road construction.
1259D. Standard Specifications for Steel Highway Bridges, adopted by the American Association of State Highway Officials and approved by the Secretary of Agriculture for use in connection with Federal-aid road work.

DEPARTMENT BULLETINS—Continued

No. 1279D. Rural Highay Mileage, Income, and Expenditures, 1921 and 1922.
1486D. Highway Bridge Location.

DEPARTMENT CIRCULARS

No. 94C. T. N. T. as a Blasting Explosive.
331C. Standard Specifications for Corrugated Metal Pipe Culverts.

TECHNICAL BULLETIN

No.55. Highway Bridge Surveys.

MISCELLANEOUS CIRCULARS

No. 62M. Standards Governing Plans, Specifications, Contract Forms, and Estimates for Federal-Aid Highway Projects.
93M. Direct Production Costs of Broken Stone.
*109M. Federal Legislation and Regulations Relating to the Improvement of Federal-Aid Roads and National-Forest Roads and Trails. 10c.

FARMERS' BULLETIN

No. *338F. Macadam Roads. 5c.

SEPARATE REPRINTS FROM THE YEARBOOK

No. 1914Y. Highways and Highway Transportation.
937Y. Miscellaneous Agricultural Statistics.

TRANSPORTATION SURVEY REPORTS

Report of a Survey of Transportation on the State Highway System of Connecticut.
Report of a Survey of Transportation on the State Highway System of Ohio.
Report of a Survey of Transportation on the State Highways of Vermont.
Report of a Survey of Transportation on the State Highways of New Hampshire.
Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio.
Report of a Survey of Transportation on the State Highways of Pennsylvania.

REPRINTS FROM THE JOURNAL OF AGRICULTURAL RESEARCH

Vol. 5, No. 17, D- 2. Effect of Controllable Variables upon the Penetration Test for Asphalts and Asphalt Cements.
Vol. 5, No. 19, D- 3. Relation Between Properties of Hardness and Toughness of Road-Building Rock.
Vol. 5, No. 24, D- 6. A New Penetration Needle for Use in Testing Bituminous Materials.
Vol 6, No. 6, D- 8. Tests of Three Large-Sized Reinforced-Concrete Slabs Under Concentrated Loading.
Vo. 11, No. 10, D–15. Tests of a Large-Sized Reinforced-Concrete Slab Subjected to Eccentric Concentrated Loads.

* Department supply exhausted.

UNITED STATES DEPARTMENT OF AGRICULTURE

BUREAU OF PUBLIC ROADS

CURRENT STATUS OF FEDERAL AID ROAD CONSTRUCTION

AS OF

MAY 31,1929

STATE	COMPLETED MILEAGE	UNDER CONSTRUCTION					APPROVED FOR CONSTRUCTION					BALANCE OF FEDERAL-AID FUNDS AVAILABLE FOR NEW PROJECTS	STATE
		Estimated total cost	Federal aid allotted	MILEAGE			Estimated total cost	Federal aid allotted	MILEAGE				
				Initial	Stage[1]	Total			Initial	Stage[1]	Total		
Alabama	1,943.4	$ 3,381,159.28	$ 1,688,553.19	242.0	21.0	263.0	$ 162,471.28	$ 81,235.64	6.4		6.4	$ 2,502,919.33	Alabama
Arizona	881.8	1,932,042.62	1,625,826.11	72.1	29.7	101.8						3,162,886.69	Arizona
Arkansas	1,759.9	2,886,446.55	1,428,642.80	88.9	6.6	95.5	782,421.38	391,209.68	27.4		27.4	2,290,448.98	Arkansas
California	1,586.9	9,597,180.66	4,386,404.64	260.7	15.0	275.7	206,390.87	122,201.27	10.2	3.7	13.9	2,340,921.39	California
Colorado	1,110.7	3,544,662.57	1,850,947.37	122.8	41.9	164.7	838,397.65	423,579.44	20.4	12.4	32.8	2,005,858.55	Colorado
Connecticut	229.3	792,275.72	217,937.99	12.5		12.5	673,564.46	272,075.09	3.6		3.6	845,766.66	Connecticut
Delaware	212.9	753,366.80	298,843.42	15.7		15.7	535,871.82	264,272.84	26.0		26.0	45,324.01	Delaware
Florida	439.5	2,970,573.29	1,253,256.11	100.7	5.4	106.1						1,965,926.62	Florida
Georgia	2,541.2	4,113,760.11	1,850,192.19	187.4	36.9	224.3	32,436.30	15,846.43	2.6		2.6	2,020,838.82	Georgia
Idaho	1,141.5	935,640.07	558,854.56	75.6	3.0	78.6	111,000.00	66,000.00	7.3		7.3	948,611.53	Idaho
Illinois	1,820.0	20,120,995.98	8,980,855.30	610.7		610.7	1,588,275.15	779,809.20	53.1		53.1	2,642,000.00	Illinois
Indiana	1,266.7	6,764,961.89	3,255,692.67	210.6		210.6	2,860,252.88	1,378,345.13	97.1		97.1	283,981.79	Indiana
Iowa	2,993.8	3,323,930.37	1,489,047.25	61.5	86.2	147.7	2,866,720.87	1,264,646.85	38.8	64.6	103.4	18,502.65	Iowa
Kansas	2,515.1	3,433,117.54	1,324,656.72	246.2		246.2	484,715.58	232,446.09	40.3	1.5	41.8	1,992,444.74	Kansas
Kentucky	1,313.6	4,533,260.34	2,162,001.39	229.3		229.3	733,971.16	366,937.81	64.5	3.4	67.9	735,867.94	Kentucky
Louisiana	1,321.3	3,754,252.83	1,869,367.79	152.0		152.0	287,899.05	105,752.71	.2	8.2	8.4	1,142,950.71	Louisiana
Maine	480.5	1,779,653.03	596,458.57	40.9		40.9	192,121.39	93,601.55	8.9		8.9	1,449,150.77	Maine
Maryland	627.9	167,810.00	62,360.00	3.7		3.7	713,847.50	343,090.00	26.0	16.8	42.8	327,345.37	Maryland
Massachusetts	570.7	4,709,006.57	1,438,854.88	83.8		83.8	749,742.50	202,905.00	10.7	2.8	13.5	1,769,009.90	Massachusetts
Michigan	1,443.2	11,050,955.95	4,706,049.11	265.1		265.1	1,857,320.00	806,206.86	50.0	9.5	59.5	1,662,249.29	Michigan
Minnesota	3,954.8	1,549,078.11	497,618.27	96.7	11.2	107.9	2,946,156.25	955,297.96	129.9	51.4	181.3	1,045,000.00	Minnesota
Mississippi	1,669.4	4,547,907.06	2,051,907.05	192.4	4.4	196.8	286,065.01	129,736.53	16.8	13.0	29.8	1,401,373.68	Mississippi
Missouri	2,274.1	9,189,332.13	3,500,796.61	211.6	55.4	267.0	3,986,695.60	1,525,336.52	21.4	104.7	126.1	132,614.46	Missouri
Montana	1,539.3	4,484,465.30	2,851,910.46	301.6	7.6	309.2	649,185.09	341,435.53	73.8		73.8	4,590,594.75	Montana
Nebraska	3,591.7	3,163,629.25	1,575,522.41	275.0	60.2	335.2	822,448.55	381,115.67	30.1	100.7	130.8	2,938,142.64	Nebraska
Nevada	1,062.8	1,446,843.05	1,268,326.27	133.3	112.4	245.7	35,408.62	31,460.54		18.9	18.9	440,174.85	Nevada
New Hampshire	331.7	279,253.01	108,613.31	7.5		7.5	363,075.44	121,035.00	7.0	1.0	8.0	284,440.54	New Hampshire
New Jersey	462.6	4,167,603.82	706,410.00	47.1		47.1	398,619.23	108,795.00	7.3		7.3	755,805.08	New Jersey
New Mexico	1,828.0	2,623,275.50	1,656,965.11	177.6		177.6	370,507.18	236,198.30	13.0		13.0	1,067,511.12	New Mexico
New York	2,148.8	23,460,531.43	5,083,030.55	339.4		339.4	8,582,886.33	1,888,155.00	126.1		126.1	5,032,084.39	New York
North Carolina	1,689.7	1,644,406.06	822,203.00	85.8	7.2	93.0	40,800.00	13,920.00	.9		.9	1,950,526.33	North Carolina
North Dakota	3,646.6	3,229,952.47	1,346,123.36	493.9	118.8	612.7	1,156,081.83	434,580.81	147.2	221.3	368.5	1,081,927.67	North Dakota
Ohio	1,991.9	11,735,361.39	4,109,579.65	250.2	.1	250.3	3,260,531.59	737,836.79	47.4	9.8	57.2	3,417,817.31	Ohio
Oklahoma	1,783.9	1,890,710.99	900,690.65	71.1	25.7	96.8	1,700,273.77	751,047.60	59.9	17.8	77.7	1,009,194.32	Oklahoma
Oregon	1,147.9	636,377.39	380,464.45	48.2		48.2	61,358.00	38,183.08	6.0		6.0	2,111,903.79	Oregon
Pennsylvania	2,062.5	12,073,958.30	3,245,057.84	194.9		194.9	1,966,809.70	568,255.67	33.5	14.1	47.6	3,079,305.13	Pennsylvania
Rhode Island	165.2	1,469,617.87	357,000.00	23.8		23.8	162,593.51	68,966.55	1.6		1.6	548,347.68	Rhode Island
South Carolina	1,813.6	4,387,081.03	1,045,814.40	126.2	37.4	163.6	265,582.70	54,000.00	18.8		18.8	958,856.17	South Carolina
South Dakota	3,294.0	2,885,781.27	1,583,300.59	446.9	24.4	471.3	978,582.90	518,120.02	88.0	39.9	127.9	588,828.37	South Dakota
Tennessee	1,125.5	3,780,828.75	1,702,282.29	93.7	25.9	119.6	608,666.35	304,333.17	23.0		23.0	1,226,276.81	Tennessee
Texas	6,104.7	15,996,953.83	6,813,454.99	583.6	223.9	807.5	4,491,496.94	2,019,318.42	123.5	118.5	242.0	1,755,946.85	Texas
Utah	915.9	1,586,997.66	1,054,449.36	67.3		67.3	177,864.34	131,789.60	18.5		18.5	526,585.13	Utah
Vermont	229.0	1,825,458.76	600,594.22	38.0		38.0	63,918.62	31,959.31	.1		.1	72,762.85	Vermont
Virginia	1,333.8	2,663,981.30	1,169,009.04	77.2	15.2	92.4	488,026.31	202,414.07	34.8	6.8	41.6	921,857.94	Virginia
Washington	847.9	3,761,513.31	1,278,675.25	79.6	18.1	97.7						1,406,300.03	Washington
West Virginia	688.6	1,915,850.26	828,504.13	51.1	12.4	63.5	1,888,388.17	715,388.03	36.9	12.3	49.2	198,544.50	West Virginia
Wisconsin	2,099.4	5,670,878.60	2,416,213.96	179.3	4.9	184.2	4,603,300.49	2,117,671.13	150.4	24.4	174.8	538,511.47	Wisconsin
Wyoming	1,683.8	1,164,226.15	724,005.49	117.7	9.4	127.1	293,646.16	188,862.76	21.5	10.2	31.7	577,887.14	Wyoming
Hawaii	39.5	402,261.10	137,426.82	6.6		6.6						1,319,923.77	Hawaii
TOTALS	77,727.5	224,179,167.32	90,860,751.59	7,899.5	1,020.3	8,919.8	56,336,388.50	21,825,374.65	1,730.9	887.7	2,618.6	71,132,050.61	TOTALS

[1] The term stage construction refers to additional work done on projects previously improved with Federal aid. In general, such additional work consists of the construction of a surface of higher type than was provided in the initial improvement.